Philosophia Britannica:

OR,

A NEW and COMPREHENSIVE

SYSTEM

OF THE

NEWTONIAN

PHILOSOPHY, ASTRONOMY,

AND

GEOGRAPHY,

In a Course of Twelve Lectures,

With NOTES;

Containing the PHYSICAL, MECHANICAL, GEOMETRICAL and EXPERIMENTAL PROOFS and ILLUSTRATIONS of all the Principal Propositions in every Branch of

NATURAL SCIENCE:

ALSO,

A Particular Account of the INVENTION, STRUCTURE, IMPROVEMENT and USES of all the considerable INSTRUMENTS, ENGINES, and MACHINES;

With NEW CALCULATIONS relating to their

NATURE, POWER, AND OPERATION.

The Whole collected and methodized from all the Principal Authors, and Public Memoirs;

And embellished with EIGHTY-ONE COPPER-PLATES.

By B. MARTIN.

THE FOURTH EDITION.

VOLUME III.

LONDON:

Printed for JOHN, FRANCIS, CHARLES RIVINGTON; and THOMAS CARNAN, in St. Paul's Church-Yard; and ANDREW STRAHAN, in New-Street.

M,DCC,LXXXVI I.

A

TABLE OF CONTENTS.

LECTURE IX, X.

OPTICS.

Of MIRROURS *and* LENSES.

CONTENTS.

Of the EYE *and* VISION.

OPTIC INSTRUMENTS.

CONTENTS.

LECTURE XII.

The USE *of the* GLOBES.

CONTENTS.

CONTENTS.

APPENDIX.

Of the LUNAR MOTIONS, &c.

APPENDIX II.

IMPROVEMENTS *in* OPTICAL INSTRUMENTS.

LECTURE IX, X.

Of OPTICS.

Of the Science of OPTICS *in general. Of* CATOPTRICS *and* DIOPTRICS. *Of diverging, converging, and parallel* RAYS. *Of the ſeveral Kinds of* MIRROURS *and* LENSES. *Of the* FOCUSSES *of Glaſſes; the* CALCULATIONS *thereof, and* THEOREMS *for every Caſe. Of* OBJECTS *and their* IMAGES, *with* THEOREMS *relating thereto for every Kind of Glaſs. The* THEORY *of* VISION *explained. The ſeveral Parts of the* EYE *deſcribed. Of the* DEFECTS *of* VISION, *and how remedied by* SPECTACLES *of ſeveral Sorts. Of* READING-GLASSES. *Of* SINGLE-MICROSCOPES *of every Sort by* Reflection *and* Refraction. *Of* DOUBLE-MICROSCOPES *by Reflection and Refraction. Their* Structure *and* Uſe *explained. A* New POCKET-MICROSCOPE *deſcribed, furniſhed with a* MICROMETER. *The* Nature, Structure, *and* magnifying Power *of a refracting* TELESCOPE *of every Sort;*

the Reason of their Imperfection *explained. Of* REFLECTING TELESCOPES, *with their Theory at large explained. Of the* CAMERA OBSCURA, *and its various Uses. Of the* SCIOPTIC BALL *and* SOCKET. *Of the* SOLAR TELESCOPE; *and* SOLAR MICROSCOPES *of several Sorts. Of the new-invented* HELIOSTATA *of* s'Gravesande, *with its* Theory, *and Manner of Use explained.*

WE are now arrived to that Part of Natural Philosophy which treats of *Vision*, and the various Phænomena of visible Objects, by Rays of Light reflected from Mirrours, and transmitted through Lenses, which constitute the Subject of the most delightful Science of OPTICS (CXXV.)

THE

(CXXV.) 1. OPTICS is divided into Two Parts, CATOPTRICS and DIOPTRICS. The former treats of Vision by Light reflected from Mirrours or polished Surfaces; and the latter of Vision effected by Light transmitted through Lenses. Of these Lenses the several Sorts in Use are the Plano-Convex A, the Double-Convex B, the Plano-Concave C, the Double-Concave D, the *Meniscus* E, (which is convex on one Side, and concave on the other) and the Hemisphere F. The Line G H, that is perpendicular to and passes through the Middle of each Lens, is called the *Axis* of the Lens, and that Middle Point the *Vertex* of the Lens.

Plate XLII. Fig. 6.

2. As

THE principal Things here to be considered are, First, *the Rays of Light*; Secondly,

2. As Rays of Light fall on these Glasses, they are variously reflected and refracted, as above described in the Lecture. The Theorems which shew the different Effects of all these Glasses in reflecting and refracting the Rays of Light, and forming the Images of Objects, are investigated several Ways; one of which is by *Algebra*. By this means Dr. *Halley* has raised a general Theorem extending to all the particular Cases of every Kind of Optic-Glasses of a spherical Form, and which I have largely applied and exemplified in my *Treatise of Optics*.

3. Another Method of doing this is by *Fluxions*, which is easy and universal, comprehending all the Cases of Mirrours and Lenses of every Form. This I propose to exhibit and illustrate here for Variety, and for the Genuineness and Excellency of this Method above all others, it depending on Principles that are more of a Philosophical than of a Mathematical Nature. It is as follows.

4. Let V B G be the Section of any curved Superficies of a Medium V G H I, V the Vertex, and A I the Axis of the Curve V G. From any Point in the Axis A let a Ray of Light A B be incident on the Medium in B, which suppose refracted to a Point F in the Axis. Then, by having given the Distance of the radiating Point A D, and the Sine of Incidence B D, we are to find the focal Distance V F after Refraction. Plate XLII. Fig. 7.

5. To do this, from the Point B let fall the Perpendicular B D to the Axis; and putting $AV = d$, $AB = z$, $BF = v$, $VD = x$, $BD = y$, and $VF = f$, then will $DF = f - x$, $AD = d + x$, $z = \sqrt{y^2 + d^2 + 2dx + x^2}$, and $v = \sqrt{y^2 + f^2 - 2fx + x^2}$; and therefore in Fluxions we have $\dot{z} = \frac{y\dot{y} + d\dot{x} + x\dot{x}}{\sqrt{y^2 + d^2 + 2dx + x^2}}$, and $\dot{v} = \frac{y\dot{y} - f\dot{x} + x\dot{x}}{\sqrt{y^2 + f^2 - 2fx + x^2}}$.

 6. But

condly, *the Glasses by which they are reflected and refracted*; Thirdly, *the Theorems or Laws relating*

9. But $\dot{z}$ and $\dot{v}$ being the Fluxions of the incident and refracted Rays, will represent their *Velocities* before and after Refraction, which Velocities we have shewn (*Annot.* CXVII.) are as the Sines of Incidence and Refraction m and n; whence $\dot{z} : \dot{v} :: m : n$. And from the Nature of Refraction (above explained) it is manifest that while the incident Ray increases, the refracted Ray decreases; therefore their Fluxions must have contrary Signs, *viz.* $+ \dot{z}$, and $- \dot{v}$. Wherefore $\dot{z} : - \dot{v} :: \frac{y\dot{y} + d\dot{x} + x\dot{x}}{\sqrt{y^2 + d^2 + 2dx + x^2}} : \frac{f\dot{x} - y\dot{y} - x\dot{x}}{\sqrt{y^2 + f^2 - 2fx + x^2}} :: m : n$.

7. Now because in those Mirrours and Lenses which are of common Use in *Optics* we regard only the Focus of those Rays which fall very near the Axis, in which Case the Arch B V is very small, and therefore V D $= x = o$ nearly; therefore xx and $x\dot{x}$ may be rejected, without sensibly affecting the Value of the Expressions; therefore $m : n :: \frac{y\dot{y} + d\dot{x}}{\sqrt{y^2 + d^2}} : \frac{f\dot{x} - y\dot{y}}{\sqrt{y^2 + f^2}}$, and so $m \times \frac{f\dot{x} - yy}{\sqrt{y^2 + f^2}} = n \times \frac{y\dot{y} + d\dot{x}}{\sqrt{y^2 + d^2}}$.

8. From which Equation we shall find $f =$ F V, in any Curve V G from the Equation expressing its Nature. Thus if V G be a CIRCLE, its Equation is $yy = 2r\dot{x} - xx$, (where C B $= r =$ the Radius) the Fluxion of which is $y\dot{y} = r\dot{x} - x\dot{x}$; and since $x = o$, we have $yy = o$, $y\dot{y} = rx$; and, substituting these Values in the general Equation above, we have $m \times \frac{f\dot{x} - r\dot{x}}{\sqrt{f^2}} = n \times \frac{r\dot{x} + d\dot{x}}{\sqrt{d^2}}$, and, dividing by ·, $m \times \frac{f - r}{f} = n \times \frac{r + d}{d}$; therefore $mdf - mdr = nrf + ndf$, and thence $\frac{mdr}{md - nd - nr} = f =$ V E.

9. If

relating to the Formation of the Images of Objects thereby; Fourthly, *the Nature of Vision*,

9. If the Medium be Glaſs, then $m : n :: 3 : 2$; therefore $\frac{3dr}{d-2r} = f$. And for parallel Rays *A* B, where d is infinite, we have $\frac{3dr}{d-2r} = \frac{3dr}{d} = 3r = f =$ V F. But in Water, where $m : n :: 4 : 3$, we have $f = \frac{4dr}{d-3r}$, and $4r = f =$ V *F*, for parallel Rays *A* B.

10. This Theorem (in *Art.* 7.) may be alſo adapted to the ELLIPSIS, the Equation of which Curve is $yy = px - \frac{pxx}{a}$, which in Fluxions is $y\dot{y} = \frac{p\dot{x}}{2} - \frac{px\dot{x}}{a}$; and, putting $x = o$, we have $yy = o$, $y\dot{y} = \frac{p\dot{x}}{2}$, which Values ſubſtituted in the general Equation give $\frac{\frac{3}{2}dp}{d-p} = f$; and when d is infinite, or the Rays parallel, then $\frac{p}{4} =$ V *F*, the focal Diſtance of the Ellipſis V G, a fourth Part of the *Latus Rectum* from the Vertex, for the Sun-Beams. The Expreſſion is alſo the ſame for an HYPERBOLA V G, becauſe only $\frac{pxx}{a}$ is affected with a different Sign, and vaniſhes in that Equation alſo.

11. If V G be a PARABOLA, its Equation is $yy = px$, and in Fluxions $2y\dot{y} = p\dot{x}$; whence, ſince $x = o$, we have $yy = o$, $y\dot{y} = \frac{p\dot{x}}{2}$, which ſubſtituted as before give $\frac{\frac{3}{2}dp}{d-p} = f$; and in caſe of parallel Rays, or the Sun-Beams, $\frac{p}{4} =$ V *F*, the Focus or Burning-Point of the Parabola.

sion, and Structure of the Eye; and Fifthly, *the Structure and Use of the principal Optical Instruments.*

THE

12. Hence we observe, that in the Circle V G, whose Radius C B is equal to half the *Latus Rectum* of the Ellipsis or Parabola, *viz*, $r = \frac{1}{2} p$, the Focus will be at the same Distance from the Vertex V, or V F will be the same in all; for then it is $\frac{\frac{3}{2} d p}{d - p} = \frac{3 d r}{d - 2 r} = f$ in all the Curves, and consequently the *Circle*, *Ellipsis*, and *Parabola*, have all the same Degree of Curvature at the Vertex V in this Case.

13. When $d = 2 r$, or $d = p$, then the focal Distance $f = \frac{3 d r}{0} = \frac{\frac{3}{2} d p}{0} =$ V F becomes infinite; that is, if the Radiant Point A be at the Distance of the Diameter of the Circle, or the Parameter of the Conic Section from the Vertex V of the Medium of Glass, then the Rays will be refracted parallel to the Axis. And, *vice versâ*, parallel Rays will be refracted from a Substance of Glass by a spherical Surface to the Distance of the Diameter of the Sphere; or from an elliptical or parabolical Surface to the Distance of the *Latus Rectum*, from the Vertex V.

14. After the same Manner we express the several Cases of a *Spherical*, *Elliptical*, or *Parabolical* reflecting Surface V B G, that is, such a one where the incident Ray A B is reflected from the Point B instead of being refracted; and then since the Angle of Incidence A B L is equal to the Angle of Reflection L B K, the Ray K B will be so reflected from the Point B as if it came from a Point F in the Axis, and therefore that Point F we must consider as the Focus of reflected Rays. In this Case the Velocities of the incident and reflected Rays are the same, *viz.* $\dot{z} = \dot{v}$, and both affirmative; also $m = n$.

Plate XLII. Fig. 8.

Whence $\frac{y\dot{y} + \overline{d + x}\,\dot{x}}{\sqrt{y^2 + d^2 + 2dx + x^2}} = \frac{y\dot{y} - \overline{f + x}\,\dot{x}}{\sqrt{y^2 + f^2 - 2fx + x^2}}$; or,

THE *Rays of Light* are diſtinguiſhed into three Sorts, *viz. Parallel, Converging, and Diverging*

or, putting $x = o$, $yy = o$, and $y\dot{y} = r\dot{x}$, or $= \frac{1}{2}p\dot{x}$, (as above) then this general Theorem becomes $\frac{dr}{r+2d} = f =$ V F, in the *Circle*; and $\frac{dp}{p+4d} = f =$ V F, in the *Ellipſis*, *Hyperbola*, and *Parabola*.

15. If d, or A V, be infinite, as in parallel Rays, or the Sun-Beams, then $\frac{dr}{r+2d} = \frac{dr}{2d} = \frac{1}{2}r = f =$ V F, in the ſpherical convex Mirrour V G; but if the ſaid Mirrour be *Elliptical*, *Hyperbolical*, or *Parabolical*, then $\frac{pd}{p+4d} = \frac{1}{4}p = f =$ V F. But becauſe the Rays B K do not actually proceed from the Point F, that Point is in this Kind of Mirrours called the *Virtual Focus*.

16. If the Radius B C $= r$ of the convex Mirrour be infinite, the ſpherical Surface V B G will become a Plane, *viz.* a plane Speculum or Looking-Glaſs, as V B G in the following Figure; and the Theorem $\frac{dr}{r+2d} = \frac{dr}{r}$ Plate XLII.
$= d = f =$ V F, that is, A N is equal to V F, or the Fig. 9.
incident Ray A B is ſo reflected at B into B K as if it came from a Point F, juſt as far behind the Glaſs as the Radiant A is before it.

17. Furthermore, if $r =$ B C be ſuppoſed greater than Infinite, or from affirmative to become negative, the Centre C will then lie on the contrary Side, the Speculum V B G will become concave, and in the Theorem above, r muſt have a negative Sign, which then will Fig. 10.
be $\frac{-dr}{2d-r} = f =$ V F, which ſhews that in concave Mirrours, when d is leſs than $\frac{1}{2}r$, that is, when A V is leſs than $\frac{1}{2}$ C V, the Focus f will be affirmative, or on the ſame Side as before; or the Ray A B will be ſo re-

Diverging Rays. *Parallel Rays* are ſuch as in their Progreſs keep always an equal Diſtance

fleſted at B into B K as if it came from a Point F behind the Speculum.

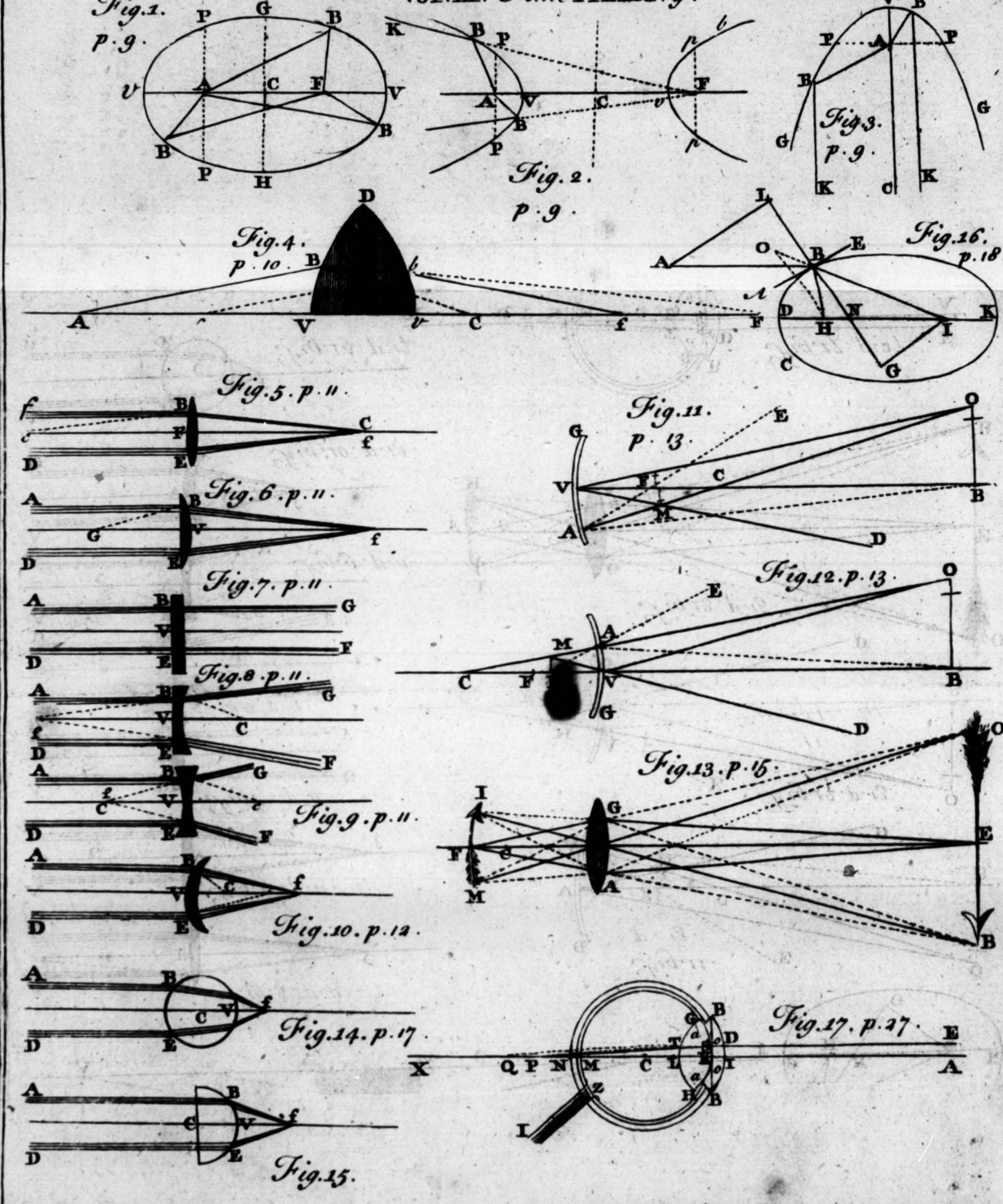
18. When $d = \frac{1}{2} r$, or A V $= \frac{1}{2}$ C V, then is the Focus F at an infinite Diſtance, the Theorem then being $\frac{-dr}{o} = f$; ſo that in this Caſe all the Rays A B will be reflected parallel to the Axis, as B *K*. But when d is greater than $\frac{1}{2} r$, then the Focus f will be negative, or it will be $\frac{-dr}{2d-r} = -f$. Wherefore in this Caſe the Focus F will be on the ſame Side with the Radiant A.

19. Laſtly, when $d = r$, then alſo $f = r$; that is, if the Radiant A be placed in the Centre C, the Focus F will be there too; or, in other Words, Rays proceeding from the Centre will be reflected back upon themſelves.

20. On the contrary, (in all theſe Caſes) *converging Rays* K B are reflected to a Point in the Axis leſs diſtant than $\frac{1}{2}$ C V, or half the Radius. *Parallel Rays K* B are reflected to that Point F of the Axis where F V $= \frac{1}{2}$ C V. This will therefore be the *Burning-Point of the Sun's Rays*, and is the *Solar Focus* above mentioned. *Diverging Rays* have their Focus at a Diſtance from the Vertex V, greater than half the Radius C V.

21. If V B G be an Ellipſis, Hyperbola, or Parabola, the Theorem is found in the ſame Manner to be $\frac{-dp}{4d-p} = f$, in concave Speculums of this Sort; and all that has been ſaid with reſpect to d and $\frac{1}{2} r$ in the ſpherical Speculums, is true of d and $\frac{1}{4} p$ in theſe. Thus when $d = \frac{1}{4} p$, the Rays will be reflected parallel to the Axis; and on the other hand, parallel Rays will be reflected to a Point in the Axis whoſe Diſtance from the Vertex V is $\frac{1}{4} p$. Thus the Sun's Rays are collected at the Diſtance of *One-fourth Part of the Parameter* (in each

Vol. III. Plate XLIII. P. 9.



tance from each other, as A B D C; such as are the Sun's Rays, in their natural State, with Plate XLVIII. Fig. 4.

each Section) from the Vertex; and as this is the *Burning-Point*, we see the Propriety of its being called the *Focus* of those Curves.

22. As within the Curve of an Ellipsis V G v H there are two of those Focus's, 'tis observable, that if the Radiant A be in one Focus, the Rays will be reflected to the other at F, wherever the Point B be taken in the Perimeter of the Ellipse. For in this Case V $v = a$, A $v = x$, A P $= y = \frac{1}{2} p$, (for P P $= p$) A V $= d = a - x$, and F V $=$ A $v = f = - x$; therefore writing $a - x$, and $- x$ for d and $-f$, in the Equation above, we shall have $\frac{p x - p a}{4 a - 4 x - p} = - x$, and so $4 a x - 4 x x = p a$, or $a x - x x = \frac{1}{4} p a$, which is the known Property of the Ellipsis. Plate XLIII. Fig. 1.

23. And the same Thing holds with respect to the Foci of the two opposite Hyperbola's V B and $v b$; for if the Radiant be in the Focus A of one, any Ray A B will be so reflected into B K, as if it came from the Focus F to the opposite Hyperbola $v b$, as is evident in the Figure. In the Parabola V B G, if the Radiant be placed in the Focus A, the reflected Rays B K, tending to the other Focus at an infinite Distance, will be all parallel to the Axis V C, agreeable to what is said above, *Article* 21. Fig. 2. Fig. 3.

24. If we resolve the Equation $\frac{d r}{r + 2 d} = f$, into an Analogy, we shall discover that the Axis of the Mirrour is divided harmonically in the Points V, F, C, and A; or that it is A V : A C :: V F : F C. For supposing it to be so, we have $d : d \pm r :: f : r \pm f$, which gives us the above Theorems; $\frac{d r}{2 d + r} = f$, in the convex Speculum; and $\frac{- d r}{2 d - r} = f$, in the concave. This

curious

with reſpect to Senſe. *Converging Rays* are ſuch as in their Progreſs approach nearer

curious Property of Speculums was firſt diſcovered by the late Mr. *Ditton*.

25. We now proceed to apply this Method to *Dioptric Problems*, that is, to find the Focus of Rays refracted through any Sort of Lenſes. To this end we muſt recollect, that in *Article* 8. we had $m\,d\,f - m\,d\,r = n\,r\,f + n\,d\,f$; whence deduce this other Equation $\frac{m}{n} = \frac{r+d}{f-r} \times \frac{f}{d} = \frac{AC}{CF} \times \frac{VF}{AV}$, which in Words is thus expreſſed: *The Ratio of the Sine of Incidence to the Sine of Refraction is compounded of the Ratio of the Diſtances of the Foci* A *and* F *from the Centre* C, *and of the Ratio of their Diſtances from the Vertex* V.

Plate XLIII. Fig. 4. 26. If then we conſider B *b* (in the double-convex Lens V D *v*) as a converging Ray refracted from Glaſs into Air, we ſhall find the Diſtance *v* f, at which the refracted Ray *b* f ſhall interſect the Axis of the Lens, by the Rule in *Article* 25. Only here we muſt conſider, that the Point A will be negative, or on the ſame Side with the Focus f, *viz.* at A. And as the Refraction is out of Glaſs into Air, we muſt uſe the Ratio $\frac{n}{m}$ inſtead of $\frac{m}{n}$; then $\frac{n}{m} = \frac{Ac}{fc} = \frac{fv}{Av}$.

27. Let the Thickneſs of the Lens be V $v = t$, and $v\,\mathrm{f} = \mathrm{f}$; alſo let the Radius of the ſecond Surface be $c\,b = \mathrm{r}$; then $\frac{n}{m} = \frac{f+\mathrm{r}-t}{\mathrm{f}+\mathrm{r}} \times \frac{\mathrm{f}}{f-t}$; whence $\mathrm{f} = \frac{n f \mathrm{r} - n f t - n t \mathrm{r}}{m f - m t + m \mathrm{r} - n f} = v\,\mathrm{f}$ the focal Diſtance required. But if the Thickneſs t be inconſiderable, as it commonly is, it may be neglected, and then $\mathrm{f} = \frac{n f \mathrm{r}}{m f + m \mathrm{r} - n f}$, whence

nearer and nearer to each other, all of them tending towards a certain Point F, where

whence $f = \frac{m \mathrm{f} \mathrm{r}}{n \mathrm{r} + n \mathrm{f} - m \mathrm{f}} = \frac{m d r}{m d - n d - n r}$, which Equation reduced gives $\mathrm{f} = \frac{n d r \mathrm{r}}{m r d - n r d + m d \mathrm{r} - n d \mathrm{r} - n r \mathrm{r}}$; and putting $\frac{n}{m - n} = q$, we have $\mathrm{f} = \frac{q d r \mathrm{r}}{r d + d \mathrm{r} - q r \mathrm{r}}$. But in Glaſs, $q = 2$; and if we ſuppoſe the Lens equally convex, or $r = \mathrm{r}$, we have $\mathrm{f} = \frac{d r}{d - r} = v\ \mathrm{f}$, the focal Diſtance of the Ray A B after paſſing through the Lens, as required.

28. If d be infinite, then $r = f$; therefore parallel Rays, or the Sun-Beams, will be collected in a Point f, whoſe Diſtances from the Lens is equal to the Radius of Convexity. Plate XLIII, Fig. 5.

29. If one of the Radii r, r, be infinite, the Lens will be a *Plano-Convex*, and $\mathrm{f} = \frac{2 d r}{a - 2 r}$; and for parallel Rays where d is infinite, $\mathrm{f} = 2 r$. Fig. 6.

30. If both the Radii be infinite, the Lens then is no other than a *plain Glaſs* terminated by two parallel Sides; and the Focus f will be at an infinite Diſtance for parallel Rays, or they will be parallel after Refraction as they were before. Fig. 7.

31. If one Radius r be infinite, and the other r negative, then will the Lens be a *Plano-Concave*; then will the Theorem be $\frac{-2 d r}{d + 2 r} = \mathrm{f}$, which is therefore negative, or the Rays proceed diverging after Refraction. When d is infinite, the Theorem is $\frac{-2 d r}{d} = -2 r = \mathrm{f}$, or parallel Rays diverge from a Point F, at the Diſtance of twice the Radius of Concavity. Fig. 8.

32. If both of the Radii be negative, the Lens becomes a *Double-Concave*; and if d be infinite, and the Radii Fig. 9.

where they all unite; as the Rays of the Sun collected by a Glass, as CDF. *Diverging*

Radii equal, *viz.* $\mathrm{r} = r$, the Theorem then is $\frac{dr}{-d-r} = \frac{dr}{-d} = -r = -f$; so that parallel Rays, or the Sun-Beams, are so refracted through a double and equally concave Lens, as if they proceeded from a Point f at the Distance of the Radius of Concavity from the Vertex of the Lens.

Plate XLIII. Fig. 10. 33. If one of the Radii, as r, be affirmative, and the other r negative, the Lens becomes a *Meniscus*, and the Theorem then is $\frac{-2dr\mathrm{r}}{dr - d\mathrm{r}} = \frac{-2r\mathrm{r}}{r - \mathrm{r}} = \mathrm{f}$; which shews that when $r = \mathrm{r}$, and d is infinite, the Focus f is at an infinite Distance, or the Rays are parallel after Refraction as before, as in the Case of a *Watch Glass*. If r be greater than r, or the Concavity less than the Convexity, the Focus f will be affirmative, or parallel Rays will be converged to a real Focus; but if r be less than r, the Focus f will be negative, or parallel Rays will proceed diverging after Refraction.

34. We now proceed to determine the Position, Magnitude, Form, &c. of the Images of Objects formed by Mirrours and Lenses, having first premised, that the Image of an Object always appears in the Place from whence the Rays diverge after Reflection or Refraction; or in other Words, the Image appears in that Place, which we have hitherto called the Focus of the Rays. This Sir *Isaac Newton* has delivered as an Axiom, as being very evident, because the Species, or several Points of the Image of an Object, are brought to the Eye by the reflected or refracted Rays.

Fig. 11, 12. 35. Let A V G be a reflected Speculum, C its Centre, V B its Axis, F the solar Focus; and let O B be an Object at the Distance V B; through the Center C draw O A, which as it is perpendicular to the Speculum

verging Rays are those which proceed from a Point, as F, and in their Progress recede

lum will be reflected back upon itself, and therefore the proper Focus of the Point O will be in the Line A O, and that of the Point B in the Line or Axis B V. Those focal Points are easily found, thus: Draw O V and V D making equal Angles with the Axis V B; also draw B A, and A E, making equal Angles with the Axis O A; then shall those two refracted Rays V D and A E intersect the Perpendiculars O A and B V in the Points M and I, which will therefore be the focal Points where the Representation of the extreme Points O and B will be made; and consequently all the Points between O and B will be represented between M and I, and therefore the Line I M will be the true Representation or Image of the Object O B.

36. Hence also 'tis easy to observe, that the Position of the Object O B is inverted in the Image I M, and consequently the same Parts of the Object and Image are on contrary Sides of the Axis in a *concave Mirrour*, where the Rays have a real Focus, or form a real Image: But in a *convex Mirrour*, where the Rays have no real but an imaginary Focus, or form not a real but an apparent Image, no such Inversion can happen, but the Object and Image both appear in an erect Position, as is easy to understand from the Figure. Plate XLIII. Fig. 11. Fig. 12.

37. Again; the Object and Image are commutable, or may be taken the one for the other in the Schemes. Thus if O B be the Object, then I M will be its Image; but supposing I M the Object, then will O B be its Image.

38. Hence also it appears, that if I M represent an Object placed before a convex Mirrour nearer to the Vertex V than the Solar Focus F, the Rays will be so reflected as to form an apparent Image O B behind the Speculum; and this Case will be every way the same with that of the convex Speculum reversed.

39. It is farther obvious, that the Object O B and Image I M subtends equal Angles, both at the Vertex V and Center C of the Mirrour, whether concave or convex;

cede from one another towards the Parts G E.

THE

convex; for at the Vertex the Object O B subtends the Angle O V B = B V D or I V M, which the Image subtends (by *Art.* 35.). And at the Center C, the Angles O C B and I C M, under which the Object and Image appear, are equal, as is evident by Inspection, they being vertical to each other.

40. Therefore the Triangles O V B and I V M, also the Triangles O C B and I C M, are similar, as having all their Angles respectively equal; therefore we have O B : I M :: V B : V I; also O B : I M :: B C : I C. That is, the Lengths of the Object and Image are proportional to the Distances from the Vertex or Center of the Speculum.

41. Hence in Symbols (putting O = Object, and I = Image) we have $O : I :: d : f$; whence $\frac{I\,d}{O} = f = \frac{d\,r}{2\,d - r}$; therefore $O : I :: 2\,d - r : r$. Wherefore, by having given the Radius of the Speculum, you may place the Object at such a Distance, that it shall bear any given Proportion to its Image, as that of m to n; for then, since $m : n :: 2\,d - r : r$, we have $m\,r = 2\,d\,n - r\,n$, and $m\,r + r\,n = 2\,d\,n$; consequently, $d = r \times \frac{m + n}{2\,n}$ for a concave Speculum, and $d = r \times \frac{m - n}{2\,n}$ for a convex one.

42. From hence it is manifest, no Object can be magnified by a convex Speculum; for, because in that Case n is greater than m, $r \times \frac{m - n}{2\,n}$ would be a negative Quantity, and so d would have a negative Value, which is impossible. And when $m = n$, then $d = o$; or the Object and Image are then only equal, when they coincide at the Vertex of the concave Mirrour.

43. In a concave Mirrour, while m is greater than n, it

THE Point F, where the Rays are collected, is call'd the *Focus*, or Burning-Point, be-

it is plain the Distance d of the Object is greater than the Radius r of the Mirrour. But when $m = n$, then $d = r$; or the Object and Image are equal in the Center of the Mirrour. When m is less than n, or the Object is magnified, then d is less than r. Now this may be done two different Ways in a concave Speculum; for n may be affirmative, or the Image real and form'd before the Glass, then $d = r \times \frac{m \times n}{2n}$; or n may be negative, or the Image only apparent and represented behind the Mirrour, then $d = r \times \frac{n - m}{2n}$; in which Case, 'tis plain, the Object cannot be diminish'd. But lastly, if n be infinite in respect of m, then $rn = 2dn$, or $r = 2d$, that is, $d = \frac{1}{2}r$. Or when the Object is placed in the Solar Focus, the Image is form'd at an infinite Distance, and infinitely large.

44. Such are the Theorems for *Specula*; those for *Lenses* are raised after a like Manner. For let GVA be a double and equally convex Lens; C its Center, or CV the Radius of Convexity $= r$; OB an Object, EV its Distance (in the Axis of the Lens) $= d$, IM the Image, and FV $= f$, the focal Distance at which it is formed. Then as the Point E in the Object is form'd in the Point F in the Axis of the direct double Pencil of Rays EGFA, so the Point O will be form'd at M in the Axis of the Pencil OGMA; and since these two Axes cross each other in the Middle of the Lens at V, therefore the Points O and M, and (for the same Reason) B and I, will be on contrary Sides of the Axis EF, and consequently the Image in respect of the Object is inverted. Plate XLIII. Fig. 13.

45. Because the Angles OVB and IVM are equal, as being vertical, the Object and Image have the same apparent Magnitude if view'd from the Vertex of the Lens V; and are in Proportion to each other as their Distances from the Lens, that is, OB : IM :: VE : VF.

46. Hence,

because there the Sun's Rays, being united within a very small Compass or Circle, are great-

46. Hence, if (as before) we make OB : IM :: m : n :: d : f, we have $\frac{nd}{m} = f = \frac{dr}{d-r}$; whence $m : n :: d-r : r$; and so $mr = dn - rn$, or $mr + rn = dn$; wherefore $d = r \times \frac{m+n}{n}$. If $m = n$, then $2r = d$; and if n be infinite in respect to m, $r = d$. And if n be negative, or on the same Side of the Lens with the Object, then $d = r \times \frac{n-m}{n}$, which shews the Object in that Case is always magnified.

47. If the Lens be a single or double concave, the Rays cannot be converged to a Focus, (as is manifest from *Art.* 32.) and consequently no real Image can be form'd, but only an imaginary one; and because it is in this Case $d = r \times \frac{-m-n}{n}$, 'tis plain when $m = n$ then $d = r \times \frac{-o}{n} = o$, that is, the Image can only be equal to the Object when they coincide at the Lens.

48. The Form of the Image IFM is not a right or strait Line, but a Curve; for let VE $= d$, VF $= f$, and VO $= \mathrm{d}$, VM $= \mathrm{f}$; then since $\frac{dr}{d-r} = f$, and $\frac{\mathrm{d}r}{\mathrm{d}-r} = \mathrm{f}$, we have $f : \mathrm{f} :: \frac{dr}{d-r} : \frac{\mathrm{d}r}{\mathrm{d}-r}$; but if IFM were a Right Line, it would be $f : \mathrm{f} :: d : \mathrm{d}$. Neither is the Image of a circular Form, unless the Object be so; because in that Case $f = \mathrm{f}$, which cannot be but when $d = \mathrm{d}$, or VE $=$ VO; so that if the Object be the Arch of a Circle, the Image will be the Arch of a Circle concentric with the Object, or else of a Conic Section, as before observed of Images form'd by Mirrours, *Art.* 36.

49. If the Object be a Surface, the Image will be a Surface similar thereto; and since Surfaces are in duplicate

greatly conſtipated and condenſed, by which means their Action or Heat is proportionably

plicate Proportion of their like Sides (*Annot.* II. *Art.* 3.) therefore $m : n :: OB^2 : IM^2$, in this Caſe. And if the Object be a Solid, the Image will be a ſimilar Solid, and they will be in the triplicate Proportion of their homologous Sides; whence $m : n :: OB^3 : IM^3$.

50. Though *Speculums* and *Lenſes* are of moſt general Uſe in *Optics*, yet it will be neceſſary to conſider the Property of a *Globe* or *Sphere*, as alſo of an *Hemiſphere*, with reſpect to their Power of converging the Rays of Light to a Focus. If therefore in the Theorem of *Art.* 27. we put $t = 2r$ = Diameter of the Globe, and becauſe $r = r$, we ſhall have $\frac{dr + 4rr}{2d - r} = f$, the Focus of diverging Rays; and when d is infinite, the Theorem is $\frac{dr}{2d} = \frac{r}{2} = f = Vf$. Therefore a Globe of Glaſs will converge the Rays of the Sun to a Focus at the Diſtance of half the Radius. Plate XLIII. Fig. 14.

51. But in caſe the Globe be Water, then in the aforeſaid Theorem we have $m = 4$, $n = 3$, and the reſt as before; then by Reduction it will become $\frac{dr - 2rr}{d - r} = f$, for diverging Rays; and for parallel Rays, where d is infinite, we have $\frac{dr}{d} = r = f$, juſt twice as large as in Glaſs.

52. In an *Hemiſphere* of Glaſs, when the convex Side is turn'd towards the Radiant, having r infinite, and $t = r$, the Theorem will become $\frac{4dr + 4rr}{3d - 6r} = f$, the focal Diſtance of diverging Rays; but for parallel Rays it becomes $\frac{4dr}{3d} = \frac{4}{3}r = f$.

53. If the plane Side of the Hemiſphere be turned towards the Radiant, the Theorem for diverging Rays

portionably increased, and therefore Objects posited in that Point will be greatly heated, burnt, or melted.

OF

will be $\frac{6dr + 4rr}{3d - 4r} = f$; and for parallel Rays $\frac{6dr}{3d} = \frac{6}{3}r = 2r = f$; which is $\frac{2}{3}r$ greater than before.

54. In an Hemisphere of Water, the convex Part being towards the Radiant, we have $\frac{9dr - 9rr}{4d - 12r} = f$; and for parallel Rays it is $\frac{9dr}{4d} = \frac{9}{4}r = f$. But if the Radiant be opposed to the plane Side, then $3r = f$, greater by $\frac{3}{4}r$ than before.

55. We have hitherto considered the Property of *spherical Bodies* only, with respect to their Power of refracting a Ray of Light; let us now consider the Nature of Refraction in Bodies whose Figures are derived from the Curves of the *Conic Sections*. In order to this, let DBKC be an Ellipsis, DK its transverse Axis, H, I, its two Foci, and AB a Ray of Light parallel to the Axis be incident on the Point B. Let BE be a Tangent in the said Point, and LG drawn perpendicular to the Tangent through the Point B; join HB and IB; make AB = IB, and from the Points A and I let fall the Perpendiculars AL, IG, on the Line LG; produce IB to O, and draw HO parallel to LG.

Plate XLIII. Fig. 16.

56. Then in the similar Right-angled Triangles ALB, ING, we have AL : IG :: AB : NI :: IB : NI, because AB = IB. But IB : NI :: IO : IH, because of the similar Triangles BNI and OHI. Again, the Angle HBG = GBI from the Nature of the Curve; whence GBI = HOB (= HBG) = BHO; therefore the Triangle HBO is Isosceles, or BH = BO. But IB + BH = DK, *per Conics*; therefore

IB

OF GLASSES there are two Kinds, *viz.* *Mirrours*, and *Lenses*. A *Mirrour* or *Speculum* is that, which from one polished Surface reflects the Rays of Light; and these are either *Convex*, *Concave*, or *Plane*, as will be shewn. A *Lens* is any transparent or diaphanous Body, as *Glass*, *Crystal*, *Water*, &c. through which the Rays of Light do freely pass, and is of a proper Form to collect or disperse them. Of these there are several Species, as a *Plane Lens*, a *Plano-Convex*, *Plano-Concave*, *Double-Convex*, *Double-Concave*, and *Meniscus*.

IB + BO = IO = DK. Consequently, AL : IG (:: IB : NI :: IO : IH.) :: DK : IH.

57. Since LG is perpendicular to the Tangent or Curve in the Point B, 'tis evident that AL is the Sine of Incidence, and IG the Sine of Refraction to the Radius AB = BI. If therefore a Solid be generated by the Revolution of an Ellipsis about its Axis, which Ellipsis has its transverse Axis DK to the Distance between the Foci in the Ratio of the Sine of Incidence to that of Refraction; then parallel Rays AB, falling on every Point B of its Surface, will be refracted to the remote Focus I.

58. After the same Manner we proceed for the *Hyperbolic Conoid*; but as Lenses made of these Forms are extremely difficult to work, and are likely never to be of Use (since the great Defect of these Glasses is owing to quite a different Cause, as we shall shew in the next *Annotation*) I shall say no more of them here, but refer the inquisitive Reader to the *Dioptrics* of M. *Des Cartes*, who treats largely of this Subject.

I SHALL now conſider the different Properties and Effects of theſe Glaſſes in reflecting and refracting the Sun's Light, and forming the Images of Objects: And this all depends *(in Reflection of Light)* on that fundamental Law, *That the Angle of Incidence is equal to the Angle of Reflection.*

Plate XLVIII. Fig. 5. LET EH be a concave Mirrour, V its Vertex, and C the Centre of its Concavity. Let A be a Ray of the Sun's Light incident on the Point E, and draw E C, which will be perpendicular to the Mirrour in the Point E; make the Angle CEF equal to the Angle AEC, then ſhall E F be the reflected Ray. Thus alſo HF will be the reflected Ray of the incident one DH, at an equal Diſtance on the other Side of the Axis B V.

IF now the Points E and H be taken very near the Vertex V, we ſhall have E F, or HF, very nearly equal to F V; but $EF = FC$; therefore $FV = FC = \frac{1}{2} CV$. That is, *the Focal Diſtance* F V *of parallel Rays will be at the Diſtance of half the Radius* C V *of the Concavity of the Mirrour, from the Vertex* V, *in the Axis* B V.

Fig. 6. AFTER the ſame Manner, a *convex Mirrour* is ſhewn to reflect the Rays A E, D H, into E F, H F, as if they came diverging from

from a Point F in the Axis C V, which is half the Radius C V diſtant from the Vertex V. But ſince the Rays do not actually come at, or from the Focus *f*, it is called the *Imaginary or Virtual Focus.*

PARALLEL Rays falling directly on a *plane Speculum* are reflected back upon themſelves; if they fall obliquely, they are reflected in the ſame Angle, and parallel as they fell. Hence there is no ſuch Thing, properly ſpeaking, as a *Focus* belonging to a *plane Speculum*, neither *real nor virtual.*

THE Focus F, or *f*, of parallel Rays, is called the *Solar Focus*; becauſe in that the Image of the *Sun* is formed, and of all Objects very remote. But the Focus of any Object ſituated near the Mirrour will have its Diſtance from the Vertex more or leſs than half the Radius; the Rule in all Caſes being as follows:

Multiply the Diſtance of the Object into the Radius of the Mirrour, and divide that Product by the Sum of the Radius and twice the Diſtance of the Object; the Quotient will be the Focal Diſtance of a Convex Mirrour.

AGAIN; for a *Concave Mirrour, the ſame Product of the Radius into the Diſtance of the Object, divided by the Difference of Radius*

 and

and twice the Distance of the Object, will give the Focal Distance V F *or* V *f*. And here we are to observe, that as twice the Distance of the Object is lesser or greater than the Radius, so the Focus will be positive or negative, that is, behind the Glass or before it.

THE Image of every Object is formed in the Focus proper to its Distance: And since the Writers on *Optics* demonstrate, that *the Angles under which the Object* O B *and its Image* I M *are seen from the Centre or Vertex of the Mirrour* C *are always equal*; it follows, that the Image I M will be always in Proportion to the Object O B, as the Focal Distance V F to the Object's Distance G V.

Plate XLVIII. Fig. 7.

THE Position of the Object will be always erect at a *positive Focus*, or behind the *Speculum*; diminished by a convex, and magnified by a concave one. Hence, since a convex has but one, *viz.* an *affirmative Focus*; so it can never magnify any Object, howsoever posited before it.

THE Position of the Image in a *negative Focus*, or that before the Glass, will be ever inverted; and if nearer the Vertex than the Centre C, it will be less; if farther from it, it will be greater than the Object;

Object; but in the Centre it will be equal to the Object, and ſeem to touch it.

THE Image formed by a *plane Speculum* is erect; large as the Life; at the ſame apparent Diſtance behind the Glaſs, as the Object is before it; and on the ſame Side of the Glaſs with the Object. Theſe Properties render this Sort of Mirrour of moſt common Uſe, *viz.* as a LOOKING-GLASS.

IF the Rays fall directly, or nearly ſo, on a plane Mirrour, and the Object be opake, there will be but *one ſingle Image formed*, or at leaſt be viſible; and that by the ſecond Surface of the *Speculum*, and not by the firſt, through which the Rays do moſt of them paſs.

BUT if the Object be luminous, and the Rays fall very obliquely on the *Speculum*, there will be more than one Image form'd, to an Eye placed in a proper Poſition to view them. The firſt Image being form'd by the firſt Surface will not be ſo bright as the ſecond, which is formed by the ſecond Surface. The third, fourth, &c. Images are produced by ſeveral Reflections of the Rays between the two Surfaces of the *Speculum*; and ſince ſome Light is loſt by each Reflection, the Images from the ſecond will appear ſtill more faint and ob-

ſcure, to the eighth, ninth, or tenth, which can ſcarcely be diſcerned at all.

WE proceed now to *Lenſes.* And here, ſince all Viſion by them is effected by the Refraction of Rays through their Subſtance, it will be too intricate an Affair to ſhew the particular Manner how Rays are collected by them to their ſeveral Focus's: It muſt ſuffice only to ſay, *That parallel Rays are refracted through a plano-convex Lens to a Point or Focus, which is the Diameter of the Sphere of its Convexity diſtant from it:*

THAT *the ſame Rays are collected by a double and equally convex Lens in a Point which is the Centre of the Sphere of its Convexity:*

THAT *parallel Rays are refracted through a plano-concave Lens in ſuch manner as tho' they came from a Point diſtant from it by the Diameter of its Concavity:*

AND *that the ſame Rays are refracted through a double and equally concave Lens, in ſuch manner as though they proceeded from a Point which is the Centre of the Concavity.*

AND in caſe of a double and equally convex Lens, we have this general Rule for finding the *Focus* of Rays univerſally, be the Diſtance of the Object and Radius of Convexity what it will, *viz.*

Multiply

Multiply the Diſtance of the Object by the Radius of Convexity, and divide that Product by the Difference of the ſaid Diſtance and Radius; the Quotient will be the Diſtance of the Focus required.

HENCE, if the Diſtance of the Object be greater than the Radius, the Focus will be *affirmative*, or behind the Lens; the Image will be inverted, and diminiſhed in Proportion of its Diſtance to the Diſtance of the Object.

AGAIN; if the Diſtance of the Object be leſs than the Radius, the Focus will be *negative*, or on the ſame Side of the Lens as the Object; and the Image will be magnified, and in an erect Poſition.

IF the Diſtance be equal to the Radius, the Focus will be at an infinite Diſtance; that is, the Rays, after Refraction, will proceed parallel, and will therefore enlighten Bodies at a vaſt Diſtance. Hence the Contrivance of the *Dark Lanthorn* for this Purpoſe.

LASTLY: If the Diſtance of the Object be equal to twice the Radius, then will the Diſtance of the Focus and Image be equal to the Diſtance of the Object; and conſequently the Image will be equal in Magnitude to the Object, but inverted. Hence the Uſe of theſe Lenſes to Painters, and Draught-

Draught-Men in general, who have often Occaſion for the Images of Objects as large as the Life, to delineate or draw from.

As to *Plano-Concaves*, they, having no real Focus, form no Images of Objects; ſo that we ſhall paſs them to proceed to the Structure of the Eye, the Manner of performing Viſion therein, the ſeveral Defects thereof, and how remedied by Glaſſes; which will be illuſtrated by the Diſſection of a *natural Eye*, and exemplified by an *artificial one*.

THE *Eye* is the *noble Organ of Sight or Viſion*: It conſiſts of various Coats and Humours, of which there are Three remarkable, *viz.* (1.) The *Aqueous or Watry Humour*, which lies immediately under the *Cornea*, and makes the Eye globular before. (2). The *Vitreous Humour*, which is by much the greateſt Quantity, filling the Cavity of the Eye, and giving it the Form of a Globe or Sphere. (3) The *Cryſtalline Humour*, ſituated between the other two, near the Fore-part of the Eye, and is the immediate Inſtrument of Sight; for being of a lenticular Form, it converges the Rays, which paſs through the Pupil, to a Focus on the Bottom of the Eye, where the Images

of

of external Objects are by that means form'd and represented. (CXXVI.)

OVER

(CXXVI.) 1. In order to exhibit a just Idea of the *true Theory of Vision*, I shall here give a more exact and particular Description of the EYE and of its several Parts, with an Account or Calculation of the various Refractions of the Rays of Light through the several Humours, for forming the Images of Objects on the *Retina* at the Bottom of the Eye.

2. To this End I have here represented a Section of the *Human Eye* in its true or natural Magnitude, which consists of two Segments of two different Spheres, *viz.* one larger, as B N B, and a lesser, B I B. The larger Segment consists of three Tunics or Coats, of which the outmost is of a hard, thick, white, opake Substance, called the *Sclerotica*, as B N B. Within this is another thin, soft, and blackish Tunic, called the *Choroides*; which serves as it were for a Lining to the other, or rather as a delicate *Stratum* for the third Tunic called the *Retina*, which is a curious fine Expansion of the Optic Nerve Y Z over all the larger Segment of the Eye, every Way to B B. Plate XLV. Fig. 17.

3. The lesser Segment consists of one Coat or Tunic, called the *Cornea*, as resembling a Piece of transparent Horn; this is more convex than the other, and is denoted by B I B. Within this Coat, at a small Distance, is placed a circular Diaphragm, as B *o*, B *o*, called the *Uvea*, or *Iris*, because of the different Colours it has in different Eyes. In this is a round Hole in the Middle called the Pupil, as *o o*, which in some Creatures is of a different Figure, *viz.* oblong, as in Cows, Cats, &c.

4. As the *Cornea*, by its Transparency, admits the Light to enter the Eye, so the Pupil is destined to regulate the Quantity of the Rays that ought to enter the interior Part of the Eye for rendering Vision distinct, and the Images of Objects properly illumined. To this Purpose it is composed of two Sets of muscular Fibres, *viz.* one of a circular Form, which, by corrugating, contract or diminish the Pupil; and the other is an *Annulus*

OVER all the Bottom of the Eye is ſpread a very fine and curious Membrane call'd the

nulus of radial Fibres, tending every where from the Circumference B B of the *Uvea* to the Centre of the Pupil, which, by contracting, dilate and enlarge the Pupil of the Eye.

5. Immediately within the *Uvea* is another *Annulus* of radial Fibres, which on the extreme Part is every where connected with the *Cornea*, where it joins the *Sclerotica* at B B; and on the other Circumference it is connected with the anterior Part of the *Capſula* including the Cryſtalline Humour; and is called the *Ligamentum Ciliare*, and ſometimes the *Proceſſus Ciliares*, and is denoted by B *a*, B *a*.

6. The Bulk or Body of the Eye is made up of three Subſtances, commonly called *Humours*, *viz.* the *Aqueous*, the *Cryſtalline*, and the *Vitreous*. The *Aqueous Humour* is properly ſo called, being every way like Water, in reſpect of its Conſiſtence, Limpidity, ſpecific Gravity, and refractive Power. It is contained between the *Cornea* and the *Ligamentum Ciliare*, as B I B *a a* B. This Humour gives the protuberant Figure to the *Cornea*, which makes the firſt Refraction of the Rays of Light.

7. The ſecond Humour (improperly ſo call'd) is the *Cryſtalline*, having its Name from reſembling Cryſtal in Clearneſs and Tranſparency. It is denoted by G K H L, and is in Form of a thick Lens unequally convex, whoſe anterior Surface G K H is the Segment of a larger Sphere, and its poſterior Surface G L H the Segment of a leſſer. This Humour is of a ſolid Conſiſtence, and very little exceeds the ſpecific Gravity of Water. *viz.* in the Proportion of 11 to 10 nearly, as I have often found by Experiment. It is contained within a moſt delicate Tunic or *Capſula*, called *Arachnoides*, every where pellucid as the Cryſtalline itſelf.

8. The third Humour is the *Vitreous* (being clear as *Glaſs*) and is largeſt of all in Quantity, filling the whole Orb of the Eye B M B, and giving it a globular Shape. This

the *Retina*, which is an Expansion of the *Optic Nerve*; upon which the Images of Objects

This Humour is exactly like the White of an Egg, and but a little exceeds the specific Gravity and refractive Power of Water.

9. We proceed now to give the Dimensions of the Eye and its several Parts (in order for Calculation) as they have been determined by actual Measurement in a great Number of human Eyes with the greatest Care and Exactness. These Measures are express'd in Tenths of an Inch, as follows.

	Tenths.
The Diameter of the Eye from Outside to Outside, taken at a Mean from six adult Eyes, — —	I N = 9,4
The Radius of Convexity of the *Cornea*,	B I B = 3,3294
The Radius of Convexity of the anterior Surface of the Crystalline, from twenty-six Eyes, — —	G K H = 3,3081
The Radius of Convexity of the hinder Surface, from the same Eyes, at a Mean, — — —	G L H = 2,5056
The Thickness of the Crystalline, from the same Eyes, — —	K L = 1,8525
The Thickness of the *Cornea* and Aqueous Humour together, —	I K = 1,0358

10. Moreover, it is found by Experiment, that the Ratio of Refraction at the *Cornea* I is as 4 to 3, being the same with that of Air into Water; the Ratio of Refraction at K as 13 to 12, and at L as 12 to 13. These Things premised, let A X be the Axis of the Eye, and E D a Ray parallel thereto, and incident on the *Cornea* very near it at D; we are to determine the Foci of the several Refractions of this Ray at the several Surfaces I, K, and L.

11. The first Focus is determined by the Theorem (in *Annot.* CXXV. *Art.* 27.) $\frac{mdr}{md - nd - nr}$; for supposing

OVER all the Bottom of the Eye is ſpread a very fine and curious Membrane call'd the

nulus of radial Fibres, tending every where from the Circumference B B of the *Uvea* to the Centre of the Pupil, which, by contracting, dilate and enlarge the Pupil of the Eye.

5. Immediately within the *Uvea* is another *Annulus* of radial Fibres, which on the extreme Part is every where connected with the *Cornea*, where it joins the *Sclerotica* at B B; and on the other Circumference it is connected with the anterior Part of the *Capſula* including the Cryſtalline Humour; and is called the *Ligamentum Ciliare*, and ſometimes the *Proceſſus Ciliares*, and is denoted by B *a*, B *a*.

6. The Bulk or Body of the Eye is made up of three Subſtances, commonly called *Humours*, *viz.* the *Aqueous*, the *Cryſtalline*, and the *Vitreous*. The *Aqueous Humour* is properly ſo called, being every way like Water, in reſpect of its Conſiſtence, Limpidity, ſpecific Gravity, and refractive Power. It is contained between the *Cornea* and the *Ligamentum Ciliare*, as B I B *a a* B. This Humour gives the protuberant Figure to the *Cornea*, which makes the firſt Refraction of the Rays of Light.

7. The ſecond Humour (improperly ſo call'd) is the *Cryſtalline*, having its Name from reſembling Cryſtal in Clearneſs and Tranſparency. It is denoted by G K H L, and is in Form of a thick Lens unequally convex, whoſe anterior Surface G K H is the Segment of a larger Sphere, and its poſterior Surface G L H the Segment of a leſſer. This Humour is of a ſolid Conſiſtence, and very little exceeds the ſpecific Gravity of Water, *viz.* in the Proportion of 11 to 10 nearly, as I have often found by Experiment. It is contained within a moſt delicate Tunic or *Capſula*, called *Arachnoides*, every where pellucid as the Cryſtalline itſelf.

8. The third Humour is the *Vitreous* (being clear as *Glaſs*) and is largeſt of all in Quantity, filling the whole Orb of the Eye B M B, and giving it a globular Shape. This

the *Retina*, which is an Expanſion of the *Optic Nerve*; upon which the Images of Objects

This Humour is exactly like the White of an Egg, and but a little exceeds the ſpecific Gravity and refractive Power of Water.

9. We proceed now to give the Dimenſions of the Eye and its ſeveral Parts (in order for Calculation) as they have been determined by actual Meaſurement in a great Number of human Eyes with the greateſt Care and Exactneſs. Theſe Meaſures are expreſs'd in Tenths of an Inch, as follows.

	Tenths.
The Diameter of the Eye from Outſide to Outſide, taken at a Mean from ſix adult Eyes, — —	I N = 9,4
The Radius of Convexity of the *Cornea*,	B I B = 3,3294
The Radius of Convexity of the anterior Surface of the Cryſtalline, from twenty-ſix Eyes, — —	G K H = 3,3081
The Radius of Convexity of the hinder Surface, from the ſame Eyes, at a Mean, — — —	G L H = 2,5056
The Thickneſs of the Cryſtalline, from the ſame Eyes, — —	K L = 1,8525
The Thickneſs of the *Cornea* and Aqueous Humour together, —	I K = 1,0358

10. Moreover, it is found by Experiment, that the Ratio of Refraction at the *Cornea* I is as 4 to 3, being the ſame with that of Air into Water; the Ratio of Refraction at K as 13 to 12, and at L as 12 to 13. Theſe Things premiſed, let A X be the Axis of the Eye, and E D a Ray parallel thereto, and incident on the *Cornea* very near it at D; we are to determine the Foci of the ſeveral Refractions of this Ray at the ſeveral Surfaces I, K, and L.

11. The firſt Focus is determined by the Theorem (in *Annot.* CXXV. *Art.* 27.) $\frac{mdr}{md - nd - nr}$; for ſuppoſing

Objects being painted and impreſs'd, they are by that means convey'd to the *Common Senſory*

poſing all behind the *Cornea* B I B were the aqueous Humour continued, then ſince in this Caſe $m = 4$, $n = 3$, $r = 3{,}3294$, and d is infinite, we have $f = 4r = 13{,}3176 = IQ$, the focal Diſtance from I by the firſt Refraction.

12. The Ray tending by this means from D to Q falls converging on the anterior Surface of the Cryſtalline Humour at S. We muſt now find the Focus of the converging Ray D S refracted through a Medium every where the ſame with the Cryſtalline Humour. This we do by the ſame Theorem; for at K we have $m : n :: 13 : 12$, and $r = 3{,}308$, and $IQ - IK = 12{,}2818 = KQ = d$, the Diſtance of the Radiant Q from the Point K; but d is in this Caſe negative, or $-d$, and the Theorem is

$$\frac{-mdr}{-md + dn - nr} = f = 10{,}06$$

$= KP$, the new focal Diſtance from K, by the ſecond Refraction.

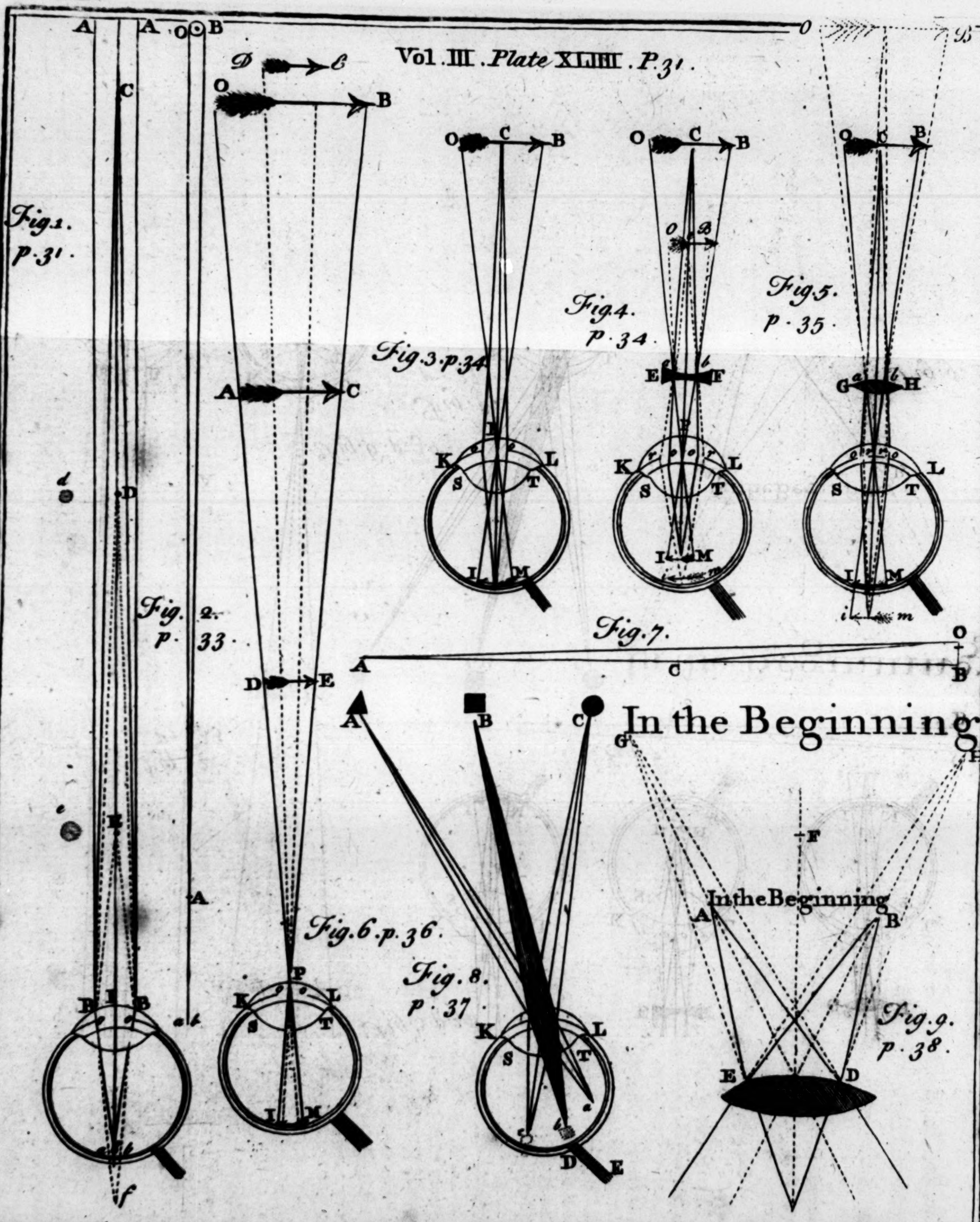
13. The Ray converging from S to P is intercepted by the hinder Surface of the Cryſtalline at T, and meeting there with a Medium of different Denſity, and a concave Surface, is again refracted by it; and here we have $m : n :: 12 : 13$, (by *Art.* 10.) alſo the Radius r is negative, as well as d; and here it is $-r = 2{,}5056$, and $-d = KP - KL = LP = 8{,}31$; therefore the Theorem is

$$\frac{mdr}{-md + nd + nr} = \frac{12dr}{d + 13r} = f = LM$$

$= 6{,}112$, the laſt focal Diſtance required.

14. The Point M therefore is that in which parallel Rays E D are collected within the Eye, and where the Images of remote Objects are formed. The Diſtance of this Point from the *Cornea* is $IM = IK + KL + LM = 1{,}036 + 1{,}852 + 6{,}112 = 9$. Then $IN - IM = 9{,}4 - 9 = 0{,}4 = NM$. Now the Thickneſs of the *Sclerotica* is by the Micrometer found to be very nearly $0{,}25$; then $0{,}4 - 0{,}25 = 0{,}15$; which is much about

Vol. III. Plate XLIII. P. 31.



Senſory in the Brain, where the Mind views and contemplates their Ideas; but this in a Manner

about equal to the Thickneſs of the *Choroides* and *Retina* together. *Hence we ſee the Forms and refractive Power of thoſe ſeveral Humours are ſuch as nicely converge parallel Rays to a Focus upon the Retina in the Bottom of the Eye.*

15. From hence it follows, that ſince parallel Rays only have their Focus upon the Retina, they alone can paint an Image there diſtinctly, or produce a diſtinct Viſion of an Object. If therefore the Object be ſo near, that the Rays from any particular Point come diverging to the Pupil, they will neceſſarily require a greater focal Diſtance than I M, and therefore, as the Rays are not united upon the Retina, that Point cannot be there diſtinctly repreſented, but will appear confuſed.

16. Thus if A B, A B, are two parallel Rays falling upon the Pupil of the Eye, then any other two Rays, as C B, C B, though really diverging, yet as the Point C, whence they proceed, is remote from the Eye, they will at the Entrance of the Eye be ſo nearly coincident with the parallel Rays, as to have nearly the ſame focal Point on the Retina; whence the Point C will there be diſtinctly repreſented by *c*. But if any other Point E be viewed very near the Eye, ſo that the Angles E B A which they contain with the parallel Rays be very conſiderable, they will after Refraction tend towards a Point *f* in the Axis of the Eye produced, and upon the Retina will repreſent only a circular indiſtinct Area like that at *e*, whoſe Breadth is equal to *a b*, the Diſtance of the Rays upon the Retina. The ſame Point at D will not be quite ſo much dilated and indiſtinct; the Rays D B, D B, having a leſs Degree of Divergence. Plate XLIV. Fig. 1.

17. It is found by Experience, that the neareſt Limit of diſtinct Viſion is about ſix Inches from the Eye; for if a Book be held nearer to the Eye than that, the Letters and Lines will immediately become confuſed and indiſtinct. Now this Cauſe of indiſtinct Viſion may be in ſome meaſure remedied by leſſening the Pupils, which we

a Manner too myſterious and abſtruſe for us to underſtand.

THE

we naturally do in looking at near Objects, by contracting the annular Fibres of the *Uvea*; and artificially, by looking through a ſmall Hole made with a Pin in a Card, &c. for then a ſmall Print may be read much nearer than otherwiſe: The Reaſon is plain; for the leſs the Diameter of the Aperture or Pupil B B, the leſs will the Rays diverge in coming from D or E, or the more nearly will they coincide with parallel Rays.

18. Beſides the Contraction of the Pupil, Nature has furniſhed the Eye with a Faculty of adapting the Conformation of the ſeveral Parts to the reſpective Poſitions of Objects as they are nigh or more remote. For this Purpoſe, the *Cornea* is of an elaſtic yielding Subſtance, and the Cryſtalline is incloſed with a little Water in its *Capſula*, that by the Contraction and Relaxation of the Ciliary Ligament, the Convexity of both the Surfaces of the *Capſula* may be a little alter'd, and perhaps the Poſition of the Cryſtalline, by which means the Diſtance from the Retina may be fitted and adjuſted to nigh Objects, ſo as to have their Images very diſtinctly formed upon the Retina.

19. I have mentioned near Objects only, (by which I mean ſuch as are near the Limit of diſtinct Viſion, as between ſix and a hundred Inches Diſtance) becauſe Objects more remote require ſcarce any Change of the Conformation of the Eye, the focal Diſtance in them varying ſo very little. Thus ſuppoſe all the Refractions of the Eye were equivalent to that of a double and equally convex Lens, whoſe Radius $r = 1$ Inch; if then the Object were 10 Inches diſtant, or $d = 10$, we ſhould have the focal Diſtance $f = \frac{d\,r}{d - r} = \frac{10}{9} = 0{,}11111$; and if another Object be diſtant 100 Inches, then $d = 100$, and $f = \frac{d\,r}{d - r} = \frac{100}{99} = 0{,}10101$. The Difference between theſe two focal Diſtances is but 0,0101, *viz.*

THE *Cryſtalline Humour* is of ſuch a Convexity, that in a ſound State of the Eye its Focus

viz. the hundredth Part of an Inch, which the Eye can eaſily provide for. If we go beyond this, ſuppoſe to an Object 1000 Inches diſtant, we have $f = \frac{dr}{d-r} = 0{,}1001001$, which is only a thouſandth Part of an Inch leſs than the former, and is therefore inconſiderable.

20. We have ſeen the natural Limit of diſtinct Viſion for *near Objects*; we ſhall now conſider what the Limit on the other hand may be for *remote Objects*; for Objects may appear indiſtinct and confuſed by being removed too far from the Eye, as well as when they are too near it. And in this Caſe we find Objects will appear diſtinct ſo long as their Parts are ſeparate and diſtinct in the Image form'd on the Retina. Thoſe Parts will be ſeparate ſo long as the Axis of the Pencils of Rays which paint them are ſo at their Incidence on the Retina; that is, ſo long as the Angle they contain is not leſs than *One-tenth of a Degree*; for it is found by Experience that Objects and their Parts become indiſtinct when the Angle they ſubtend at the Pupil of the Eye is leſs than that Quantity.

21. Thus ſuppoſe O B be a Circle $\frac{1}{10}$ of an Inch Diameter, it will appear diſtinct with its central Spot till you recede to the Diſtance of 6 Feet from it, and then it becomes confuſed; and if it be $\frac{1}{5}$ of an Inch, it will begin to be confuſed at 12 Feet Diſtance, and ſo on; in which Caſes the Angle ſubtended at the Eye, *viz.* O A B, is about $\frac{1}{10}$ of a Degree, or 6 Minutes. And thus all Objects, as they are bigger, appear diſtinct at a greater Diſtance; a ſmall Print will become confuſed, at a leſs Diſtance than a larger; and in a Map of *England* the Names of Places in ſmall Letters become firſt indiſtinct, where thoſe in Capitals are very plain and legible; at a bigger Diſtance theſe become confuſed, while the ſeveral Counties appear well defined to a much

Plate XLIV. Fig. 2.

Focus falls precisely on the *Retina*, and there paints the Objects; and therefore Vision

greater Distance. These also at last become so indistinct as not to be known one from another, when at the same time the whole Island preserves its Form very distinctly to a very great Distance; which may be so far increased, that it also at last will appear but a confused and unmeaning Spot.

22. We have seen the Causes of indistinct Vision in the *Objects*, and shall now enquire what may produce the same in the *Eye* itself. And first it is to be observed, that there is a proper Degree of Convexity in the *Cornea* KPL, and *Crystalline* ST, for converging parallel Rays to a Focus on the Bottom of the Eye in a sound State; hence every distant Object OB will have its Image IM accurately depicted on the *Retina*, and by that means produce distinct Vision.

Plate XLIV. Fig. 3.

23. But if the *Cornea* KPL, or Crystalline ST, or both, should chance to be a little more convex than just, it will cause the Pencil of Rays *o* C *o*, which comes to the Pupil *o o*, from any Point C in the Object O B, to unite in a Focus before they arrive at the Retina in the Bottom of the Eye; the Image IM of the Object OB will be formed in the Body of the Vitreous Humour, and will therefore be very confused and indistinct on the Retina at *i m*. A Person having such an Eye is called a *Myops*, in Allusion to the *Eye of a Mouse*, by reason of its great Convexity.

Fig. 4.

24. To remedy this Defect of the Eye, a concave Lens EF is applied before it; for by this means the Rays C *a*, C *b*, which fall diverging on the Lens, will, after Refraction through it, be made to proceed still more diverging, *viz.* in the Direction *a r*, *b r*, (instead of *a o*, *b o*, as before) as if they came from the Point *C* instead of C. All which is plain from the Nature of a concave Lens above described.

25. Hence it follows, that since the Rays are made to fall with greater Divergence upon the Eye, they will require

ſion is not diſtinct, unleſs by Rays which are parallel, or nearly ſo; for thoſe only will

require a greater focal Diſtance to be united in the Axis, and conſequently the Focus may be made to fall very nicely on the Retina, by uſing a Lens E F of a proper Degree of Concavity; and therefore diſtinct Viſion will be effected in the ſame Manner as in an Eye of a juſt Conformation, by painting the Image on the Retina.

26. Since the Point *C* is nearer to the Eye than the Point C, the *apparent* Place of Objects ſeen through a concave Lens is nearer than the *true* Place; or the Object will appear at *O B*, inſtead of O B. And alſo ſince converging Rays O *a*, B *b*, proceed leſs converging after Refraction than before, the Object appears under a leſs Angle, and therefore the apparent Magnitude of Objects ſeen by a concave Lens is leſs than the true.

27. The Object is leſs luminous or bright ſeen thro' ſuch a Lens than without it; becauſe the Rays being rendered more divergent, a leſs Quantity enters the Pupil of the Eye than otherwiſe would do. But the Picture is always more or leſs bright or enlightened, according as it is made by a greater or leſs Quantity of Rays.

28. Laſtly, it appears from what has been ſaid, that when a concave Lens E F cannot be applied, we may ſtill effect diſtinct Viſion by leſſening the Diſtance between the Object and the Eye; for it is plain, if O B be ſituated at *O B*, the Image at I M will recede to *i m* upon the Retina, and be diſtinct in the ſame Manner as when made ſo by the Lens E F. Plate XLIV. Fig. 5.

29. On the other hand, when the *Cornea* or Cryſtalline is too flat (as often happens by Age), an Object O B, placed at the ſame Diſtance from the Eye P C as before, will have the Rays C *o*, C *o*, after Refraction in the Eye proceed to a Focus beyond the Bottom of the Eye, in which if a Hole were made (in an Eye taken out of the Head) the Rays would actually go on, and form the Image *i m*; which Image muſt therefore be very confuſed and indiſtinct on the Retina.

 30. To

will have their Focus at the Bottom of the Eye: Now Rays proceeding from any Point more

30. To remedy this Defect, a convex Lens G H is applied, which causes the diverging Rays C *a*, C *b*, to fall less diverging upon the Eye, or as if they came from a Point more remote, as *C*; by which means the focal Distance is shortened, and the Image duly formed on the Retina at I M, by which distinct Vision is produced.

31. Hence the apparent Place of the Object is at *C*, more distant than the true Place at C; and its apparent Magnitude *O B* is greater than the true, because the converging Rays O *a*, B *b*, are by this Lens after Refraction made to unite sooner than before, and so to contain an Angle O P *B* greater than the true O P B. The Object appears through a convex Lens brighter than without, because by this means a greater Quantity of Rays enter the Pupil; for the Rays *a o*, *b o*, are by the Lens made to enter in the Directions *a r*, *b r*, which are nearer together, and leave Room for more to enter the Pupil all around between *o* and *r*.

Plate XLIV. Fig. 6.

32. As the Image of the Object painted on the Retina is greater or less, so will the apparent Magnitude of the Object be likewise; or, in other Words, the Angle I P M subtended by the Image is always equal to the Angle O P B subtended by the Object at the Eye, and therefore the Image I M will be always proportional to the Object O B. Hence it follows, that the Angle O P B under which an Object appears is the Measure of its apparent Magnitude.

33. Therefore Objects of different Magnitudes, as O B, A C, D E, which subtend the same Angle at the Eye, have the same apparent Magnitude, or form an equal Image in the Bottom of the Eye. Hence it is that Objects at a great Distance have their Magnitude diminished proportionally. Thus the Object D E removed to *D E* appears under a less Angle *D* P *E*, and makes a less Image on the Retina, as is shewn by the dotted Lines.

34. The

more than 6 Inches diſtant from the Eye, will, when they enter the Pupil, be very nearly

34. The Angles of apparent Magnitude O A B, O C B, when very ſmall, are as their Sines, and therefore as the Sides O C and O A, or B C and B A; that is, the apparent Magnitude of the Object O B, at the Diſtance B C and B A, is inverſely as thoſe Diſtances; or its Magnitude at C is to that at A as A B to C B.

35. The more directly any Object is ſituated before the Eye, the more diſtinctly it will appear; becauſe thoſe Rays only which fall upon the Eye near its Axis can be convened to a Point in the Bottom of the Eye on the Retina, and therefore that Part of the Image only which is formed by the direct Pencil of Rays can be clear and diſtinct; and we are ſaid to *ſee* an Object by ſuch a Pencil of Rays, but only to *look at it* by the others which are oblique.

36. Suppoſe A, B, C, repreſent three Pieces of Paper ſtuck up againſt the Wainſcot of a Room at the Height of the Eye; if then a Perſon places himſelf ſo before them, and ſhutting his Right Eye views them with his Left, it is very remarkable that the Paper B, whoſe Pencil of Rays fall upon the Inſertion D of the Optic Nerve D E, will immediately vaniſh or diſappear, while the two extreme Papers C and A are viſible; and by altering the Poſition of the Eye, and its Diſtance, any of the Papers may be made to vaniſh, by cauſing the Pencil of Rays to fall on the Point D. Plate XLIV. Fig. 8.

37. Why the Rays of Light ſhould not excite the Senſation of Viſion in that Point D where the Fibres of the Nerves begin to ſeparate and expand every way to form the Retina, is not known. But 'tis highly worth our notice, that the Nerve D E is for that Reaſon placed *on one Side of the Eye*, where only the oblique Rays come, the Loſs of which is not conſiderable, and no way affects or hinders the Perfection of Sight. Whereas had it entered in the Middle of the Bottom of the Eye, it had render'd uſeleſs all the direct Rays, by

nearly co-incident with parallel Rays; and therefore to a ſound Eye diſtinct Viſion cannot

by which the moſt perfect and diſtinct Viſion is effected; and we could have had only a confuſed and imperfect Perception of Objects by oblique collateral Rays. How glaring an Inſtance is this of Contrivance and Deſign in the Conſtruction of this admirable Organ!

38. I ſhall conclude this Head with obſerving, that the Nature of a *Reading-Glaſs* is the ſame with that of common *Spectacles*; only in the latter Caſe we uſe a Lens to each Eye, but in the former one Lens is made large enough for both. Alſo in the Uſe of them we have different Ends to anſwer; for by the Spectacles we only propoſe to render Objects diſtinct at a given Diſtance, but the Reading-Glaſs is applied to magnify the Object, or to render the reading of a ſmall Print very eaſy, which otherwiſe would be apt to ſtrain the Eye too much. Therefore the Size of a Lens for Spectacles is not required larger than the Eye; but that of a Reading-Glaſs ought to be big enough to take in as large a Part of the Object, at leaſt, as is equal to the Diſtance between both the Eyes. I ſhall treat of the Viſual Glaſſes in the Appendix.

39. In the Reading-Glaſs E C D the Object or Print Plate XLIV. Fig. 9. A B is always nearer to the Glaſs than its Focus F; becauſe in this Caſe it is neceſſary the Image or magnified Print G H ſhould be erect, and on the ſame Side of the Glaſs with the Object; that is, the Diſtance d is negative in the Equation $\frac{dr}{d-r}=f$. Hence the Pencil of Rays A E D, proceeding from any Point A, will after Refraction through the Lens be divergent, but leſs ſo than before, and therefore will ſeem to come from a Point G. Thus alſo the Point B will be referr'd to H, and the Print at G H will be magnified in Proportion of G C to A C. All which is evident from the latter Part of the laſt *Annotation*.

cannot be effected at less than 6 or 8 Inches Distance, as is evident to any who tries the Experiment.

SINCE then there is a certain and determinate Degree of Convexity in the *Cornea* and *Cryſtalline Humour*, for forming the Images of Objects on the *Retina*; if it happens that the Convexity of thoſe Parts ſhould be more or leſs than juſt, the Focus of Rays will fall ſhort of, or beyond the *Retina*, and in either Caſe will cauſe indiſtinct Viſion. The firſt is the Caſe of ſhort-ſighted or *purblind* People, the latter of the *Aged*.

A *purblind Perſon*, having the Convexity of the Eye and Cryſtalline Humour too great, will have the Rays united in a Point before they reach the Bottom of the Eye, and conſequently the Images of Objects will be formed, not upon the *Retina*, (as they ſhould be) but above it in the Glaſſy Humour, and therefore will appear indiſtinct or confuſed.

THIS Defect of the Eye is remedied two Ways, *viz.* (1.) By diminiſhing the Diſtance between the Object and the Eye; for by leſſening the Diſtance of the Object, the Diſtance of the Focus and Image will be increaſed, till it falls on the *Retina*, and

appears diſtinct. (2.) By applying a concave Glaſs to the Eye; for ſuch a Glaſs makes the Rays paſs more diverging to the Eye, in which Caſe the Diſtance of the Focus will be alſo enlarged, and thrown upon the *Retina*, where diſtinct Viſion will enſue.

HENCE the Uſe of *Concave Spectacles:* And the *Myops*, or purblind Perſon, who uſes them, has the three following Peculiarities, *viz.* (1.) To him Objects appear nearer than they really are, or do appear to a ſound Eye. (2.) The Objects appear leſs bright, or more obſcure to them than to other People, becauſe a leſs Quantity of Rays of Light enter the Pupil. (3.) Their Eyes grow better with Age; for whereas the Fault is too great a Convexity of the Eye, the *aqueous Humour*, and alſo the *Cryſtalline*, waſting with Age will grow flatter, and therefore more fit to view diſtant Objects.

THE other Defect of the Eyes ariſes from a quite contrary Cauſe, *viz.* the *Cornea* and *Cryſtalline Humour* being too flat, as is generally the Caſe of an old Eye. This Defect is remedied by *Convex Lenſes*, ſuch as are the common *Spectacles*, and *Reading-Glaſſes*. For ſince the Rays, in theſe Eyes,

go

go beyond the Bottom of the Eye, before they come to a Focus, or form the Image; a convex Glaſs will make the Rays fall more converging to the Pupil, and on the Humours, by which means the focal Diſtance will be ſhorten'd, and adjuſted to the *Retina*; where diſtinct Viſion of Objects will then be effected.

By convex Spectacles Objects appear *more bright*, becauſe they collect a greater Quantity of Rays on the Pupil. And they appear at a greater Diſtance than they are; for the nearer the Rays approach to parallel ones, the more diſtant the Point will be to which they tend.

I have already obſerved, that if the Object be placed nigher to the convex Glaſs than its Focus, it will appear erect and magnified; which makes them of ſuch general Uſe as *Reading-Glaſſes*.

If an Object be placed in the Focus of a convex Lens, the Rays which proceed from it, after they have paſs'd through the Glaſs, will proceed parallel; and therefore an Eye placed any where in the Axis will have the moſt diſtinct View of the Object poſſible; and if it be a Lens of a ſmall focal Diſtance, then will the Object appear as much larger as it is nearer, than when

you

you view it with the naked Eye. And hence their Uſe as *Single Microſcopes*: To give an Inſtance of which, ſuppoſe the focal Diſtance of a Lens were One-tenth of an Inch, then will the Diameter or Length of an Object appear 60 times larger than to the naked Eye at 6 Inches Diſtance: Alſo the Superficies of an Object will be 3600 times larger; and the whole Magnitude or Bulk will be 216000 times larger than to the naked Eye it will appear at the abovefaid Diſtance (CXXVII).

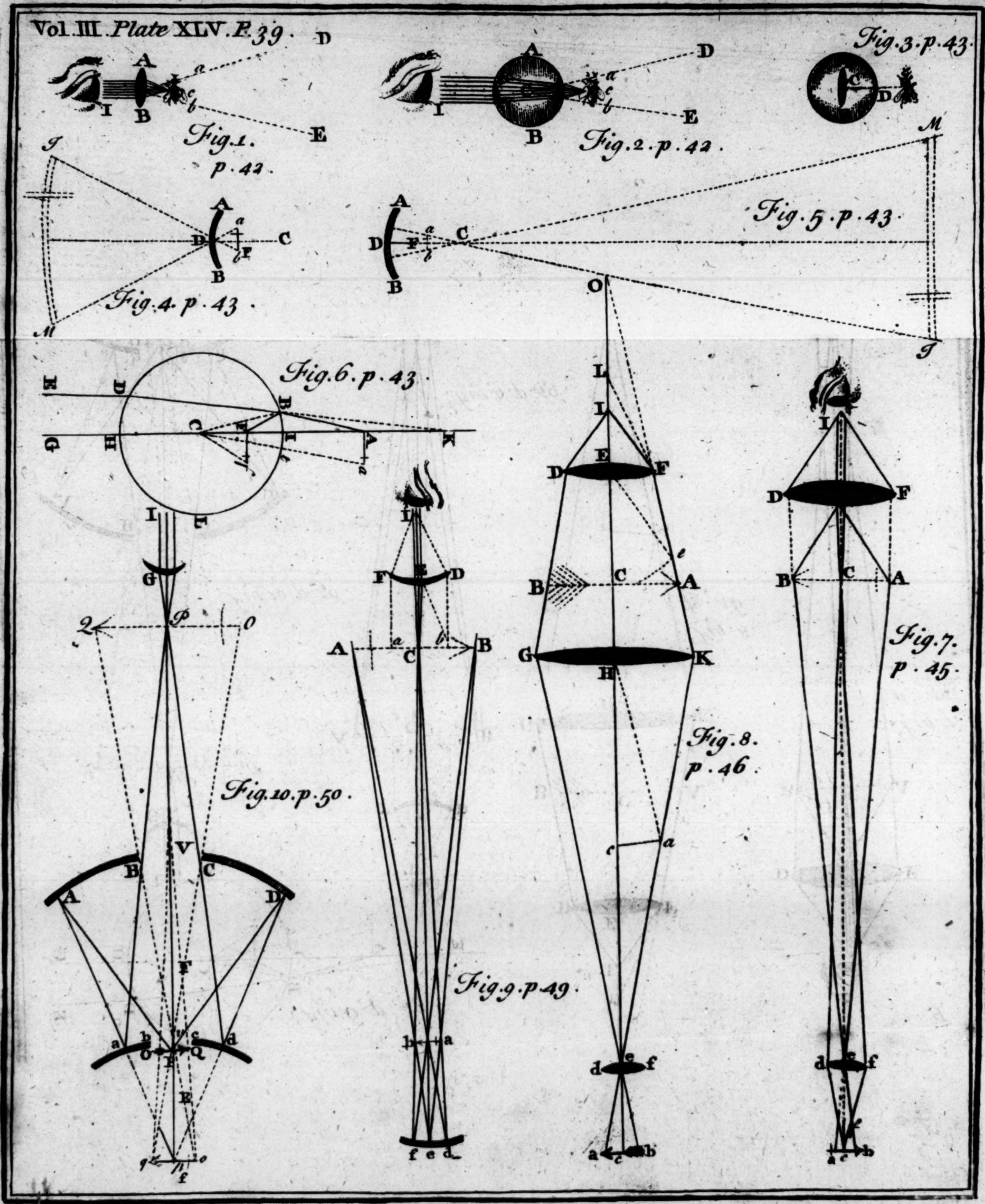
COMPOUND

(CXXVII.) 1. I ſhall here give a ſuccinct Account of every Sort of Microſcope, with reſpect to their Nature and Theory. MICROSCOPES are diſtinguiſhable into two Kinds, *viz.* *Dioptric* by Refraction, and *Catoptric* by Reflection; and each of theſe is either *Single*, as conſiſting of one Glaſs only, or *Compounded* of two or more.

Plate XLV. Fig. 1. 2. A SINGLE MICROSCOPE, of the Dioptric or Refracting Sort, is either a *Lens* or a *Spherule*. Thus if any Object *a b* be placed in the Focus *c* of a ſmall Lens A C B, the Rays proceeding from thence will after Refraction go parallel to the Eye at I, and produce diſtinct Viſion; and the Object will be magnified in the Proportion of ſix Inches to the focal Diſtance C *c*, according to the Example above.

Fig. 2. 3. Again; if an Object *a b* be applied to the Focus *c* of a Spherule A B, it will produce diſtinct Viſion thereof by means of parallel Rays (by *Annotat.* CXXVI. *Art.* 15.) and it will appear under an Angle equal to D C E, and be magnified in Proportion of 6 Inches to the focal Diſtance C *c* from the Centre. And here it is remarkable, that if the focal Diſtance of the Lens and Spherule be the ſame, the Object will be three times farther diſtant

Vol. III. Plate XLV. P. 39.



Vol III. *Plate* XLVI. P. 43.

The POCKET-MICROSCOPE

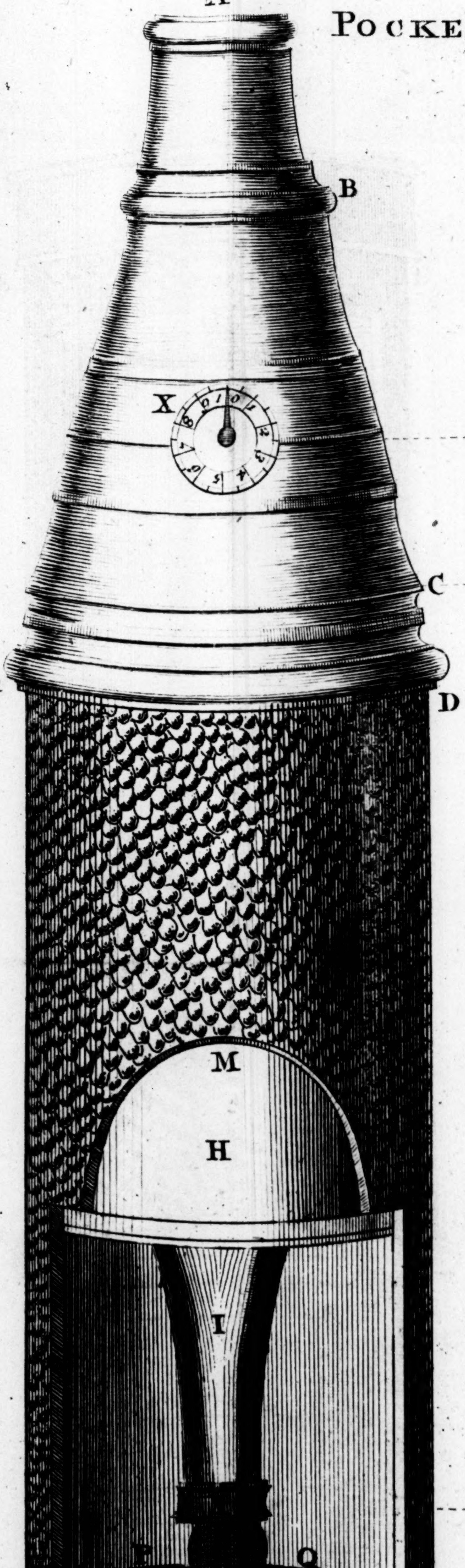


Fig. I. *p*. 44

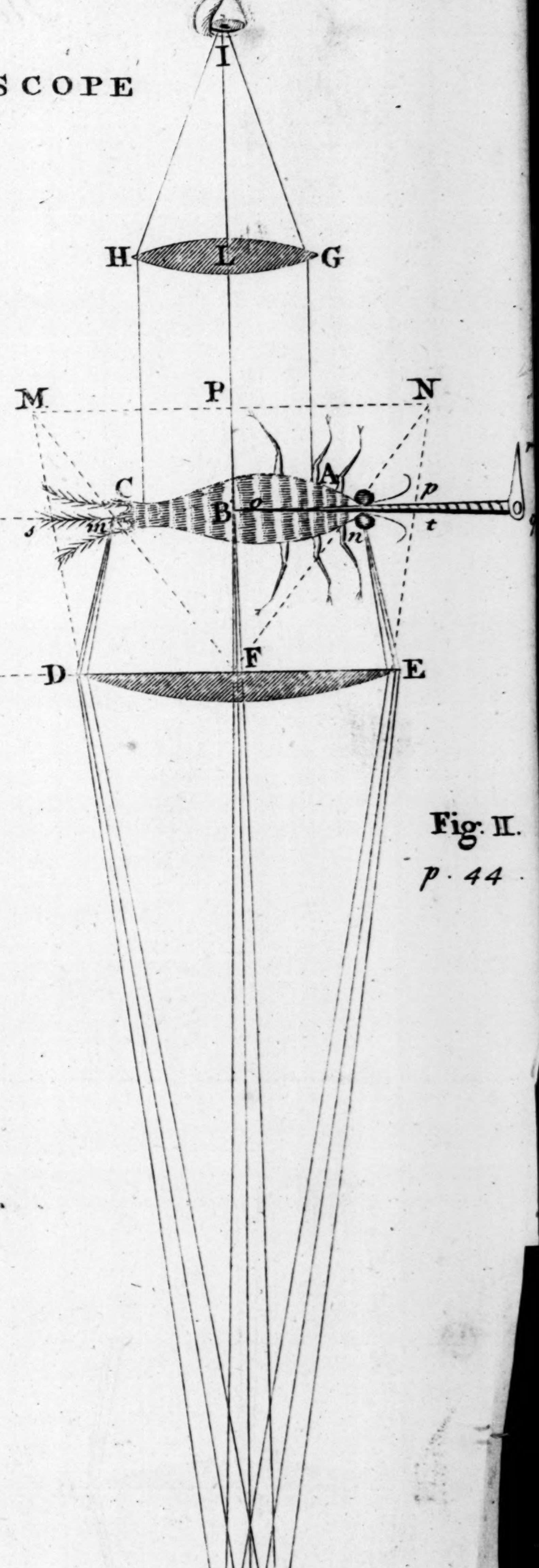


Fig. II. *p*. 44.

COMPOUND Microſcopes, eſpecially the common Sort, are conſtructed with three Glaſſes,

diſtant from the Lens than from the Spherule, becauſe CD the Semidiameter of the Sphere is $\frac{2}{3}$ of C *c*, the Diſtance of the Lens AB; conſequently an Object is viewed by a Lens to a much greater Advantage than by a Sphere, in regard to the Light, *&c*. Plate XLV. Fig. 3.

4. A Single Microſcope of the Catoptric Kind is a ſmall concave Mirrour as ADB, having the Object *a b* placed before it, nearer to the Vertex than its Focus F. In that Caſe the Image *I M* will be formed behind the Speculum, very large, erect, and diſtinct, as has been already ſhewn. Such a Speculum is of admirable Uſe to view the Eye; for being turned to the Light, the *Uvea* or *Iris*, the *Pupil*, the *Cornea*, and all the viſible Part of the *Tunica Albuginea*, with the fine Ramifications of the Blood-Veſſels, and Part of the *Glandula Lachrymalis*, are all by this means greatly magnified, and rendered curious Subjects of our Sight. Fig. 4.

5. Alſo if the Object *a b* be placed any where between the Centre C and Focus F of the ſaid ſmall Speculum A B, then will its Image *I M* be formed at a great Diſtance from the Glaſs, and may be made to bear any aſſign'd Proportion to the Object by placing the Object nearer to or farther frrom the Focus F: But for common Objects the Room ought to be dark, or the Object extremely lucid, as a Candle, *&c*. But more of this when I come to ſpeak of the *Solar Microſcope*. Fig. 5.

6. The next Sort of Single Microſcope is a Catadioptric one, which performs its Effect by Reflection and Refraction; the Theory of which being curious, I ſhall give the Reader as follows. D B L H is a Globule of Water; and it was ſhewn that an Object A *a*, placed in its Focus A, would be ſeen diſtinct and magnified by refracted Rays A B D E. (See *Art*. 3.) Now 'tis evident we may conſider the Ray B D either as the refracted Ray of A B, or the reflected Ray of F B, the Angle C B F being equal to the Angle C B D; and ſince Fig. 6.

Plate XLVI. Fig. 1, 2. Glasses, *viz.* the Object-Lens *de*, and two Eye-Glasses DE and GH. The Object *a b c*

since in each Case the Ray BD is at D refracted into DE parallel to the Axis GK, it follows, that distinct Vision will be produced of an Object F *f* placed in the Focus by Reflection F within the Drop, as the Object A *a* in its Focus A by Refraction without it.

7. In order to determine the focal Distance IF by Reflection from the Concave BI *i*, for converging Rays D *b*, we have $HK = 4 HC$, or $4 IC$, (by *Annotat.* CXXV.) whence $IK = 2 IC$, that is $d = 2r$, in the Theorem $\frac{dr}{2d - r} = f$ (in the same *Annotation.*) Also because the reflecting Surface is here concave, and the Rays converging, the Theorem will become $\frac{dr}{-2d - r} = f = \frac{2rr}{-5r} = \frac{2}{5} r = IF$; whence $CF = \frac{3}{5} r$. Also it has been shewn that $AI = IC$, or $CA = 2r$; and therefore $CF : CA :: \frac{3}{5} : 2 :: 3 : 10 :: 1 : 3\frac{1}{3}$.

8. And since the same Object will appear as much larger at F than at A, as the Angle FC *f* is greater than AC *a*, or the Distance CA greater than CF, it follows that an Object in a Globule of Water seen by Reflection is magnified $3\frac{1}{3}$ times more than it would be in the Focus A by Refraction. Suppose then $CA = \frac{1}{5}$ of an Inch, then will $CF = \frac{3}{50}$ of an Inch; and therefore, since $6 : \frac{3}{50} :: 100 : 1$, it appears that the Diameter of an Object at the Focus F is seen 100 times larger than at the Distance of 6 Inches from the Eye.

9. In a Glass Globule, $HK = 3r$, $IK = 1,5\,r = d$; and the Theorem $\frac{dr}{-2d - r} = \frac{1,5\,rr}{-4r} = \frac{3}{8} r = IF$; whence $CF = \frac{5}{8} r$. And because $IA = \frac{1}{2} r$, we have $CA = \frac{3}{2} r$; consequently, $CA : CF :: \frac{3}{2} : \frac{5}{8} :: 2\frac{1}{5} : 1$; that is, an Object is magnified $2\frac{1}{5}$ times more at the Focus F within, than at the Focus A without a Glass Globe:

a b c being placed at a little more than the Focal Distance from the Lens *d e* will have its

Globe: And hence it appears, that equal Globes of Glass and Water magnify by Reflection in Proportion of $2\frac{1}{5}$ to $3\frac{1}{3}$. Also because CF $= \frac{3}{5}$ in Water, and CA $= \frac{3}{2}$ when the same Globe is Glass, it appears that Objects are magnified in the Water Globule more than when seen through the Glass Globule, in the Proportion of $\frac{3}{2}$ to $\frac{3}{5}$, or $2\frac{1}{2}$ to 1.

10. A DOUBLE MICROSCOPE is composed of two convex Lenses, *viz.* an *Object* and an *Ocular* Lens. The Object Lens is d f, placed a little farther distant from the Object a b, than its focal Distance e *f*; because then its Image A B will be form'd at the required Distance e C; and as e *c* : e C :: a b : A B. If this Image A B be view'd by a Lens D F placed at its focal Distance from it, it will appear distinct, because the Rays will thence go parallel to the Eye at I. Plate XLV. Fig. 7.

11. Now the Object a b and the Image A B appear under equal Angles from the Vertex e of the Lens d f; that is, the Angle a e b = A e B. But the Angle under which the Image is seen from the Vertex E of the Eye-Glass D E is B E A = D I F; therefore the Image view'd by the Eye-Glass is to the Object view'd by the Object Lens, as the Angle B E A to the Angle B e A, or as C e to C E nearly. Suppose C E = 1 Inch, and C e = 4 Inches, then will the Object be magnified 4 times in Diameter by the Means of the Eye-Glass D F; but it is magnified by the Lens d f in Proportion of 6 Inches to e *c*, that is, if e *c* = 1 Inch, six times. Wherefore by both the Glasses the Diameter is magnified $4 \times 6 = 24$ times.

12. But in this Case of a single Eye-Glass, the *visible Area* (call'd the *Field of View*) though larger than in the Lens d e alone, yet is not so large as it may be render'd by the Addition of a second Eye-Glass, as G K; for by the Lens D F alone the visible Area is equal to the Aperture of the said Lens, because no more of an Object or its Image can be seen than what is

its Image form'd at a greater Diſtance on the other Side, and proportionably large, as

is contain'd between the Rays D d and F f parallel to the Axis; that is, the viſible Part of the Image is df = D F, and is ſeen under the Angle D L F (L being the Focus of the Lens D F, or L E = C E) and this is call'd the *Optic Angle*, or Angle of viſible Magnitude.

Plate XLV. Fig. 8.

13. But this Angle is greatly increaſed by the Addition of the Glaſs G K; for let a K and b G be the Axes of two Pencils of Rays which come from the extreme Parts of the Object a and b, and falling diverging upon the Lens G K are refracted by it towards the Point O in the Axis of the Glaſs produced. But in their Paſſage they are intercepted by the Lens D F, which refracts them to a Point I nearer the Lens than the Focus L; and therefore the Angle D I F is greater than the Angle D L F, and conſequently a larger Field of View is by this means obtain'd.

14. To find in what Proportion the Angle D I F is greater than D L F, we muſt conſider e as a Radiant Point, whence the Pencil of Rays G e K diverges to the Lens G K; then if H e = d, H a = r Radius of the double and equally convex Lens G K, we have $\frac{dr}{d-r} = y =$ H O the Focus after Refraction; and if H E = p, then $y - p$ = E O = d = Diſtance for converging Rays K F and G D. And let r = Radius of the double and equally convex Lens D F; then $\frac{\mathrm{d\,r}}{\mathrm{d+r}} = \mathrm{f} = \mathrm{EI} = \frac{y\mathrm{r} - p\mathrm{r}}{y - p \times \mathrm{r}}$. Wherefore L E : I E :: r:

$$\frac{y\mathrm{r} - p\mathrm{r}}{y - p + \mathrm{r}} :: 1 : \frac{y - p}{y - p + \mathrm{r}} = \frac{\frac{dr}{d-r} - p}{\frac{dr}{d-r} - p + \mathrm{r}}.$$

Therefore when the ſeveral Quantities d, r, p, r, are given, the Ratio

as at M N; which large Image is contracted into one A B C somewhat less, by the lower

Ratio of the Angle D L F to that of the amplified Angle D I F is given also.

15. For Example; let $He = d = 5$ Inches, $Hc = r = 3,5$, $p = HE = 2$, and $r = EC = LE = 1,5$; then $y = 11,33$; therefore $\frac{y-p}{y-p+r} = \frac{9,33}{10,83} = 0,86 = IE$. Therefore the Angle DLF : DIF :: IE : LE :: 0,86 : 1,5. Therefore the visible Area by the single Eye-Glass D F will be to the amplified Area by both the Lenses G K and D F, as the Square of 0,86 to the Square of 1,5; that is, as 0,7396 to 2,25, or as 7396 to 22500; which Augmentation in the Ratio of 3 to 1 nearly is very considerable, and affords great Advantage and Pleasure in viewing Objects.

16. The magnifying Power of a Compound Microscope is thus computed: The Angle under which the Object a b appears at the Distance of 6 Inches, is to that under which it appears when seen through the Lens d f, as e c to 6; wherefore the first Part of the compound Ratio of the magnifying Power will be $\frac{6}{ec}$.

17. Let Hc be the focal Distance of the Lens G K; with the Radius Hc describe the Arch ca, and draw aH; this will be the Axis of all the Rays which go from the Point a to the Lens G K; consequently, the Ray aK will after Refraction be parallel to the Axis, *i.e.* the Ray K O is parallel to aH; therefore the Image of the Object being in the Focus c of the Lens G K, will be seen under the Angle K O H, which is equal to the Angle aHc; but it is seen from the Lens d f under the Angle aec. But the Angle aHc : aec :: ec : cH. Wherefore the second Part of the Ratio for magnifying is that of ec to cH, or $\frac{ec}{cH}$.

18. Lastly; let C be the Focus of the Lens D F, and with the Radius E C describe the Arch C e; then will

lower Eye-Glaſs D E; and this Image is viewed by the Eye through the upper Eye-Glaſs

will e E be the Axis of the Pencil of Rays proceeding from the Point e to the Lens D F, of which e F being one, it will be refracted into F I parallel to e E; and ſo the Angle F I E = C E e. But F I E is the Angle under which the Image is viewed through the Lens D F, which is to the Angle C O A as C O to C E. Therefore the third and laſt Part of the Ratio for magnifying is $\frac{CO}{CE}$.

19. If now we compound the ſeveral Parts of the Ratio now found into one, it will make the Ratio of $\frac{CO}{CE} \times \frac{ec}{cH} \times \frac{6}{ec}$ to 1. For Example, let e c = $\frac{1}{2}$ an Inch, c H = $3\frac{1}{2}$, e c = 2, C E = $1\frac{1}{2}$; then H E being 2, and H O = 11,66, we have C O = 10,66; whence the above Ratio in Numbers will be $\frac{10{,}66}{1{,}5} \times \frac{2}{3{,}5} \times \frac{6}{0{,}5} = 40{,}87$. Therefore the Diameter of any Object is magnified near 41 times by ſuch a Compound Microſcope.

20. If this Calculation be enquired into, we ſhall find that the Glaſs G K diminiſhes the magnifying Power, which is greater by the Eye-Glaſs D F alone, and more diſtinct. Thus in Fig. 11. of the large Plate XLVI. if the lower Glaſs D E were taken away, the Rays would go on and be united in a Focus at the Points M, P, N, and there form an Image of the Length M N; but by replacing the Glaſs D E we ſhall have the large Image M N contracted into a leſſer $m\,n$. Now this larger Image M N may be conſidered as formed by the Lens D E at a negative Focus from an Object $m\,n$, whoſe Diſtance F B is leſs than the focal Diſtance of the ſaid Lens: All which is eaſy to underſtand from the foregoing Theory of *Dioptrics*.

Glaſs GH; where it alſo diſtinctly views the MICROMETER $o\,p\,q$, paſſing over a minute

21. Now $a\,c$: M N :: $b\,f$: fP; and drawing M F and N F, we have M N : $m\,n$ :: F P : F B, becauſe the Object and its Image do in every Caſe ſubtend the ſame Angle from the Vertex of the Lens, as was ſhewn before. Since F P is given, ſo alſo is F B, from the common Theorem $\frac{dr}{d-r}=f$, for a double and equally convex Lens; or $\frac{2dr}{d-2r}=f$, for a Plano-Convex one. For ſince the Focus f is negative, or $\frac{dr}{d-r}=-f=$ F P, therefore $dr=-df+rf$, and ſo $dr+df=rf$; therefore $\frac{rf}{r+f}=d=$ F B; and $\frac{fr}{2r+f}=d$, in a Plano-Convex.

22. It is evident from the Scheme, that no more of the large Image M N, or of the contracted one $m\,n$ can be viewed through the Eye-Glaſs H G, than what is contained between the perpendicular Lines H C and G A; and that therefore a much greater Part of the Object can be ſeen in the Image $m\,n$, than in the Image M N, which is wholly owing to its being contracted by the large Lens D E; and this is all the Reaſon of its Uſe.

23. The next Sort of Microſcope I ſhall take notice Plate
of is a *Catadioptric* one, *i. e.* ſuch an one as performs its XLV.
Effects by *Reflection and Refraction jointly*; for it is con- Fig. 9.
ſtructed with a ſmall Object Speculum f e d, whoſe Focus is at F; and it has been ſhewn, that if a ſmall Object a b be placed a little farther from the Speculum than the Focus f, there will be formed a large Image thereof A B; which Image will be inverted, and in Proportion to the Object as the Diſtance C e to the Diſtance e e, as when an Object Lens was uſed.

 24. Part

nute Part of the Image in meaſuring it. But what is farther neceſſary in the Theory and

24. Part of this Image is viewed by an Eye-Glaſs F D, which is or ought to be a *Meniſcus*, as here repreſented; becauſe the Image being formed by Reflection, it will be more perfect, and admit of a deeper Charge in the Eye-Glaſs D F; and thoſe of the *Meniſcus* Form are beſt for this Purpoſe, becauſe the Errors of the Rays, and conſequently the Confuſion cauſed thereby, in the Refraction made at the convex Surface, are in a great meaſure rectified by the contrary Refraction at the concave Surface, as is eaſy to underſtand from what has been ſaid of refracted Light, *Annot.* CXVII.

Plate XLV. Fig. 10. 25. Another Sort of *Catoptric* or *Reflecting* Microſcope is conſtructed with two Speculums, a b c d and A B C D, with a central Hole in each. The large Speculum is concave, the other convex, and both of equal Sphericity. They have their Focus at one Inch Diſtance, and placed at the Diſtance of $1\frac{1}{2}$ Inch from each other, that ſo an Object O P Q, being placed a little before the ſmall Speculum, might be nearer to the large one than its Centre E.

26. This being the Caſe, the Rays, P A, P D, which flow from the Point P to the Speculum A D, will be reflected towards a Focus *p*, where an Image *o p q* would be formed, if the Rays were not intercepted by the convex Speculum a b; and the Point *p* being nearer than its Focus f, the Rays A a, D d, which tend towards it, will be reflected to a Focus *P*, where the laſt Image *O P Q* will be formed, to be viewed by the Eye-Glaſs G, tranſmitting parallel Rays to the Eye at I.

27. The Power of magnifying in this Microſcope is thus eſtimated. (1.) The Object O P ſeen from the Vertex V of the Speculum A D is to the ſame ſeen at the Diſtance of 6 Inches from the naked Eye as 6 to V P, or as $\frac{6}{VP}$. (2.) The firſt Image *o p q*, (to be conſidered now as a *virtual Object*) ſeen from the Vertex V of

and Structure of these Microscopes may be found in my *Micrographia Nova*, together with

of the Mirrour A D, is to the same seen from the Vertex v of the Mirrour a d as $v\,p$ to $V\,p$, or as $\frac{V\,p}{v\,p}$.

(3.) Lastly, the Image $O\,P\,Q$, seen from the Vertex v of the Speculum a d, is to the same seen through the Eye-Glass G, as G P to $P\,v$, or as $\frac{P\,v}{G\,P}$. Where the whole magnifying Power is as $\frac{6}{V\,P} \times \frac{V\,p}{v\,p} \times \frac{P\,V}{G\,P}$ to 1. This Contrivance we owe to Dr. *Smith* of *Cambridge*.

28. But a better Form and easier Method of constructing a Catoptric Microscope, with *two reflecting Mirrours*, is that which follows. A B C D E F is a Case or Tube, in one End of which is placed a concave Speculum G H, with a Hole I K in the Middle; the Centre of this Speculum is at c, and its Focus at O, so that V O = O c. At the open End of the Tube is placed a small convex Speculum d e f, on a Foot e f, by which it is moveable nearer to or farther from the larger Speculum, G H, as Occasion requires. Plate XLVII. Fig. 10.

29. If now an Object a b be posited in the Centre c of the large Speculum, the Image thereof *a b* will be formed in the same Place, as has been shewn already; and this Consideration is all the Reason of this Form of a Microscope; for, if now we look upon the Image *a b* as an Object nearer to the convex Speculum d f than its Focus *f*, 'tis plain a larger Image *A B* will be formed thereby at the Focus *C*; or that Rays *c* G, *c* H, proceeding from any Point c in the Object a b, will be reflected back upon themselves, as being perpendicular to the Speculum; but the refracted Rays meeting with, or impinging on, the convex Surface of the Speculum d f, will (as they tend to a Point *c*, nearer than the Focus *f*) be reflected to a Focus *C*, which is found by the Theorem $\frac{d\,r}{r + 2\,d} = f$ (*Annot.* CXXV.).

with a large Account of all Kinds of *Microscopic Objects*, and a Description of the *Solar*

30. For in this Case $f = ec$, and $d = eC$; and since $dr = rf + 2df$, we have $\frac{rf}{r-2f} = d$. Thus if we put the Radius of the small Speculum $r = 2$ Inches, then $ef = 1$, and let $ec = f = 0{,}8$; then $\frac{rf}{r-2f} = \frac{1{,}6}{2-1{,}6} = \frac{1{,}6}{0{,}4} = 4$ Inches $= eC$; and $ab : AB :: 0{,}8 : 4 :: 1 : 5$, or the Image AB will be 5 times longer than the Object a b. This Image AB is viewed by the *Meniscus* Eye-Glass L M, whence 'tis easy to observe that this Form of a Microscope is the same with that in *Article* 23, 24. only there is but one Reflection, and here is two; and there a small Concave was used, but here a Convex; because by this means the Instrument is shorter by twice the focal Distance ef nearly, which is very considerable, as being $\frac{1}{3}$ Part of the Whole.

31. I shall shew in the next *Annotation* how both these Microscopes may be had very conveniently in the reflecting Telescope, and conclude this with an Account of the Nature and Use of the Micrometer for measuring the smallest Parts of natural Bodies; and here I shall not take notice of the several uncertain conjectural Methods described by others, but only such as I use in my own Microscopes, which is strictly Mechanical, and gives the Measurement absolutely.

Plate XLVI. Fig. 1, 2. 32. The MICROMETER consists of a graduated circular Plate X, of a Screw $q\,o$, and its Index $q\,r$. The Threads of the Screw are such, that 50 make the Length of one Inch exactly. When it is to be used, the Point o is set to the Side of the Part to be measured, and then the Index is turned about with the Finger, till the Eye perceives the Point has just passed over the Diameter of the Part; then the Number of Turns, and Parts of a Turn, shewn by the graduated Circle, will give the Dimensions

Solar Microſcope. Yet, that the Reader may have an Idea of the *two new Forms* of Microſcopes deſcribed in that Book, I have here annexed the Plates, *viz.* of the POCKET and UNIVERSAL MICROSCOPE reſpectively.

THE TELESCOPE is of two Sorts, *viz. Dioptric*, or *Refracting*; or *Cata-Dioptric*, by *Reflection* and *Refraction* conjointly. A *refracting Teleſcope* conſiſts of an Object-Glaſs *x z*, by which the Image *f d* of an Plate XLVIII. Fig. 8.
Object

Dimenſions in Parts of an Inch, as I ſhall ſhew by the following Example.

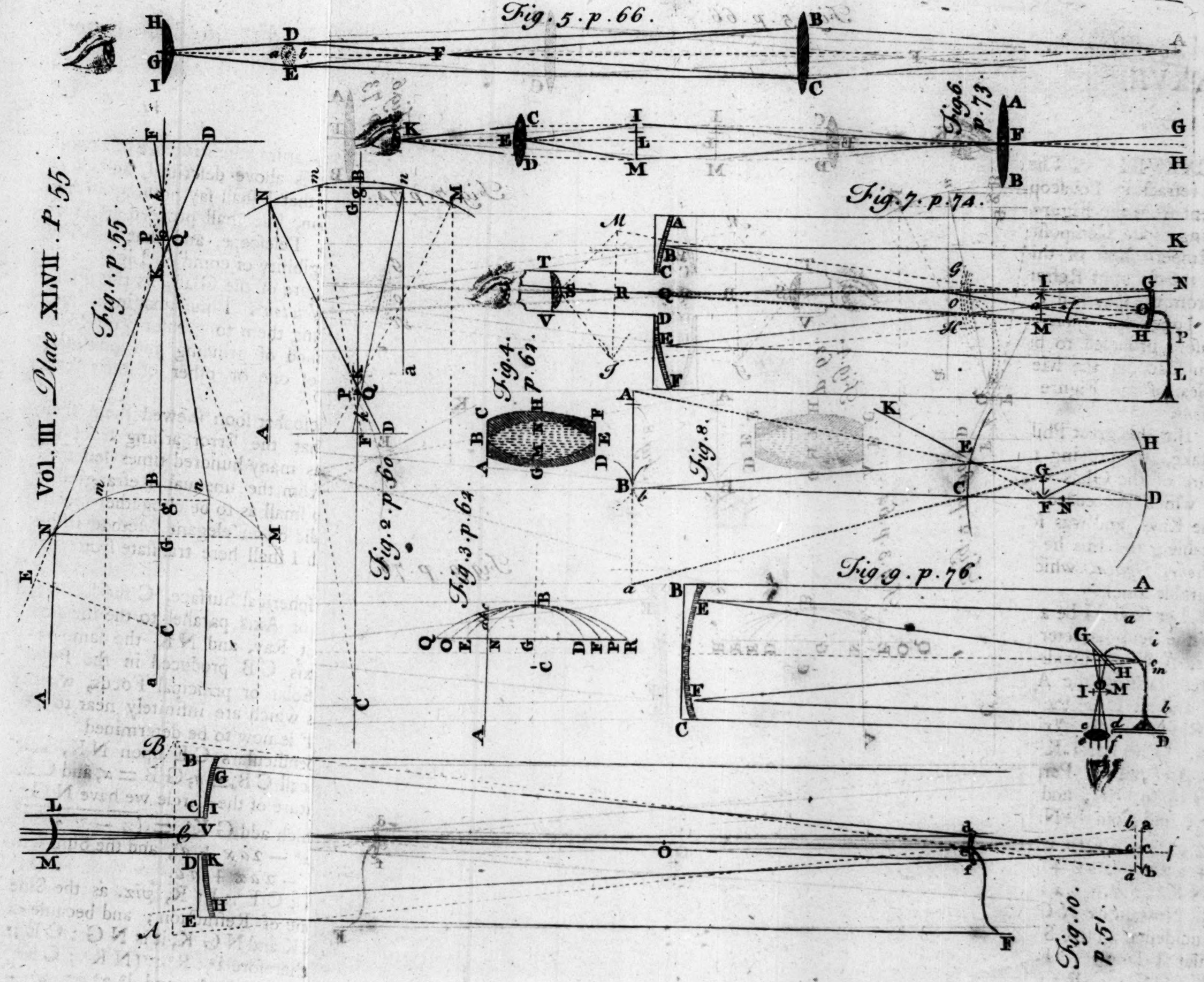
33. Suppoſe it required to meaſure the Diameter of an human Hair, and I obſerve the Index is turned juſt once round while the Point paſſes *o* over it. Then 'tis plain the Diameter of the Hair in the Image is $\frac{1}{50}$ of an Inch. Now if the Microſcope magnifies 6 times, or makes the Image 6 times larger in Diameter than the Object, then is the Diameter of the Hair itſelf but $\frac{1}{6}$ of , that is, but $\frac{1}{300}$ Part of an Inch.

34. Alſo it is to be obſerved, that as there are ten large Diviſions, and twenty ſmall ones, on the Micrometer Plate, ſo each of thoſe ſmall Diviſions are the $\frac{1}{20}$ of $\frac{1}{50}$, or the $\frac{1}{1000}$ Part of an Inch. Therefore, if in meaſuring any Part of an Object, you obſerve how many of theſe ſmaller Diviſions are paſs'd over by the Index, you will have ſo many 1000dth Parts of an Inch for the Meaſure required. All which is ſo plain, that nothing can be ſaid to illuſtrate the Matter.

35. In *Plate* XLVIII. I have given a Print of the Form of my NEW POCKET-MICROSCOPE furniſhed with the MICROMETER above deſcribed. This Microſcope is of the moſt ſimple Structure, moſt eaſy and expeditious for Uſe, and comes at the leaſt Price of any hitherto invented of the compound Sort.

Object O B, at a Diſtance, is formed in the Focus *e* of the ſaid Glaſs, and in an *inverted Poſition.* This Image may be view'd by a ſingle Lens *a b*, placed at its Focal Diſtance, as is uſually done for viewing the heavenly Bodies, becauſe in them we regard not the Poſition: But for viewing Objects near us, whoſe Image we would have erect, we muſt for that Purpoſe add a ſecond Lens *p q*, at double its Focal Diſtance from the other, that the Rays which come from *a b* may croſs each other in the Focus O, in order to erect the Image *g n*, which it will form in its own Focus *m*, becauſe the Rays come parallel from the firſt Lens *a b*. Laſtly, a third Lens *i c* is added, to view that ſecondary Image *g n*. Theſe three Lenſes, or Eye-Glaſſes, are uſually of the ſame Size and Focal Length; and the Power of magnifying is always *as the Focal Length of the Object-Glaſs* e w *divided by the Focal Length of the Eye-Glaſs* l m *or* h e. For inſtance: Suppoſe $e w = 10$ Feet or 120 Inches, and $h e$ or $l m = 3$ Inches; then will the Length of the Object appear to the Eye through ſuch a Teleſcope 40 times larger than to the naked Eye; and its Surface will be magnified 1600 times, and

Vol. III. Plate XLVII. P. 55.



and its Bulk or Solidity 64000 times. (CXXVIII.)

IF

(CXXVIII.) 1. The Nature and Structure of a common refracting Telescope is above described, and is so evident from the Figure, that I shall say nothing farther relating to its Composition, but shall proceed to shew the Imperfection of this Telescope, and that it arises from the different Refrangibility of common Light, and not from the spherical Figure of the Glass, as the *Opticians* before Sir *Isaac Newton*'s Time imagined, and therefore proposed to bring them to greater Perfection by introducing the Method of grinding and polishing Glasses of the Figure of one or other of the *Conic Sections*.

2. But this great Philosopher soon shewed them their Mistake, by proving that the Error arising from the Figure of the Glass was many hundred times less than that which proceeded from the unequal Refrangibility of the Rays, and was so small as to be altogether inconsiderable; and this he did by an elegant Method in his *Lectiones Opticæ*, which I shall here translate from that admirable Book.

3. Let N B M be a spherical Surface, C the Centre, C B the Semidiameter or Axis parallel to the incident Rays, A N an incident Ray, and N K the same refracted, cutting the Axis C B produced in the Point K; and let F be the Solar or principal Focus, where the Rays meet the Axis which are infinitely near to the Axis. The Error K F is now to be determined. Plate XLVII. Fig. 1.

4. Let fall the Perpendiculars C E upon N K, and N G upon C K; and call C B $= a$, G B $= x$, and C K $= z$; and from the Nature of the Circle we have N G^2 $= 2ax - xx$, to which add G K^2 $= \overline{(z - a + x)}^2$ $z^2 + 2xz - 2az + x^2 - 2ax + a^2$, and the Sum will be N K^2 $= z^2 + 2xz - 2az + aa$.

5. Now since N G : C E :: I : R, *viz.* as the Sine of Incidence to the Sine of Refraction; and because of similar Triangles C E K and N G K, it is N G : C E :: N K : C K :: I : R; therefore $I^2 : R^2 ::$ (N K^2 : C K^2) $z^2 + 2xz - 2az + a^2 : z^2$; and $I^2 z^2 = z^2 +$

If instead of a *convex Eye-Glass* we should use a *concave* one of the same Focal Length, it

$2xz - 2az + a^2 \times R^2$, and by Reduction $zz = \frac{2azR^2 - 2xzR^2 - a^2R^2}{R^2 - I^2}$; and (putting $\frac{R^2 a - R^2 x}{R^2 - I^2} = s$, we have $z^2 = 2zs - \frac{R^2 a^2}{R^2 - I^2}$, and $z^2 - 2sz = -\frac{R^2 a^2}{R^2 - I^2}$, and compleating the Square, $z^2 - 2sz + ss = ss - \frac{R^2 a^2}{R^2 - I^2}$; and extracting the Root, $z = s + \sqrt{ss - \frac{R^2 a^2}{R^2 - I^2}}$; whence by Substitution we have) $z = \frac{R^2 a - R^2 x + R\sqrt{I^2 a^2 - 2R^2 ax + R^2 x^2}}{R^2 - I^2}$.

6. And, reducing the radical Part to an infinite Series, we have $z = \frac{Ra}{R - I} - \frac{R^2 x}{IR - I^2} - \frac{R^2 x^2}{2I^3 a} - \frac{R^6 x^3}{2I^6 a^2}$ &c. Now when $x = 0$, $z = \frac{Ra}{R - I} = CF$; whence $CF - CK = KF = \frac{R^2 x}{IR - I^2} + \frac{R^3 x^2}{2I^3 a}$, &c. which is the Value of the Error required.

7. Hence when B G or x is exceeding small, $\frac{R^2 x}{IR - I^2} = KF$ nearly, because in that Case the other Terms, where the ascending Powers of x are found, become extremely small, and nothing in regard to the first Term where x is single.

8. Again; putting $NG = y$, we have $\frac{R^2 y^2}{2IRa - 2I^2 a} = KF$ nearly; for $NG^2 = BG \times \overline{BC + CG} = BG \times 2BC$ nearly, (from the *Elements*) that is, $y^2 = 2ax$ nearly, or $\frac{y^2}{2a} = x$. If then for x in the Equation of the

it would represent tne Object erect, equally magnified, and more distinct and bright; but

the last *Article* we substitute its Value $\frac{y^2}{2a}$, it gives the Equation above in this.

9. Hence also it follows, that the Error KF is always as the *Sagitta* or versed Sine GB, or the Square of the Semichord NG.

10. If the Ray ANK be given in Position, and an be any other parallel Ray nearer to the Axis, and on the other Side; of which let nk be the refracted Part cutting the Axis in k, and the refracted Ray NK in Q, and from Q draw Qo perpendicular to the Axis: Then will the Line Ko become greatest of all, or a *Maximum*, when the Ray an is about half the Distance of the Ray AN from the Axis.

11. For draw ng perpendicular to the Axis, and put $\text{ng} = v$, $\text{K}o = s$, $\text{GK} = f$, and $\text{KF} = h$; and since, by *Art.* 9, we have $\text{NG}^2 : \text{ng}^2 :: \text{KF} : k\text{F}$, or $y^2 : v^2 :: h : \frac{v^2 h}{y^2} = k\text{F}$, therefore $\text{KF} - k\text{F} = \text{K}k = h - \frac{hv^2}{y^2} = \frac{hyy - hvv}{yy}$.

12. Moreover, $\text{GK} : \text{GN} :: \text{K}o : \text{Q}o$; wherefore $\text{Q}o = \frac{ys}{f}$. Also $\text{g n} : \text{GK}$ ($= \text{g}k$ nearly) $:: \text{Q}o : ok = \frac{ys}{v}$; therefore $ko + \text{K}o = \frac{ys}{v} + s = \frac{ys + vs}{v} = \text{K}k = \frac{hyy - hvv}{yy}$; and dividing by $v + y$, and reducing the Equation, we have $s = \frac{hvy - hvv}{yy}$.

13. Now to determine s a *Maximum*, we must make its Fluxion $= o$, that is, $\dot{s} = \frac{h\dot{v}y - 2hv\dot{v}}{yy} = o$; whence we get $h\dot{v}y - 2h\dot{v}v = o$, that is, $hy = 2hv$, or

but the Disadvantage of this Class is, that it admits of but a small *Area*, or *Field of View*,

or $2v = y$, or $2ng = NG$, when s or $K o$ is greatest of all.

14. Therefore $K o$, when greatest, is equal to about $\frac{1}{4}$ of K F; for if in the Equation expressing the Value of s (in *Article* 12.) you write $2v$ for y, there will arise $\frac{1}{4} h = s$.

15. Also because $CF - CB = BF = GK$ nearly, therefore $GK = \frac{Ra}{R-I} - a = \frac{Ia}{R-I}$. Whence since $GK \left(= \frac{Ia}{R-I}\right) : GN \ (= y) :: K o \ (= \frac{1}{4} KF = \frac{R^2 y^2}{8 I R a - 8 I^2 a}) : Q o = \frac{R^2 y^3}{8 I^2 a^2}$.

16. If the Arch B M be taken equal to the Arch B N, and $Bm = Bn$, and Rays incident on M, and m are refracted intersecting each other in the Point P, then 'tis evident $PQ = 2 Q o = \frac{R^2 y^3}{4 I^2 a^2}$; and it is also plain, that all the Rays which fall on the Curve between N and M are so refracted as to pass through the Space P Q, and that the said circular Space P Q is the least possible in which all the Rays can be congregated; and therefore that this Space is the Focus or Place of the Image of an Object, which sends parallel Rays upon the whole Surface of the Lens N B M.

17. For no Rays can be refracted without this Space, because since $Q o$ is in a given Ratio to $K o$, it will be at the same time a *Maximum* with it; and therefore the Point Q is the most remote from the Axis, in which any of those refracted towards F, can possibly intersect the external Ray N K. Neither can they be refracted into a less Space, because the Rays M K, N K, cut the external Rays $n k$ and $m k$ in the Points P and Q, by which the Space P Q is terminated.

18. If the Radius of the Circle (or Lens) N B M be constant, the lateral Error P Q will be as y^3, or as the

View, and therefore not to be uſed when we would ſee much of an Object, or take in

the Cube of the Breadth of the Aperture N M. Alſo, if the Aperture of the Lens remain the ſame, the ſaid Error P Q will be reciprocally as aa, or as CB^2, and therefore as BF^2, ſince C B and B F are in a given Ratio. But if neither the Magnitude of the Circle nor of the Aperture be conſtant, the Error P Q will be as $\frac{y^3}{a^2}$, or as $\frac{NM^3}{BF^2}$, as is evident from its Value $\frac{R^2y^3}{4I^2a^2}$, wherein the Part $\frac{R^2}{4I^2}$ is conſtant, and therefore omitted. Thus far Sir *Iſaac*.

19. In all that has been ſaid in the preceding *Articles*, we are to underſtand Sir *Iſaac*'s Deſign is to ſhew what the Quantity of the Error is, and in what Proportion it varies, that ariſes from the circular Figure of the Glaſs only in refracting the ſame Ray as it is nearer to or farther from the Axis. And therefore we are to underſtand that the Rays here meant are homogeneal, or all of the ſame Sort, and which admit of no Error from a different Refrangibility.

20. Hence we are able to compare the Errors ariſing from the different Refrangibility of the Rays, and from the ſpherical Figure of the Glaſs (ſuppoſing it a *Plano-Convex*, as it commonly is) in a Teleſcope of any given Length. For Example: In a refracting Teleſcope of 100 Feet Length, that is, where $BF = 2BC = 2a = D =$ Diameter of the Sphere $= 1200$ Inches, $y = NG = 2$ Inches, and let $I : R :: 20 : 31$ out of Glaſs into Air. Then will the Expreſſion for the lateral Error from the Figure of the Glaſs be $\frac{R^2y^3}{4I^2a} = \frac{31\times31\times8}{4\times20\times20\times600\times600} = \frac{961}{72000000}$ Parts of an Inch, the Diameter of the circular Space P Q.

21. The Diameter of the little Circle through which the Rays are ſcatter'd by unequal Refrangibility is about the

in a great Scope; but it is used to great Advantage in viewing the *Planets* and their *Satellites*,

the 55th Part of the Breadth of the Aperture of the Object-Glass (as we have already shewn) that is, in the present Case, a 55th Part of 4 Inches, or $\frac{4}{55}$. Wherefore the Error arising from the spherical Figure of the Glass is to that arising from the different Refrangibility of the Rays as $\frac{961}{72000000}$ to $\frac{4}{55}$, that is, as 1 to 5449; and therefore being in comparison so very small, deserves not to be considered in the Theory of Telescopes.

Plate XLVII. Fig. 2. 22. Let us now see, according to Sir *Isaac*'s Method, what the Value of his lateral Error P Q is in the Rays reflected from a spherical Surface, where every Part is denoted by the same Letters as before; only now the refracted Ray N K is the reflected Ray: And here also $NG^2 = 2ax - xx$, and $GK^2 = (a - z - x^2 =)\ a^2 - 2ax - 2az + x^2 + 2zx + zz$, as before, (*Article* 4.) therefore $NG^2 + GK^2 = NK^2 = a^2 - 2az + 2xz + zz = zz$; because $NK = CK$, from the Law of *Reflection*. Whence $a^2 = 2az - 2xz$, and therefore $z = \frac{a^2}{2a - 2x} = CK$; but $CF = \frac{1}{2}a$, therefore $CK - CF = FK = \frac{a^2}{2a - 2x} - \frac{1}{2}a = \frac{ax}{\overline{a - x} \times 2}$.

23. Hence, when x is indefinitely small, $FK = \frac{ax}{2a} = \frac{1}{2}x = \frac{1}{2}GB$ nearly; and because $yy = 2ax$ nearly, (see *Article* 8.) therefore $\frac{y^2}{4a} = \frac{1}{2}x = FK$; and hence it appears, that the Error K F is always as x or the versed Sine G B, or as y^2, or Square of the Sine or Semi-Aperture N G.

24. Again;

Satellites, *Saturn's Ring*, *Jupiter's Belts*, &c. This is called the *Galilean Telescope*, from *Galileo*,

24. Again; every thing in *Art.* 10, 11, 12, 13, and 14, is the same here as there; and so K $o = \frac{1}{4}$ K F $= \frac{y^2}{16\,a}$. And because G K is nearly equal B F $= \frac{1}{2}\,a$, therefore G K : G N :: K o : Q o; that is, $\frac{1}{2}\,a : y :: \frac{y^2}{16\,a} : \frac{y^3}{8\,a\,a} =$ Q o; consequently, 2 Q o = P Q $= \frac{y^3}{4\,a\,a}$. *Q. E. I.*

25. Hence if we put a = B C = Radius of the reflecting Sphere, N B M, we shall have P Q in the refracting Surface or Lens, to P Q in the reflecting Surface or Mirrour, as $\frac{R^2\,y^3}{4\,I^2\,a\,a}$ to $\frac{y^3}{4\,\mathrm{a\,a}}$, or (if $a = \mathrm{a}$) as $\frac{R^2}{I^2}$ to 1, that is, as 2,4 to 1; so that *the Error by Refraction is near twice and a half greater than that by Reflection*, when the Radius of the Sphere is the same in both.

26. If the Medium be given, or the Ratio of I to R, and also the Aperture N M = 2 y; then the Error by Reflection is to that by Refraction as $\frac{1}{\mathrm{a\,a}}$ to $\frac{1}{a\,a}$. Hence, since if the focal Distance of a reflecting Telescope and a refracting one be equal, we have $a = 4\,\mathrm{a}$, therefore $\frac{1}{\mathrm{a\,a}}$ to $\frac{1}{a\,a}$ as $\frac{1}{16}$ to 1, it appears that the Error P Q in the Refractor is to that of the Reflector as 16 to 1.

27. Again; it appears, that in the Reflector, as well as the Refractor, the Error is (*cæteris paribus*) proportional to y^3, or the Cube of the Aperture of the Object-Metal N B M.

28. Lastly, we observe in the refracting Telescope, if the Radius C B = a, and Semi-Aperture N G = y, be given, the Error P Q will be as $\frac{R^2}{I^2}$. Hence, if the

Lens

Galileo, the Inventor, and is the first Sort of Telescope ever made.

THE

Lens be Glass, we have $\frac{R^2}{I^2} = \frac{31 \times 31}{20 \times 20} = 2,4$; and if the Lens be Water, we have $\frac{R^4}{I} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9} = 1,777$. Therefore the Error by Refraction in a Glass-Lens is to that in a Water-Lens (*cæteris paribus*) as 2,4 to 1,777, or as 4 to 3 nearly.

29. Before Sir *Isaac Newton*, all Opticians imagined the Indistinctness or Imperfection of Telescopes was owing wholly to the Figure of the Glass or Lens; which put them upon introducing the Figures of the *Conic Sections*, because, being acquainted with the Ratios of Incidence and Refraction, they could find by Geometry that an Aberration of Rays from the principal Focus F would be occasioned by the Curvature of the Glass, and that was always less of course as the Curvature was less; and that therefore if N B D, E B F,
Plate XLVII. Fig. 3. O B P, and Q B R, represent the curved Surface of a *Circle*, an Ellipsis, a *Parabola*, and an *Hyperbola*, whose common Focus is C, 'tis plain, if a parallel Ray A N be incident on each of these Curves in the Points N, *a*, *b*, *c*, the Aberration or Error caused in the Ray by Refraction in each will be as the Curvature is less, or as the Radius of Curvature in the Points N, *a*, *b*, *c*, increases; and it has been shewn to be as the Square of that Radius inversely. (See *Art.* 18. and 26.) Consequently, since the Aperture and principal Focus is the same in all those Lenses, the Errors of the Rays will be lessened in each of them respectively.

30. But if the Imperfection of the refracting Telescope had been owing only to the spherical Figure of the Glass, Sir *Isaac Newton* proposed a Remedy without Recourse to the *Conic Sections*, which was by composing the Object-Glass of two Meniscus-Glasses, with Water
Fig. 4. between them. Thus let A D F C represent the Object-Glass composed of two Glasses A B E D and B E F C, alike convex on the Outsides A G D and C H F,

and

THE *Cata-Dioptric* or *Reflecting Telescope* is the most noble and useful of all others; the

and alike concave on the Insides BME, BNE, with Water in the Cavity BMEN.

31. Now let the Sines of Incidence and Refraction out of Glass into Air be as I to R, and out of Water into Air as K to R; then out of Glass into Water they will be as I to K (*Annot.* CXVII.). And let the Diameter of the Sphere to which the convex Sides are ground be D, and the Diameter of the Sphere to which the concave Sides are ground be to D as the Cube Root of $\overline{K-I} \times K$ to the Cube Root of $\overline{K-I} \times R$. Then the Refractions on the concave Sides of the Glasses will be very much corrected by the Errors of Refractions on the convex Sides, so far as they arise from the Sphericalness of the Figure.

32. But since those compound Lenses of Glass and Water are with Trouble and Difficulty made, Opticians have applied themselves to invent the best Figure of Lenses for this Purpose, that is, such that the Refraction at the second Surface might correct the Errors of Refraction at the first Surface (arising from the Figure of the Glass only) as much as possible: And the famous *Huygens* has given us a Theorem by which he proves the following Particulars.

33. *First*, That when parallel Rays fall upon the plane Side of a plano-convex Lens, the (longitudinal) Aberration of the extreme Ray is $\frac{9}{2}$ of the Thickness, and is less than the like Aberration caused by any Meniscus-Glass whose concave Side is exposed to the incident Rays.

34. *Secondly*, When the said Glasses have their convex Sides turned to the incident Rays, the Aberration of the extreme Ray in the Plano-Convex is $\frac{7}{6}$ of its Thickness, and is less than the like Aberration of any Meniscus in this Position.

35. *Thirdly*, That a double convex Glass, whose Radius of the first Surface, on which the Rays fall, is to that of the second Surface as 2 to 5, is just as good as

the *Mechanism* whereof is as follows: A B E H is the large Tube or Body of the Instrument,

as the Plano-Convex in its beſt Poſition, the Error being in both $\frac{7}{6}$ of their common Thickneſs.

36. *Fourthly*, When the Radii of a Double Convex are equal, the Aberration is $\frac{5}{3}$ of the Thickneſs; and therefore ſuch a Lens is not ſo good as a Plano-Convex of the ſame Thickneſs in its beſt Poſition.

37. *Fifthly*, But if the Radius of the firſt Surface be to that of the ſecond as 1 to 6, it is then the beſt Glaſs of all, its Aberration then being the leaſt poſſible, *viz.* $\frac{15}{14}$ of its Thickneſs. But if this beſt Glaſs be turned with its other Side to the Rays, the Aberration will be $\frac{145}{42}$, and therefore becomes much worſe than before.

38. *Sixthly*, When a Plano-Concave has its plane Side turned towards parallel Rays, the Aberration of the extreme Ray is alſo $\frac{9}{2}$ of the Thickneſs; and when inverted it is only $\frac{7}{6}$. In a Double Concave likewiſe, whoſe Radii of the firſt and ſecond Surfaces are as 1 to 6, the Aberration is the leaſt poſſible, *viz.* $\frac{15}{14}$, as above in the like Convex.

39. Hence the Glaſſes of common Spectacles ought to have the Figure of the Convex in *Art.* 37. and thoſe Hand-Glaſſes which ſhort-ſighted People uſe ought to be ſuch Concaves as are laſt mention'd.

40. In all the abovementioned Glaſſes the ſame Aperture, Thickneſs, and focal Diſtance is ſuppoſed, and that they differ in nothing but the Figure ariſing from the various Magnitude and Poſition of their Radii reſpectively. But after all, ſince, as we have ſhewn, the Aberration cauſed by the Figure bears ſo ſmall a Proportion to that by the different Refrangibility of Rays, the Perfection of refracting Teleſcopes becomes deſperate, and can only admit of Improvement by increaſing their Length.

41. From hence long Teleſcopes became of common Uſe; and ſo great were the Improvements of this Sort, that for viewing the celeſtial Bodies the Tube of the Teleſcope

ſtrument, in which BE is a large reflecting Mirrour, with a Hole in the Middle CD. This

Teleſcope was thrown aſide, and a Method invented by *Hugenius* of managing them with much greater Eaſe, and of a greater Length. For he contrived to fix the Object-Glaſs upon the Top of a long upright Pole, and directed its Axis towards any Object by means of a Silk-Line coming down from the Glaſs to the Eye-Glaſs below. In this Manner were Teleſcopes made to the Length of 123 Feet.

42. Theſe were called *Aerial Teleſcopes*, as being uſed without a Tube in a dark Night; for the Uſe of a Tube is not only to direct the Glaſſes, but alſo to make the Part dark where the Images of Objects are formed; for in Teleſcopes, as well as in the *Camera Obſcura*, we ought to have no other Light come to the Eye than what proceeds from the Pictures made of the Objects abroad.

43. In order to underſtand in what Proportion Teleſcopes are to be lengthened, ſo that they ſhall magnify in any propoſed Degree with the ſame Diſtinctneſs and Brightneſs of the Object, we are to conſider, that the Indiſtinctneſs of Viſion conſiſts in this, *that the ſenſible Image of a lucid Point in the Object is not a Point in the Image, but a circular Area*; and that two contiguous Points in the Object make two of thoſe Areas in the Image, whoſe Centres are contiguous; and therefore as thoſe two Areas are mixed almoſt entirely with each other, the Repreſentation of the ſaid two Points in the Object is not diſtinct but confuſed.

44. And ſince this is the Caſe with reſpect to every other Point in the Object, 'tis evident there will be a Mixture of ſo many Points of an Object in every Point of the confuſed Picture, as there are Points in the Circle of Aberration; ſince the Centre of any one Circle of Aberration will be covered by all other Circles of Aberration, whoſe Centres fall within the Perimeter of the firſt mentioned Circle; or, in other Words, there will be ſuch a Number of Points in the Object mixed in

This Mirrour receives the Rays *a c*, *b d*, coming from the Object at a Distance, and reflects

any one Point in the confused Image, as is proportional to the Area of the Circle of Aberration.

45. Hence, since this confused Representation of several Points in one is impressed on the Retina by the Eye-Glass, and from thence conveyed to the Common Sensory, it appears *that the Indistinctness of an Object is as the Area of a Circle of Aberration in the Focus of a Telescope*, or as the Square of its Diameter.

Plate XLVII. Fig. 5.

46. To illustrate this Matter, let A be a given Point, B C an Object-Glass of a Telescope, B C A a Pencil of Rays coming from the Point upon the Glass; each Ray, A B, A C, will be so refracted through the Lens, as that the most refrangible Part of each will meet and intersect each other in the Point F in the Axis, the mean refrangible Part will go to *c*, and the least refrangible Part will meet and intersect the most refrangible on each Side in the Points D and E; therefore D E will be the Diameter of the confused Image or Circle of Aberrations *a* D *b* E, and *c* its Centre.

47. Let H I be the Eye-Glass, and G its Centre; then will the Angle D G E be that under which the Circle of Aberrations is seen at the Eye-Glass, and consequently at the Eye (as we have shewn already). But this Angle is as the Subtense D E directly, and as the Perpendicular G *c* inversely, that is, D G E is as $\frac{DE}{Gc}$; for it increases as D E increases while G *c* remains the same, and as G *c* decreases while D E is constant; wherefore, since D E is always as the Angle D G E, we have $DE : \frac{DE}{Gc}$, and so $DE^2 : \frac{DE}{Gc^2}$. But DE^2 is as the Area of the Circle of Aberration, and therefore as the Indistinctness of Vision; consequently, the apparent Indistinctness of a given Object will be as $\frac{DE^2}{Gc^2}$.

reflects them converging to its Focus *e*, where they cross each other, and form the inverted

48. Therefore the Distinctness of Vision will be as $\frac{\mathrm{G}c^2}{\mathrm{DE}^2}$; or, because $\mathrm{DE} = \frac{1}{55}\,\mathrm{CB}$ the Diameter of the Aperture of the Object-Glass, therefore DE^2 will be as CB^2; and so the Distinctness of a given Object will always be as $\frac{\mathrm{G}c^2}{\mathrm{CB}^2}$, that is, *as the Square of the focal Distance of the Eye-Glass directly, and as the Square of the Diameter or Area of the Aperture inversely.*

49. If then in any one refracting Telescope the Distinctness of an Object be represented by $\frac{\mathrm{G}c^2}{\mathrm{BC}^2}$, and in any other Telescope of the same Sort by $\frac{Gc^2}{BC^2}$; then if $\frac{\mathrm{G}c^2}{\mathrm{BC}^2} = \frac{Gc^2}{BC^2}$, we have $\mathrm{BC}^2 \times Gc^2 = BC^2 \times \mathrm{G}c^2$, or $\mathrm{BC} \times Gc = BC \times \mathrm{G}c$; and therefore $\mathrm{BC} : BC :: \mathrm{G}c : Gc$; that is, *two refracting Telescopes shew an Object equally distinct, when the Diameters of the Apertures of the Object-Glasses are as the focal Distances of the Eye-Glasses.*

50. In reflecting Telescopes the Diameter of the Circle of Aberrations was $\mathrm{PQ} = \frac{y^3}{4aa} = \frac{y^3}{\mathrm{D}^2}$, (supposing $\mathrm{D} = 2a$ = Diameter of the Sphere; see *Article* 24.) whence $\mathrm{PQ}^2 = \frac{y^6}{\mathrm{D}^4}$. Let F = focal Distance of the Eye-Glass, then the Indistinctness of Vision will be as $\frac{\mathrm{PQ}^2}{\mathrm{F}^2}$ (*Article* 47.) $= \frac{y^6}{\mathrm{D}^4 \times \mathrm{F}^2}$.

51. Therefore if the same Parts in another Telescope of this Sort be represented by $\frac{PQ^2}{F^2} = \frac{y^6}{D^4 \times F^2}$

inverted Image I M. *x y* is a small concave Mirrour, whose Focus is at *f*, at a smal Distance

and since the Distinctness in each will be as $\frac{\mathrm{D}^4\ \mathrm{F}^2}{y^6}$, and $\frac{D^4\ F^2}{\mathrm{y}^6}$; then if we suppose the Object seen equally distinct in both, we shall have $\mathrm{D}^4\,\mathrm{F}^2 \times \mathrm{y}^6 = D^4\,F^2 \times y^6$, or $\mathrm{D}^2\,\mathrm{F}\,\mathrm{y}^3 = D^2\,F\,y^3$. Hence $\mathrm{F} : F :: \frac{y^3}{\mathrm{D}^2} : \frac{\mathrm{y}^3}{D^2}$; that is, *Reflecting Telescopes shew an Object equally distinct, when the focal Distances of the Eye-Glasses are as the Cubes of the Diameters of the large Specula or Object-Metals, divided by the Square of the Diameter of the Spheres to which they are ground, or by the Square of the focal Distance of the Metals.*

52. In any Telescope, or Double Microscope, the Brightness of a given Image will be as the Quantity of Light by which it is shewn; that is, as the Area of the Aperture of the Object-Glass, or as the Square of the Diameter.

53. Also, if the Area of the Aperture of an Object-Glass be given, the Brightness of the Image will be inversely as its Area, or Square of its Diameter or Breadth: For the less the Area of the Picture is, the greater will be its Brightness by the same Quantity of Light.

54. Therefore when neither the Apertures of the Glasses, nor the Amplifications of the Picture are given, or the same, the Brightness is as the Square of the Diameter of the Apertures directly, and the Square of the linear Dimensions of the Pictures inversely.

55. Hence in all Sorts of Telescopes a given Object appears equally bright, when the Diameters of the Apertures are as the linear Dimensions of the Pictures: But the Picture is larger as the focal Distances of the Object-Glasses is so, and also as the focal Distance of the Eye-Glass is less; therefore the linear Dimensions of Pictures are as the focal Distances of the Object-Glasses directly,

Distance from the Image. By this Means the Rays coming from the Images are reflected

directly, and as the focal Distances of the Eye-Glasses inversely. Let these be represented by F and *F*, and f, *f*, in any two Telescopes; let D, *d*, be the Diameters of the Apertures, and L, *l*, the linear Dimension of the Pictures; then we have $D : d :: L : l :: \frac{F}{f} : \frac{F}{f}$, when Objects appear equally bright in both.

56. Hence, since the Brightness of a Picture or Image is as $\frac{D^2}{L^2}$ (*Art.* 54.) $= \frac{D^2 f^2}{F^2}$ because $L = \frac{F}{f}$ by the last) therefore if D or f be each increased in any Ratio, the Distinctness will remain the same as before, (by *Art.* 49.) and the linear Dimensions of the Image will be diminished in the same Ratio, (since L is inversely as f) but the Brightness of the Image will be increased in the quadruplicate Ratio of what it had before. For,

57. Suppose F or the focal Length of the Telescope given, then the Brightness of the Picture will be in this Case as $D^2 f$; and if D and f be increased each in the Ratio of 1 to *m*, then will the Brightness be in this Case as $m^2 D^2 f^2 m = D^2 f^2 m^4$; so that the former Brightness is to this as $D^2 f^2$ to $D^2 f^2 m^4$, that is, as 1 to m^4; which Ratio is quadruplicate of the Ratio 1 to *m*.

58. Because we had $D : \frac{F}{f}$, or $D f : F$, when Objects appear equally bright, (by *Art.* 55.) and when they are shewn equally distinct we had $D : f$ (by *Art* 49.); therefore in refracting Telescopes of various Lengths, that Objects may appear equally bright, and equally distinct, it is requisite that $D^2 : F$, and $f^2 : F$, or that $D : f : \sqrt{F}$; that is, *the Diameter of the Aperture and also the focal Length of the Eye-Glass should each be as the Square Root of the focal Distance or Length of the Telescope.*

flected back through the central Hole CD of the large Mirrour, where they fall on the

59. In this Case likewise *the linear Dimensions of the Picture or Image are in the same subduplicate Ratio of the Length of the Telescope*; because, as was shewn, (*Art.* 55.) the linear Dimensions are directly as the Diameter of the Aperture, which is here shewn to be as the Square Root of the Length of the Telescope.

60. In reflecting Telescopes, when the Distinctness is given, we have $F : \frac{y^3}{D^2}$, and therefore $y^3 : D^2 F$. (See *Article* 51.) Also when the Brightness is given we have $y : \frac{D}{F}$, (*Art.* 55.) therefore $F : \frac{D}{y}$. Hence, when the Distinctness and Brightness are both given, we have $y^3 : (D^2 F) : \frac{D^3}{y}$, or $y^4 : D^3$, or $y : D^{\frac{3}{4}}$.

61. The linear Dimensions of the Picture $\frac{D}{F}$ were as y; that is, in this Case, $\frac{D}{F} : D^{\frac{3}{4}}$, and therefore $D : FD^{\frac{3}{4}}$; whence $F : \frac{D}{D^{\frac{3}{4}}} : D^{\frac{1}{4}}$. *Hence in reflecting Telescopes of different Lengths a given Object will appear equally distinct and bright, when the Diameters of the Object-Metals are as the Biquadrate Roots of the Cubes of the Diameters of the Spheres or focal Lengths of the Specula*; or, *when the focal Distances of the Eye-Glasses are as the Biquadrate Root of the focal Distance of the Specula.*

62. According to the Theorems in *Art.* 48, 49, HUGENIUS calculated a Table of the linear Aperture of the Object-Glass, the focal Distance of the Eye-Glass, and the linear Amplification or magnifying Power of the Telescope from one which he found by Experience was constructed in the best Manner. I have reduced his *Rhinland* Measures to *English* Feet, Inches, and Decimal Parts, as follows.

Focal

the plano-convex Lens W X, and are by it converged to a Focus, and there form a second

Focal Diſtance of the Object-Glaſs.	*Linear Aperture of the Object-Glaſs.*	*Focal Diſtance of the Eye-Glaſs.*	*Magnifying Power.*
Feet.	Inch Dec.	Inch Dec.	
1	0,545	0,605	20
2	0,76	0,84	27,6
3	0,94	1,04	33,5
4	1,08	1,18	39,5
5	1,21	1,33	44
6	1,32	1,45	49
7	1,43	1,58	53
8	1,53	1,69	55
9	1,62	1,78	59
10	1,71	1,88	62
15	2,10	2,30	76
20	2,43	2,68	88
30	3,00	3,28	108
40	3,43	3,76	125
50	3,84	4,20	140
60	4,20	4,60	152
70	4,55	5,00	164
80	4,83	5,35	176
90	5,15	5,65	187
100	5,40	5,95	197
120	5,90	6,52	216

63. Since it has been ſhewn that the Errors ariſing from the different Refrangibility of Rays, and of Conſequence the Indiſtinctneſs of Viſion by refracting Teleſcopes is ſo very great, a Queſtion may be put, How it comes to paſs Objects appear through ſuch Teleſcopes ſo diſtinct as they do? To which it may be anſwer'd, 'Tis becauſe the erratic Rays are not uniformly ſcatter'd over

ſecond Image R S, very large and erect, which is viewed by a *Meniſcus Eye-Glaſs* Y Z

all the Area of the Circle of Aberration, but collected infinitely more denſely in the Centre than in any other Part of that circular Space, growing rarer and rarer towards the Circumference, where, in Compariſon, they are infinitely rare, and affect not the Senſe any where but in the Centre, and very near it, on that Account.

64. 'Tis farther to be obſerved, that the moſt luminous of all the Priſmatic Colours are the Yellow and the Orange. Theſe affect the Senſes more ſtrongly than all the reſt put together; and next to theſe in Strength are the Red and Green. The Blue compared with theſe is a faint and dark Colour, and the Indigo and Violet are much darker and fainter; ſo that theſe, compared with the ſtronger Colours, are little to be regarded.

65. The Images of Objects are therefore to be placed not in the Focus of the mean refrangible Rays, which are in the Confine of Green and Blue, but in the Focus of thoſe Rays which are in the Middle of the Orange and Yellow, there where the Colour is moſt luminous and fulgent; that is, the brighteſt Yellow, that Yellow which inclines more to Orange than to Green.

66. Now it has been ſhewn (*Annot.* CXVIII. 9.) that the Diameter of the Circle in which both thoſe Colours will be contained is but the 260th Part of the Diameter of the Aperture of the Object-Glaſs; and farther, about $\frac{3}{5}$ of the brighter Halves of the Red and Green (on each Side) will fall within this Circle, and the remaining $\frac{2}{5}$ without it, which will be ſpread over twice the Space nearly, and therefore become much rarer. Of the other Half of the Red and Green, about one Quarter will fall within this Circle, and $\frac{3}{4}$ without, and be ſpread through four or five times the Space, and therefore become much rarer. Alſo this extreme Red and Green is much rarer and darker than the other Parts of the ſame Colours; and the Blue and Violet being

Y Z by the Eye at P, through a very ſmall Hole in the End of the Eye-Piece Y C D Z.

If

being much darker Colours than theſe, and more rarified, may be quite neglected.

67. Hence the ſenſible confuſed Image of a lucid Point is ſcarce broader than a Circle whoſe Diameter is the 260th Part of that of the Aperture of the Glaſs, if we except the dark miſty Light round about, which we ſcarce regard. And therefore in a Teleſcope whoſe Aperture is 4 Inches, and Length 100 Feet, it exceeds not $2\frac{1}{4}''$ or $3''$; and in a Teleſcope whoſe Aperture is 2 Inches, and Length 20 or 30 Feet, it may be about $5''$ or $6''$, and ſcarce above. And this anſwers well to Experience; for it is obſervable that in Teleſcopes of 20 or 30 Feet long, the Diameters of the fixed Stars appear to be about $5''$ or $6''$, or at moſt not more than $8''$ or $10''$.

68. Now ſuppoſe the ſenſible Image of a lucid Point to be even a 250th Part of the Diameter of the Aperture of the Glaſs, yet will this be much greater than if it were only from the ſpherical Figure of the Glaſs, *viz.* (in an 100 Foot Teleſcope) in the Ratio of $\frac{4}{250}$ to $\frac{961}{72000000}$, or of 1200 to 1. (See *Art.* 20, 21.) Therefore the Image of a lucid Point would ſtill be a Point, but for the various Refrangibility of the Rays; and this alone is the invincible Obſtacle to perfect Viſion by any refracting Inſtruments.

69. The magnifying Power of a refracting Teleſcope is thus eſtimated. Let A B be the Object-Glaſs, and C D the Eye-Glaſs; and let H F I and G F M be two Rays coming from the extreme Parts of a diſtant Object, and croſſing each other in the Centre F of the Glaſs A B. Then is the Angle G F M = I F M that under which the Object appears to the naked Eye; but I E M = C K D is that under which the Image appears as magnified by the Eye-Glaſs C D. But the Angle I E M is to the Angle I F M as L F to L E, or *as the* Plate XLVII. Fig. 6.

focal

If the first Lens W X were taken away, the Image would be formed somewhat larger at

focal Distance of the Object-Glass to the focal Distance of the Eye-Glass; and in that Proportion is the Object magnified, as was observed before in *Art.* 55.

Plate XLVII. Fig. 7. The magnifying Power of a reflecting Telescope is thus computed. The parallel Rays K B and L E are reflected by the large Object-Metal A F to its Focus *a*, where the Image I M is formed; which Image is defined by two other Rays N Q, P Q, coming from the extreme Parts of the Object at a remote Distance, and meeting in the Centre of the large Speculum at Q; for it has been shewn that the Object and its Image both appear under the same Angle from the Vertex of the Mirrour. (*Annot.* CXXV.)

71. Now if *f* be the Focus of the small Mirrour G H, supposing the Image were formed in the said Focus *f*, (that is, that both the Foci *a* and *f* were coincident) then the Rays proceeding from the Image I M will proceed parallel after Reflection, and produce distinct Vision of the Image, which will then subtend an Angle I O M at the Centre O of the Speculum G H; which is to the Angle I Q M, under which the Object appears to the naked Eye, as *a* Q to *a* O or *f O*. So that the magnifying Power would in this Case be as

$$\frac{aQ}{fO}.$$

72. But to increase this magnifying Power, the Image I M is not placed in the Focus of the small Speculum, but at a small Distance beyond it; by which means the Rays coming from the Image to the Speculum G H will be reflected converging to a distant Focus R, where a secondary large Image *I M* is formed from the first Image I M; which Image *I M* is seen under the same Angle *I O M* with the former from the Centre of the Speculum G H, but from the Centre of the Eye-Glass T V it is seen under the large Angle *I* S *M*. But the Angle *I* S *M* is to the Angle *I* O *M* as O R to S R; wherefore

at *Mg*; but the *Area*, or *Scope*, would be less, and therefore the View not so pleasant.
At

wherefore the second Ratio or Part of the magnifying Power is that of $\frac{OR}{SR}$.

73. Consequently, the whole magnifying Power of the Telescope is $\frac{aQ}{aO} \times \frac{OR}{SR}$ (because in this Case fO becomes aO). Or, in other Words, the Angle NQP, under which the Object appears to the naked Eye, is to the Angle ISM, under which the large magnified secondary Image IM appears to the Eye through the Eye-Glass, as $\frac{aQ \times OR}{aO \times SR}$. Such is the Theory of the Telescope first contrived by Dr. *J. Gregorie*, and therefore call'd the *Gregorian Telescope*; but it received its last Improvement from the late Mr. *Hadley*, and is now in common Use.

74. A small Alteration was made in the Structure of this Telescope by Mr. *Cassegrain*, viz. in using a convex Speculum GH, instead of the concave one GH. Now if they are equally spherical, that is, if they are Segments of the same Sphere, then will f be also the virtual Focus of the Convex GH; and if all other Things remain the same, the first Image IM will be virtually the same as before, and the last Image IM will be really the same; so that the magnifying Power of this Form of the Telescope is $\frac{aQ \times OR}{aO \times SR}$, which is equal to that of *Gregorie*'s Form.

75. And since to shew this, is a curious Proposition, I shall give the following easy Demonstration thereof. Let HD be a concave Speculum, and EC a convex one, both described with the same Radius CD, on the common Axis BCD: The Point N, bisecting the Radius CD, will be the Solar Focus to each Speculum. Let F be a radiant Point, from whence a Ray FH is incident Pl. XL. Fig. 8.

At T V is placed a circular Piece of Brass, with a Hole of a proper Size to circum-
scribe

incident upon the concave Mirrour in H, or to which the Ray K E incident upon the convex Mirrour tends; both those Rays will be reflected to the same Point B in the Axis, and in the same Line E B. Lastly, let G F be an Object; the Imege thereof ab formed by the Concave is equal to the Image A B made by the Convex. This is evident from the Theorems $\frac{dr}{2d\times r}$ $=f$, and $\frac{dr}{r-2d}=f$, those Specula respectively.

76. For as $d=$F C, C B$=f$ in the Convex; so in the Concave let F D$=D$, and D B$=F$; and then we have in the former, $d:f::2d+r:r$, and in the latter, $D:F::r-2D:r$. But $D=d+r$, therefore $2D=2d+2r$; whence $r-2D=2d+r$, consequently $d:f::D:F$; that is, C F : C B :: D F : D B. But the Object and Image are to each other in the same Ratio in either Glass, and therefore since the Object is the same in both, the Image will be so likewise, or A B$=ab$.

Plate XLVII. Fig. 9.

77. Sir *Isaac Newton* ordered this Telescope to be made in a different Form or Manner, as follows. A B C D was a large octogonal Tube or Case; E F a large polished Speculum, whose Focus is at o; G H a plane Speculum truly concenter'd, and fixed at half a Right Angle with the Axis of the large one. Then parallel Rays a E, b F, incident on the large Speculum E F, instead of being reflected to the Focus o, were intercepted by the small plane Speculum G H, and by it reflected towards a Hole $c\,d$ in the Side of the Tube, crossing each other in the Point O, which is now the true focal Point; and from thence they proceed to an Eye-Glass $e\,f$ placed in that Hole, whose focal Distance is very small, and therefore the Power of magnifying may be very great in this Form of the Telescope; because the Image I M is made by one Reflection, (for that

ſcribe the Image, and cut off all ſuperfluous or extraneous Rays, that ſo the Object may appear as diſtinct as poſſible.

As the Image is formed by Reflection, the Rays of every Sort will be united nearly in one Point, and will therefore admit of an Eye-Glaſs Y Z of a deep Charge, or ſmall focal Diſtance; and ſo the Power of magnifying will be proportionally greater; for it will be in a Proportion compounded of $\frac{Qe}{eG}$ and $\frac{Gk}{kt}$, if only one Eye-Glaſs Y z be uſed. Thus, in Numbers, ſuppoſe $Qe = 12$ Inches, $eG = 3{,}5$; $Gk = 18$, and $kt = 1$; then will $\frac{12}{3{,}5} \times \frac{18}{1} =$ 216

that of the plane Speculum only alters the Courſe of the Rays, and adds nothing to the Confuſion of the Image) and will for that Reaſon bear being view'd by a Glaſs of a very deep Charge, in compariſon of an Image form'd by differently refrangible Rays.

78. This Teleſcope is a very good one, as to its Effect or Performance, but is not ſo commodious for common Uſe as thoſe of the *Gregorian* Form, and is therefore now pretty much laid aſide. They who would ſee a larger Account hereof may conſult Sir *Iſaac*'s *Optics*, and ſeveral *Philoſophical Tranſactions*, where he deſcribes it at large, and the Reaſons which induced him to make choice of this Structure rather than that of Dr. *Gregorie*: Or ſee a compendious Account of the Whole in the laſt Edition of Dr. *Gregorie*'s *Elements of Optics*.

$\frac{216}{3,5} = 61,71$ nearly; whence by ſuch a Teleſcope the Length of an Object will be magnified 50 times, the Surface 2500 times, and the Solidity 125000 times; yet the Teleſcope not above 20 Inches long; an Effect equal to that of a refracting Teleſcope 16 Feet in Length.

As to the *Camera Obſcura*, and *Magic Lanthorn*, they both perform their Effects by a ſingle Lens; the former being only the Object-Glaſs of a long Teleſcope applied in a *Scioptric Ball* to the Hole of a Window-Shutter in a darken'd Room; which gives a lively Picture of all the Objects which lie before it, in true Perſpective, but in an *inverted Poſition*, on a white Sheet or Plane held at the focal Diſtance of the ſaid Glaſs: And on the other hand, the *Magic Lanthorn* is only a large convex Lens, with a ſhort focal Diſtance, which by being placed at a proper Diſtance from ſmall tranſparent-colour'd Pictures or Figures, forms a large and ſurprizing Image thereof at a great Diſtance; in order to which, it is neceſſary to illuminate them very ſtrongly with the Light of the Candle thrown on them by another

another *very large and very convex Lens* (CXXIX.).

The

(CXXIX) 1. The CAMERA OBSCURA, or *Darken'd Room*, is made after two different Methods; one is the *Obſcura Camera* or Darken'd Chamber at large, and properly ſo call'd; that is any large Room or Chamber made as dark as poſſible, ſo as to exclude all light but that which is to paſs through the Hole and Lens in the Ball fixed in the Window of the ſaid Room.

2. The other is in ſmall, and made in various Ways, as that of a Box, a Book whoſe ſides fold out, *&c.* for the Conveniency of carrying it from Place to Place, for taking an Optic View in Picture of any propoſed Place or Part of the Country, Town, *&c.* and hence it is call'd the *Portable Camera Obſcura*.

3. The following Particulars are to be attended to in this Philoſophical Contrivance. *Firſt*, That the Lens be extremely good, or free from any Veins, Blebs, *&c.* which may diſtort and blemiſh the Picture.

4. *Secondly*, That the Lens be always placed directly againſt the Object whoſe Picture you would have perfectly form'd to contemplate; for if the Glaſs has any other Poſition to the Object, the Image will be very imperfect, indiſtinct, and confuſed.

5. *Thirdly*, Care ought to be taken, that the Ball be ſufficiently large, and the Frame in which it is placed not too thick, that ſo there may be ſufficient Room for turning the Ball every Way, to take in as many Objects as poſſible, and to render the Uſe thereof moſt compleat.

6. *Fourthly*, The Lens ought to be of a juſt Magnitude or Aperture; for if it be too ſmall, the Image will be obſcure, and the minute Parts not viſible at a Diſtance for want of requiſite Light. On the other hand, if the Aperture be too large, the Image will be confuſed, and become indiſtinct by too much Light.

7. Therefore, *Fifthly*, if by Experience I find that an Aperture of 2 Inches Diameter is beſt for a Lens of 6 Feet focal Diſtance, I know (from what has been ſaid in the laſt *Annotation*) that the Diameter of any other

THE *Solar Microscope* is of the same Kind with the *Magic Lanthorn*; only here the Objects

other Lens of a different focal Distance ought to be in the subduplicate Ratio of 6 to the said focal Distance, that the Object, or its Image rather, may be equally bright and distinct in both.

8. *Sixthly*, We ought not to attempt to exhibit a Picture of Objects in a dark Room, unless the Sun shines upon or strongly illuminates the Objects; for mere Daylight is not sufficient for this Purpose, the greatest Beauty in this Phænomenon being the exquisite Appearance and Contrast of Light and Shadows, none of which can appear but from an Object placed in the Sun-Beams; without which every thing looks dark and dull, and makes a disagreeable Figure.

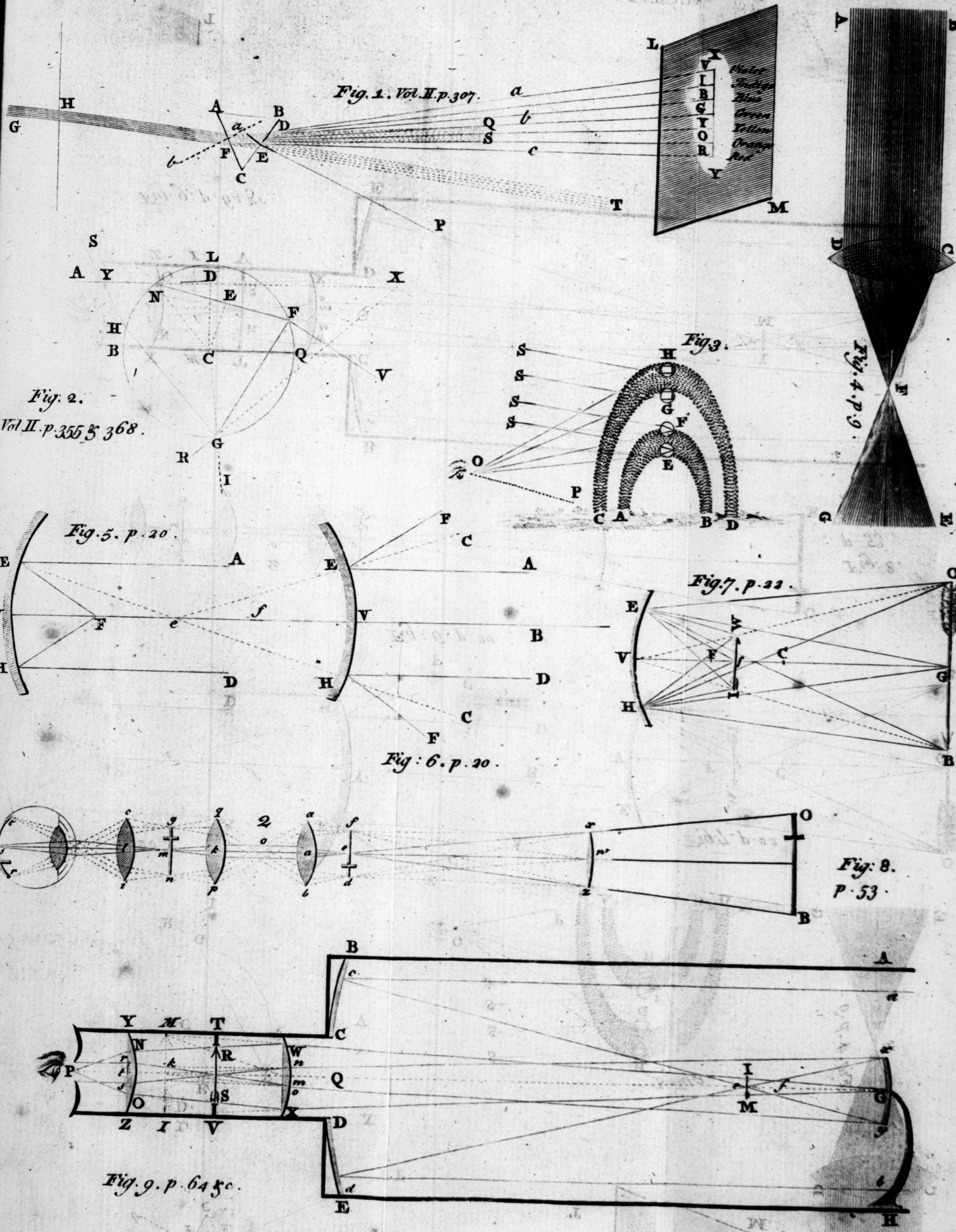
9. Therefore, *Seventhly*, the Window, or that Side of the Room where the Scioptric Ball is used, ought to look towards that Quarter directly upon which the Sun shines, that so the illuminated Sides of Objects may present themselves to the Lens, and appear more glorious in the Picture.

10. *Eighthly*, Hence it is easy to infer, that the best Time of the Day for this Experiment is about Noon, because the Sun-Beams are then strongest, and of course the Picture most luminous and distinct: Also that a North Window is the best; though or viewing the Shadows in greatest Perfection, an East or West Window will answer the end best.

11. *Ninthly*, As the Image is formed only by the reflected Rays of the Sun, so due Care should be taken that none of the Sun's direct Rays fall on the Lens in the Window; for if they do, they will, by mixing with the former, greatly disturb the Picture, and render it very confused and unpleasant to view.

12. *Tenthly*, As white Bodies reflect the incident Rays most copiously, and black ones absorb them most; so to make the Picture most perfect, it ought to be received upon a very white Surface, as Paper, a painted Cloth, Wall, &c. bordered round with Black, that so the collateral

Vol. III. *Plate* XLVIII. *P.* 80.



Objects are very small, and strongly en-lighten'd by the Sun through a concave Lens;

collateral Rays which come from on each Side the Object may be stifled, and not suffered to disturb the Picture by Reflection.

13. These are the necessary Precautions for the due ordering of the various Circumstances of this Experiment. I shall now enumerate the several *principal Phænomena* of the Dark Chamber. The *First* of which is, that an exact and every-way similar Image is formed of an external Object; for Pencils of Rays coming from all Points of the Object will represent those Points in such a Manner and Position as will be very proportional and correspondent to their respective Positions and Distances in the Object, so that the Whole in the Image shall bear an exact Similitude or Likeness of the Object in every Respect.

14. The *Second Phænomenon* is, that the Image will bear the same Proportion to the Object, whether a Line, Superficies, or Solid, as their Distances from the Glass respectively: This is evident from what has been said relating to the Effect of a convex Lens. Hence the larger the focal Distance of the Glass, the more ample will be the Picture of the same Object, but the less will be the Space or Compass of the Plan or perspective View.

15. The *Third Phænomenon* is, that the Image or Picture of the Object is *inverted*; and this is not the Effect of the Glass, but the crossing of the Rays in the Hole through which they pass into the Room; for if a very small Hole were made in the Window-Shutter of a darken'd Room, the Objects without would be all seen inverted, those which come from the upper Part of the Object going to the lower Part of the Image, and *vice versâ*. All that the Glass does is to render the Image distinct, by converging the Rays of every Pencil to their proper Focus in the Picture, the Position of each Point being the same as before.

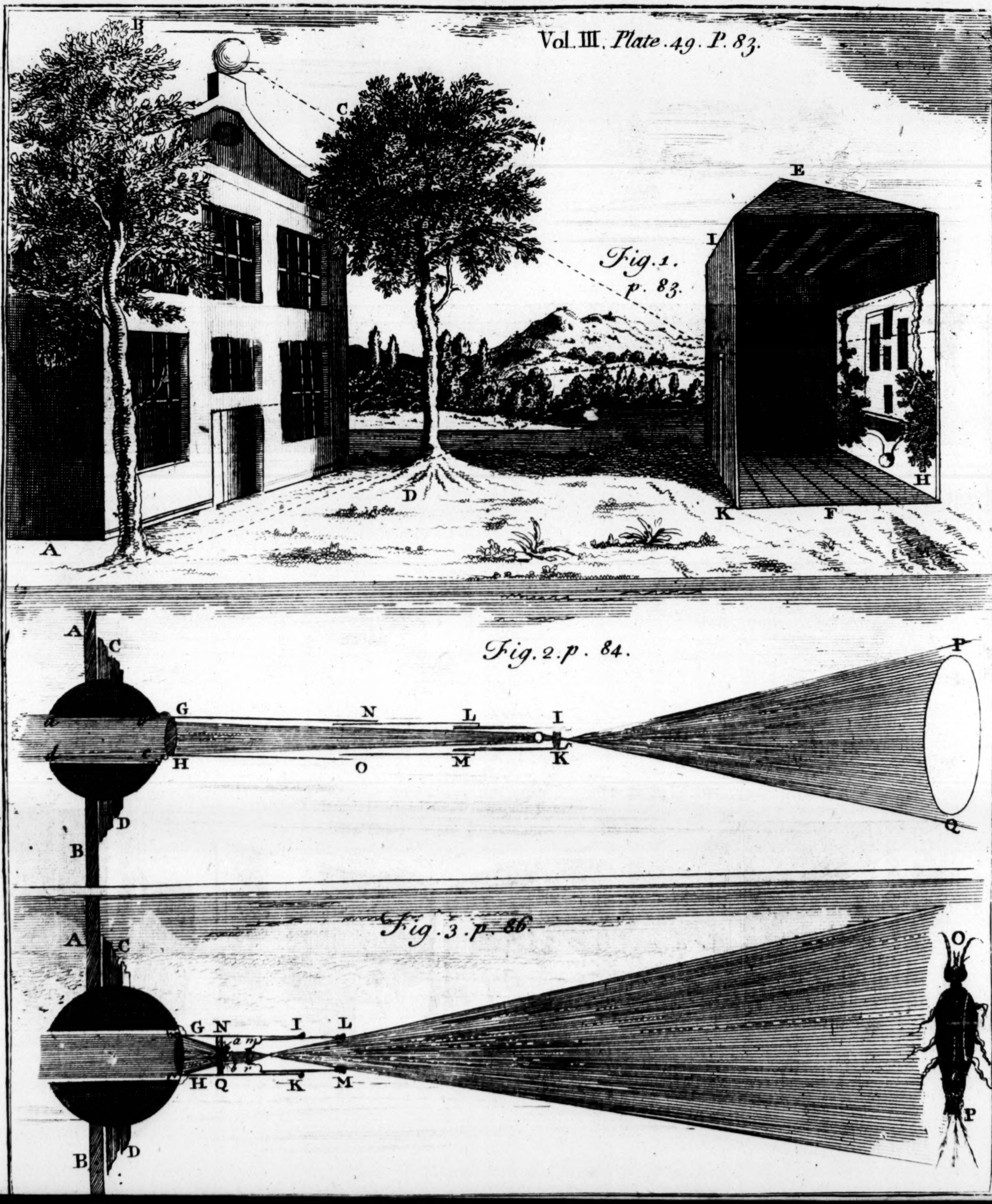
Lens; they are alſo magnified by a ſmall Lens of a very ſhort Focal Diſtance, that the

16. The *Fourth Phænomenon* is the Motion or Reſt of the ſeveral Parts of the Picture, according as thoſe of the Object are in either State. The Reaſon of this is very obvious; and this it is that gives Life and Spirit to the Painting and Portraits of Nature, and is the only Particular inimitable by Art. And indeed a more critical Idea may be formed of any Movement in the Picture of a darken'd Room, than from obſerving the Motion of the Object itſelf. For Inſtance, a Man walking in a Picture appears to have an undulating Motion, or to riſe up and down every Step he takes; whereas Nothing of this Kind is obſerved in the Man himſelf, as viewed by the bare Eye.

17. The *Fifth Phænomenon* is the *Colouring of the Optic Picture*. Every Piece of Imagery has its proper Tints and Colours, and thoſe always heighten'd and render'd more intenſe than in the Object; ſo that in this Reſpect it is an Improvement of Nature itſelf, whereas the Art of the greateſt Maſter can only pretend to a diſtant Reſemblance and faint Imitation. The Reaſon why the Image is coloured is becauſe the ſeveral Points of the Object reflecting ſeveral Sorts of colour'd Rays to the Glaſs, thoſe Rays will give a Repreſentation of thoſe ſeveral Points reſpectively, and in their own Colour, and therefore in thoſe of the Object; but thoſe Colours will be heighten'd, becauſe they are crouded into a leſs Space.

18. The *Sixth Phænomenon* is the *Claro Oſcuro*, as the *Italians* call it; that is, the Intenſity of *Light and Shadow* in the Picture: And this, as well as the Colouring, is greatly heighten'd above what it is in the Object, by reaſon of the leſſer Area of the Picture. Here every Light and every Shade is expreſs'd in its proper Degree, from the moſt brilliant in the one, to the moſt jetty black of the other, incluſive of a wonderful Variety in the ſeveral Parts, ariſing from the different Situations of the ſeveral Parts of the Object, and

Vol. III. *Plate* . 49. P. 83.



the Images may be thrown large and distinctly on the opposite Wall of a darken'd Room:

and the different Angles of Reflection. A just Imitation of Nature in the Distribution of Light and Shadows is perhaps the most difficult Part of the Art of Painting, and on which its greatest Perfection depends.

19. The *Seventh Phænomenon* is the *Optical Perspective*, or Projection of the Image, which is not *in Plano*, or on a Plane, as in common Perspective, but on a Surface described by the Revolution of a *Conic Section* about its Axis, as is evident from what was observed in *Annot.* CXXV. Therefore, though in general a plane Surface is made use of, and may do very well in large Representations, yet in smaller ones, as those of the *Portable Camera's*, it is necessary to have the Image or Picture compleat, or every where well defined, that it be received upon the Surface of an *Elliptic Figure*, and such as is suited to the middle Distance of the Objects. But this is a Nicety which few will think worth regarding, who do not aim at a very great Accuracy indeed in what they do.

20. I shall finish this Subject with an Observation that may be useful to Persons concerned in Drawing, and that is, *That if an Object be placed just twice the focal Distance from the Glass without, the Image will be formed at the same Distance from the Glass within the Room, and consequently will be equal in Magnitude to the Object itself.* The Truth of this is demonstrated in *Annot.* CXXV.

21. Although every Thing that has been said of the *Camera Obscura* is plain enough in itself to be understood, yet as a Representation thereof may facilitate the Idea, I have here given a Diagram for that Purpose; where A B C D is the Prospect of a *House*, *Trees*, &c. E F a darken'd Room, or *Camera Obscura*; on one Side is the Picture G H of the said View inverted, formed by a convex Lens in the Ball fixed before a Hole in the other Side I K at V. All which is so easy that nothing more remains to be said to explain it. Plate XLIX. Fig. 1.

Room: Which, if well perform'd, is one of the moſt exquiſitely curious and moſt delightfully ſurpriſing Effects that can be produced by any Optical Inſtrument whatſoever. (CXXX.)

(CXXX.) 1. The SOLAR TELESCOPE and SOLAR MICROSCOPE, as they ought to make a Part of the Amuſement of every Virtuoſo and Gentleman, ſo they deſerve a particular Account, and the ſeveral Ways in which they are uſed merit a particular Deſcription, which I ſhall illuſtrate by a Draught of each.

2. The SOLAR TELESCOPE is applied to Uſe in the following Manner. A B repreſents a Part of the Window-Shutter of a darken'd Room, C D the Frame, which (by means of a Screw) contains the Scioptric Ball E F, placed in a Hole of the ſaid Shutter adapted to its Size. This Ball is perforated with a Hole *a b c d* through the Middle; on the Side *b c* is ſcrew'd into the ſaid Hole a Piece of Wood, and in that is ſcrew'd the End of a common refracting Teleſcope G H I K, with its Object-Glaſs G H, and one Eye-Glaſs at I K; and the Tube is drawn out to ſuch a Length, as that the Focus of each Glaſs may fall near the ſame Point.

Plate XLIX. Fig. 2.

3. This being done, the Teleſcope and Ball are moved about in ſuch manner as to receive the Sun-Beams perpendicularly on the Lens G H, through the cylindric Hole of the Ball; by this Glaſs they will be collected all in one circular Spot *m*, which is the Image of the Sun. The Lens I K is to be moved nearer to or farther from the ſaid Image *m*, as the Diſtance at which the ſecondary Image of the Sun is to be formed requires, which is done by ſliding the Tube I K L M backwards and forwards in the Tube L M N O. Then of the firſt Image of the Sun *m* will be formed a ſecond Image P Q, very large, luminous, and diſtinct.

4. In this Manner the Sun's Face is view'd at any time without Offence to weak Eyes; and whatever Changes happen therein may be duly obſerved. The *Spots* (which make ſo rare an Appearance to the naked Eye, or through a ſmall Teleſcope in the common Way) are

are here all of them conſpicuous, and eaſy to be obſerved under all their Circumſtances of Beginning to appear, Increaſe, Diviſion of one into many, the uniting of many into one, the Magnitude, Decreaſe, Abolition, Diſappearance behind the Sun's Diſk, &c.

5. By the *Solar Teleſcope* we alſo view an Eclipſe of the Sun to the beſt Advantage, as having it in our Power by this means to repreſent the Sun's Face or Diſk as large as we pleaſe, and conſequently the Eclipſe proportionably conſpicuous. Alſo the Circle of the Sun's Diſk may be ſo divided by Lines and Circles drawn thereon, that the Quantity of the Eclipſe eſtimated in Digits may this way be moſt exactly determined: Alſo the Moments of the Beginning, Middle, and End thereof, for finding the Longitude of the Place: With ſeveral other Things relating thereto.

6. The Tranſits of *Mercury* and *Venus* over the Face of the Sun are exhibited moſt delightfully by this Inſtrument. They will here appear truly round, well defined, and very black; their comparative Diameters to that of the Sun may this way be obſerved, the Direction of their Motion, the Times of the Ingreſs and Egreſs, with other Particulars for determining the Parallax and Diſtance of the Sun more nicely than has hitherto been done.

7. By the *Solar Teleſcope* you ſee the Clouds moſt beautifully paſs before the Face of the Sun, exhibiting a curious Spectacle according to their various Degrees of Rarity and Denſity. But the beautiful Colours of the Clouds ſurrounding the Sun, and refracting his Rays, are beſt ſeen in the Picture made by the *Camera-Glaſs*. The fine Azure of the Sky, the intenſely ſtrong and various Dyes of the Margins of Clouds, the *Halo's* and *Corona's*, are this way inimitably expreſs'd. And ſince the Priſmatic Colours of Clouds, ſo variouſly compounded here, make ſo noble and delightful a Phænomenon, I have often wonder'd to ſee no more Regard had thereto by Painters, whoſe Clouds (though near the Sun) are ſeldom or never ſeen tinged or variegated with thoſe natural Tints and Colours.

8. I cannot here omit to mention a very *unuſual Phænomenon* that I obſerved about ten Years ago in my darken'd Room. The Window looked towards the

G 3 Weſt,

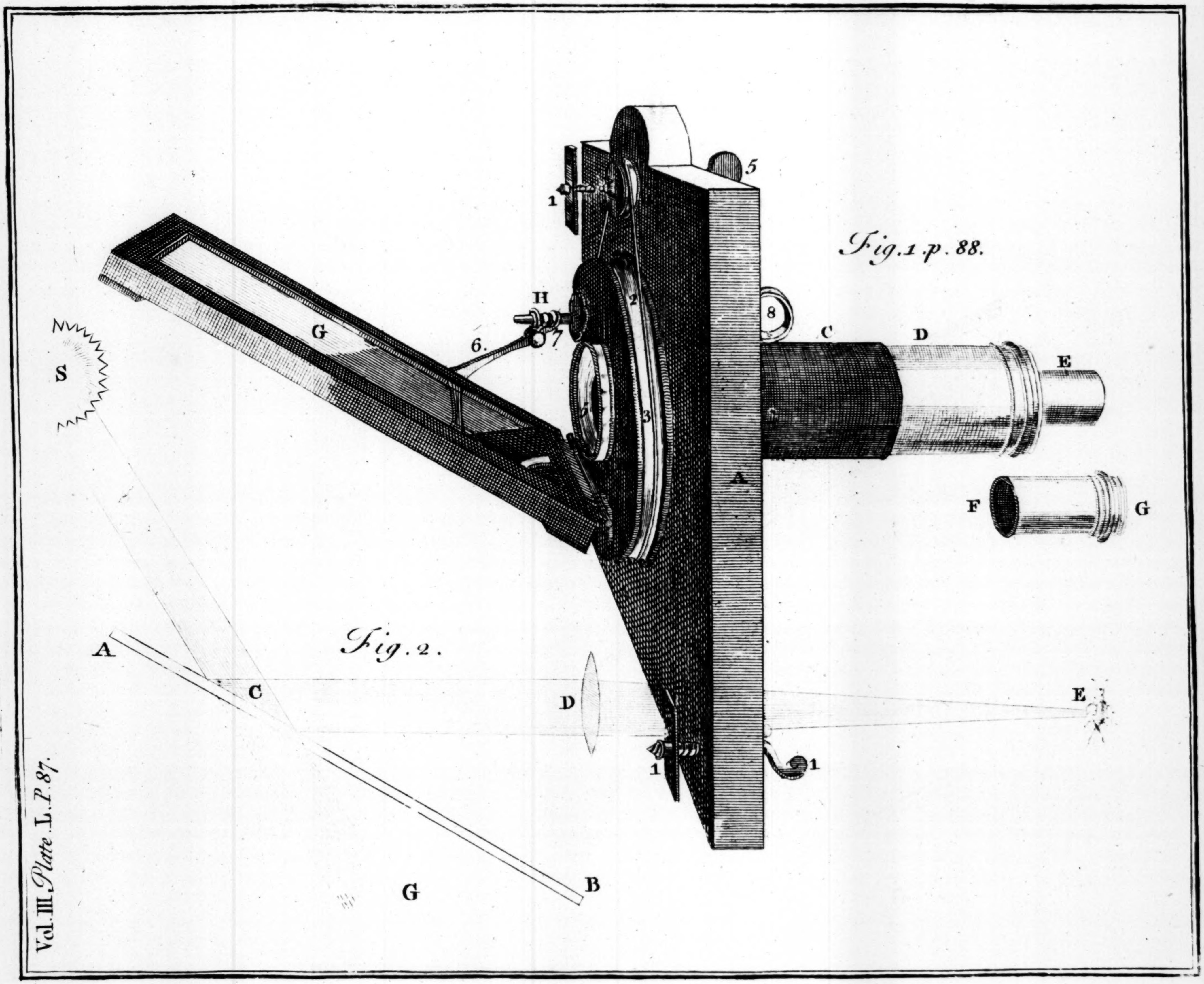
West, and the Spire of *Chichester* Cathedral was directly before it, at the Distance of about 50 or 60 Yards. I used very often to divert myself in observing the pleasant Manner in which the Sun pass'd behind the Spire, and was eclipsed by it for some time; for the Image of the Spire and Sun were very large, being made by a Lens of 12 Feet focal Distance. And once as I observed the Occultation of the Sun behind the Spire, just as the Disk disappear'd, I saw several small, bright, round Bodies or Balls running towards the Sun from the dark Part of the Room, even to the Distance of 20 Inches. I observed their Motion was a little irregular, but rectilinear, and seem'd accelerated as they approach'd the Sun. These luminous Globules appear'd also on the other Side of the Spire, and preceded the Sun, running out into the dark Room, sometimes more, sometimes less, together in the same Manner as they followed the Sun at its Occultation. They appear'd to be in general about $\frac{1}{20}$ of an Inch in Diameter, and therefore must be very large luminous Globes in some Part of the Heavens, whose Light was extinguished by that of the Sun, so that they appeared not in open Daylight; but whether of the Meteor-Kind, or what Sort of Bodies they might be, I could not conjecture.

9. The SOLAR MICROSCOPE (said to be the Invention of Dr. *Liburkun*, a *German*) is a most curious Improvement in *Optics*, and deserves to be greatly valued; as it is the best Method which Nature will admit of, or Art can furnish, for magnifying and exhibiting very small transparent Objects to the View of Spectators.

Plate XLIX. Fig. 3.

10. To this end it has been contrived very commodiously in several different Forms, two of which I shall here illustrate by Diagrams. The first is as follows: A B is a Section of the Window-Shutter of a dark Room, C D of the Frame containing a Scioptric Ball E F; in the Fore-part whereof is screw'd the Tube G I K H, at one End of which is a Lens G H, which by converging the Sun-Beams into a narrow Compass does strongly enlighten the small Object *a b* placed upon a Slip of Glass or otherwise in the Part of the Tube N Q, where a Slit is made on each Side for that Purpose. Within this Tube there slides another L *m n* M, which contains a small magnifying Lens *m n*.

11. By



Vol. III. Plate L. P. 87.

11. By moving the exterior Tube I G H K one way and the other, the Glaſs G H will be brought to receive the Rays of the Sun directly, and will therefore moſt intenſely illuminate the Object *a b*. The other Tube L M being ſlid backwards and forwards will adjuſt the Diſtance of the ſmall Lens *m n*, ſo that the Image of the Object *a b* ſhall be made very diſtinct on the oppoſite Side of the Room at O P; and the Magnitude of the Image will be to that of the Object as its Diſtance from the Lens *m n* is to the Diſtance of the Object from it, as has been ſhewn in *Annot.* CXXV.

12. Thus for Example: Suppoſe the focal Diſtance of the Lens *m n* to be 1 Inch $= r$, and let the Diſtance at which it is placed from the Object be $1,1 = d$; then if the Lens be double, and equally convex (as uſual) the Diſtance of the Image will be $\frac{dr}{d-r} = f = 110$; therefore the Image will be 110 times larger than the Object in its linear Dimenſions, and $110 \times 110 = 12100$ times larger in Surface, and in Solidity it will be $110 \times 110 \times 110 = 1331000$ times larger than the Object.

13. If the Lens, inſtead of 1 Inch, were but $\frac{1}{2}$ an Inch focal Diſtance, then would the Diameter of the Image be twice as large, or 220 times larger than the Object; and the Superficies 4 times larger, *viz.* $4 \times 12100 = 48400$; and the Solidity 8 times larger, *viz.* $8 \times 1331000 = 10648000$, that is, above 10 Millions of times larger than the Object.

14. Once more: For very ſmall Objects we may uſe a Lens $\frac{1}{4}$ of an Inch focal Diſtance, and then the Image at the ſame Diſtance as before will be in Diameter $4 \times 110 = 440$ times larger than the Object; in Superficies, $16 \times 12100 = 193600$ times larger; and in Solidity, $64 \times 1331000 = 85184000$ times larger; that is, any ſolid ſmall Object will at the Diſtance of 9 Feet 2 Inches, by means of a Lens $\frac{1}{4}$ Inch focal Diſtance, be magnified above 85 Millions of times.

15. Or more directly thus: Let the focal Diſtance of the double Convex *m n* be $\frac{1}{4} = r$, and let the Diſtance at which the Image is formed be 12 Feet or 144

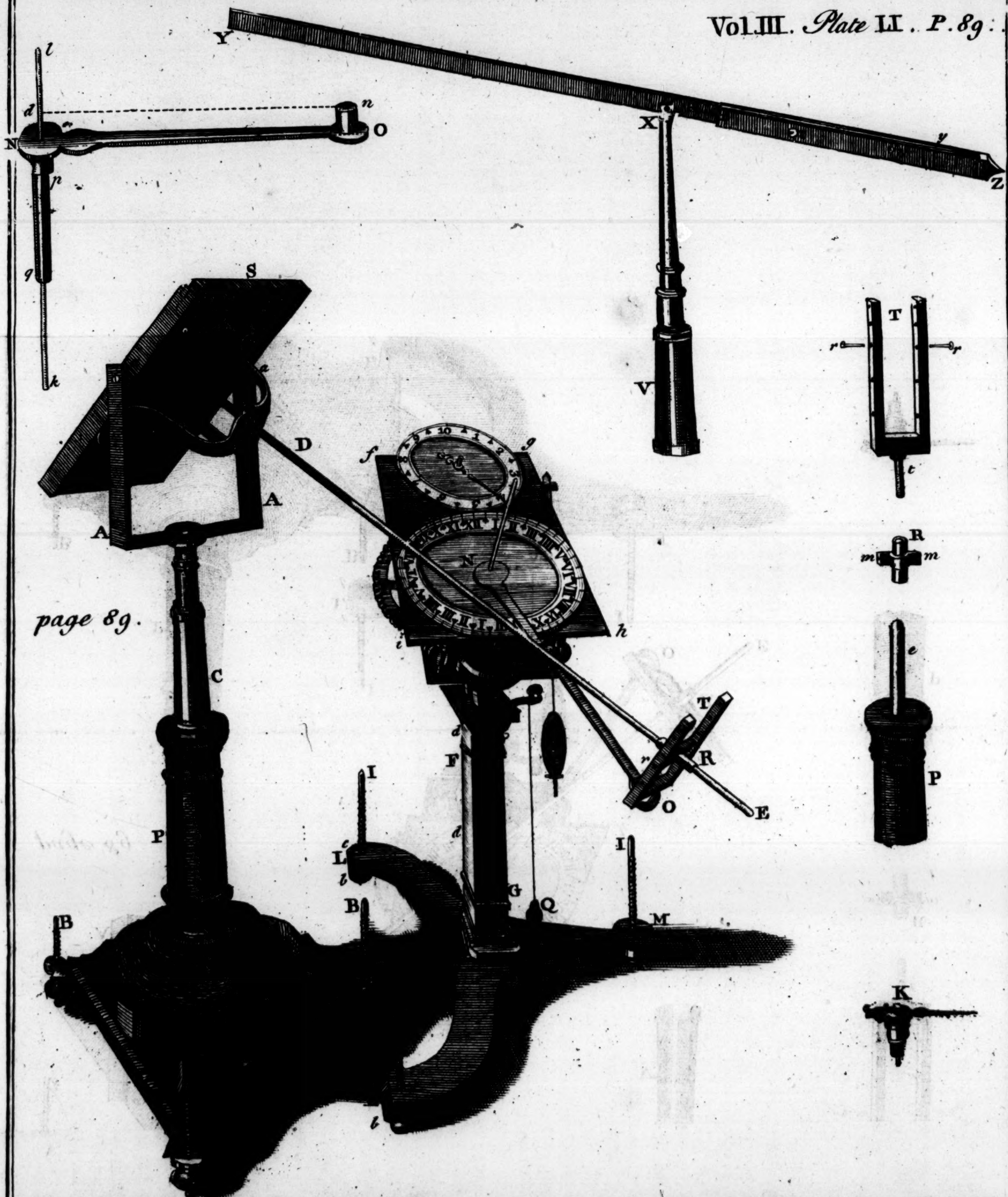
Inches $= f$; then $\frac{rf}{f-r} = d = 0{,}2504$, which therefore may be taken for $\frac{1}{4}$ of an Inch; consequently the Diſtance of the Image is 576 times the Diſtance of the Object from the Lens, and ſo much larger will it be in Diameter, and in Surface it will be $576 \times 576 = 331776$ times larger, and in Solidity it will be $576 \times 576 \times 576 = 191102976$ times larger; Or, a ſmall Blood-Globule, or other ſolid Particle, will be magnified above 191 Millions of times; an Effect prodigious, and incredible to thoſe who are not converſant with Glaſſes, or underſtand not the Rules of *Optics*.

16. If the linear Dimenſions of the Image be nicely taken by a By-Stander with a graduated Scale of equal Parts, the Dimenſion of the Object will be known of courſe from the Diſtances of the Image and Object from the Lens; and in exceeding ſmall Objects, ſuch as the Pores of Cork, the Particles of Blood, *Animalcula in Semine*, &c. there is no other way of meaſuring them ſo well: And thus the *Solar Microſcope* becomes a *Micrometer* in the laſt Degree of poſſible Menſuration.

17. The Form of this Inſtrument, as it has been hitherto deſcribed, is that which I have contrived for my own Uſe, and for theirs who regard more the general Convenience than the Grandeur of an *Apparatus*. However, that thoſe of a different Taſte may be gratified, the common Form is to be very much commended for their Uſe; of which it will be ſufficient to give a bare Deſcription, illuſtrated by a Print.

Plate L. Fig. 1. 18. This Inſtrument conſiſts of ſeveral Parts, *viz.* A, a ſquare Frame of Mahogany to be fixed to the Shutter of a Window by means of the Screws 1, 1. To this Frame is applied a circular Collar B of the ſame Wood, with a Groove on its Periphery on the Outſide, denoted by 2, 3. This Collar is connected by a Cat-Gut to the Pulley 4 on the upper Part, which is turned round by the Pin 5 within. On one Part of the Collar, on the Outſide, is faſtened by Hinges a Looking-Glaſs G in a proper Frame, to which is fixed the jointed Wire, 6, 7; by which means, and the Screw H 8, it may be made to ſtand in an Angle more or leſs inclined to the Frame.

In



Y
X
y
Z
l
d
n
N
O
p
q
k
S
a
D
A
A
V
T
r
r
t
R
m
m
f
g
N
i
h
C
e
T
r
R
O
E
d
F
d
I
L
e
b
B
P
G
Q
I
M
B
K
b

page 89.

In the Middle of the Collar is fixed a Tube of Brafs C, near two Inches in Diameter; the End of which on the Outfide, has a convex Lens 5 to collect the Sun-Beams thrown on it by the Glafs G, and converging them towards a Focus in the other Part, where D is a Tube fliding in and out, to adjuft the Object to a due Diftance from the Focus. To the End G of another Tube F is fcrew'd one of *Wilfon's Single Pocket Microfcopes*, containing the Object to be magnified in a Slider; and by the Tube F, fliding on the fmall End E of the other Tube D, it is brought to a due focal Diftance.

19. The great Artifice and Conveniency of this Solar Microfcope is, that by means of the Glafs G the oblique Rays of the Sun are made to go ftrait along the dark Room parallel to the Floor, inftead of falling upon it. Thus let A B denote a Section of the Looking-Glafs, and S C the Rays of the Sun impinging upon it at C, by which they are reflected to the Lens D, and from thence converged towards E to illuminate the Object to be magnified; fo that the Beam of Light goes from C to E in the Direction parallel to the Floor, inftead of falling on it in the Direction S G. By the Pulley 4, 5, the Glafs is turned directly to the Sun, and by the jointed Wire and Screw at H it is elevated or deprefs'd, fo as to bring the Glafs into the Pofition A B required, where the Angle of Incidence A C S is equal to the Angle of Reflection B C E. Mr. *Liburkun*, a *Pruffian* Gentleman, was the firft who invented this Method of magnifying Objects, but without the Looking-Glafs, which was afterwards added to it. The Theory of this Contrivance and the *Magic Lanthorn* is the fame; only here we make ufe of Sun-Beams inftead of Candle-Light, and the Object and magnifying Lens are of the fmalleft Size.

20. Another moft egregious Contrivance of this Sort we have from the late learned Dr. *s'Gravefande*, which he calls by the Name of HELIOSTATA, from its Property of *fixing* (as it were) *the Sun-Beam* in one Pofition, *viz.* in an *horizontal Direction* acrofs the dark Chamber all the while it is in ufe. It is an *Automaton*, or Piece of Clock-work, whofe Parts are as follow. A A is a Frame in which a metalline Speculum S is fuf- Plate LI.
pended,

pended, moveable about its Axis by means of two ſmall Screws at *a a*. This Frame is fixed to the Piece C, which being hollow is moveable upon the cylindric Shaft P about the Iron Pin *e*. (See the Part by itſelf.) This Pillar P is fixed to a triangular Baſe or Foot ſet perpendicular by the three Screws B, B, B.

21. On the Back-part of the Speculum is fixed a long cylindric Wire or Tail D, in a perpendicular Poſition. By this it is connected to the ſecond Part of the *Helioſtata*, which is a common Thirty-Hour Clock, repreſented at H; the Plane of which Clock is parallel to that of the Equator in any given Place. This Clock is ſuſtained on the Column F G, in which it is moveable up and down by a thin *Lamina* or Plate that enters it as a Caſe, and fixed to a proper Height by the Screws *d*, *d*, at the Side. The Whole is truly adjuſted to a perpendicular Situation by means of the three Screws I, I, I, in the Tripod L L M, and the Plummet Q, whoſe *Cuſpis* muſt anſwer to the Point *o* beneath.

22. The Axis of the Wheel, which moves the Index N O over the Hour-Circle, is ſomewhat large, and perforated with a cylindric Cavity verging a little to a conical Figure; and receives the Shank *p q* of the ſaid Index N O very cloſe and tight, that by its Motion the Index may be carried round. In the Extremity O of the Index is a ſmall cylindric Piece *n*, with a cylindric Perforation to receive the Tail *t* of the Fork T, yet ſo as to admit a free Motion therein. In each Side of the Fork are ſeveral Holes exactly oppoſite to each other, in which go the Screws *r*, *r*, upon whoſe ſmooth cylindric Ends moves the tubular Piece R on its Auricles *m*, *m*.

23. When the Machine is to be fixed for Uſe, another Part is made uſe of to adjuſt it; which is call'd the *Poſitor*, and is denoted by the Letters V X Y Z. The Cylinder C is removed with the Speculum from the Foot P, and the Braſs Column V X put on in its ſtead, and adheres more ſtrictly to the Pin *e*, that it may keep its Poſition while the Machine is conſtituted.

24. On the Top of the Column, about X as a Centre, moves the Lever Y Z, ſo that it may be any how inclined to the Horizon, and keep its Poſition. The

The Arm YX may be of any Length at Pleaſure, but the Arm YZ is of a peculiar Conſtruction, and of a determinate Length. To this Arm, which extends no farther than *y*, is adapted a Sliding-Piece Z*x* ſharp-pointed at Z. By this the Arm XZ is determined to a given Length, the Piece Z*x* being fixed by the Screws *z z*.

25. Upon this Arm is drawn the ſhort Line *v x*, by which it may be lengthened in the Whole, and is $\frac{9}{100}$ of the whole Length XZ when ſhorteſt. The Reaſon is, this Arm is always to increaſe and decreaſe in Proportion to the Secant of the Sun's Declination to the Radius XZ when ſhorteſt; but the Radius is to the Secant of 23° 30′ (the Sun's greateſt Declination) as 10000000 to 10904411, or as 100 to 109.

26. Now the Reaſon of this Conſtruction of the Arm XZ is to find for any given Day the Diſtance of the Centre of the Speculum S from the Top *l* of the Style *l*N, which muſt ever be equal to the Secant of the Sun's Declination; for it muſt always be equal to the Diſtance of the Top of the ſaid Style *l* from the Centre of the Cylinder R in the Fork T, and that is ever equal to the ſaid Secant of Declination.

27. For ſince the Style *l*N and the Fork T are in a Poſition parallel to each other, therefore the middle Hole in the Sides of the Fork being (as they muſt be) of the ſame Height above the End of the Index O as is the Height of the Style NT, 'tis evident that on an equinoctial Day the Sun's Rays will paſs directly through the Perforation of the Piece R, if it be put in a Poſition parallel to the Plane of the Ecliptic, or that of the Clock; and alſo that the Top of the Shadow of the ſaid Style will fall exactly on the ſaid Hole.

28. In this Caſe the Top of the Style is at the leaſt Diſtance from the central Point of R, and therefore may be repreſented by *Radius*, while in any other Poſition above or below, the Diſtance will increaſe in Proportion to the Secant of the Angle which the Rays make with this firſt or middle Ray, that paſs by the Top of the Style, and through the Hole R.

29. Now it may be demonſtrated, that on any Day of the Year, if the Clock and its Pedeſtal be ſo fixed that

that the Line of XII be exactly in the Meridian, and that the Position of R in the Fork be such that the Sun's Rays go directly through it, and the Shadow of the Style's Top fall just upon the Hole; moreover, if the Distance of the Centre of the Speculum S from the Top of the Style *l* be made equal (by the *Positor*) to the Distance of the central Point of R therefrom; and lastly, the Tail of the Speculum DE passing through R; if then the Clock be put into Motion, the Index NO shall carry about the Tail of the Speculum in such a Manner, that at all Times of that Day when the Sun can come upon the Speculum, it will reflect the Rays constantly in one and the same Position and Direction all the time without Variation.

30. The Machine thus constituted is placed in a Box or Case, and set in a Window with one Side open, exposed to the Sun, and all the other Parts close; so that when the Room is made dark, and the Solar Microscope fixed to the Forepart of the Box in which the *Heliostata* is placed, just against the Centre of the Speculum to receive the reflected horizontal Beam, all the Experiments of the darken'd Room are then performed as usual. This is a very ingenious Construction of a *Solar-Microscope Apparatus*, and full of Art, but, I fear, too expensive and troublesome for common Use. However, 'tis easy to see that this Machine is capable of being greatly reduced; for it may be made to answer the End very well without a Clock; also the Speculum may be Glass instead of Metal, and all fix'd on one Foot or Pedestal: But this I leave to the Ingenuity of the Mechanical Reader.

LECTURE XI.

Of ASTRONOMY; *and the Use of the* ORRERY *and* COMETARIUM.

Of the UNIVERSE; *an* INFINITY *of* SYSTEMS; *of the* PTOLEMAIC SYSTEM; *the* TYCHONIC SYSTEM; *of the* COPERNICAN *or* SOLAR SYSTEM *of the World. The* Extent *and* Constituent PARTS *thereof.* ARGUMENTS *for the Truth thereof.* DEMONSTRATIONS *of its Truth. Of the* SUN; *the* PRIMARY PLANETS; *the* Secondary Planets, *or* MOONS. *The* COMETS. *Of the* Magnitude, Motion, Maculæ, *&c. of the* SUN. *Of the* Number, Order, Magnitude, Distances, *&c. of the* PLANETS; *their Periods*; *of the* Nodes, Inclination, *and* Aphelia *of their* ORBITS. *Of the* MOON, *its* Phases, Period, Distance, Magnitude, *and* Light. *Of the* SATELLITES *or* Moons *of* Jupiter *and* Saturn. *Of* Saturn's RING. *The* MATHEMATICAL THEORY *of the* CELESTIAL MOTION, *with* CALCULATIONS *and* EXAMPLES.

EXAMPLES. *Of the* ORRERY: *an historical Account of the* Invention *and* Improvements *thereof. A Description of the* ARMILLARY SPHERE. *Of the* MOTION *of the* EARTH *about its* Axis, *and about the* Sun. *The* VICISSITUDES *of the* SEASONS *explained. Of the various* LENGTHS *of* DAYS *and* NIGHTS. *The* Third Motion *of the* Earth; *the great* PLATONIC YEAR; *the* RECESSION *of the* EQUINOXES *explained. A Calculation of the hottest Time of the Day. The* Doctrine *of* Solar *and* Lunar ECLIPSES *fully explained, by Calculations on a* Mathematical Theory. *An Explanation of the* ASTRONOMY *of* COMETS. *A* new Method *for* Construction *of their* ORBITS. Calculations *relating to the whole* THEORY *of Comets. An* Analytical Investigation *of their* Elliptic Orbits. *Of their* TAILS, *and all other* Phænomena *accounted for on the* genuine Principles *of* Physics.

I SHALL in this Lecture endeavour to exhibit to you a *just and natural Idea of the Mundane or Solar System*, that is, the System of the World; consisting of the *Sun*; the *Primary Planets*, and their *Secondaries* or *Moons*; the *Comets*; and the *Fixed Stars*;

Stars; according to the Hypothesis of *Pythagoras* among the Ancients, and revived by *Copernicus*: Which System is fully proved, and established on the justest Reasoning, *viz.* *Physical* and *Geometrical Conclusions*, by all our modern Astronomers. (CXXXI.)

THE

(CXXXI.) 1. By the UNIVERSE we are to understand the whole Extent of Space, which, as it is in its own Nature every way infinite, gives us an Idea of the Infinity of the Universe, which can therefore be only in Part comprehended by us: And that Part of the Universe which we can have any Notion of, is that which is the Subject of our Senses; and of this the Eye presents us with an Idea of a vast extended Prospect, and the Appearance of various Sorts of Bodies disseminated through the same.

2. The infinite Abyss of Space, which the *Greeks* called the τὸ πᾶν, the *Latins* the *Inane*, and we the *Universe*, does undoubtedly comprehend an Infinity of Systems of moving Bodies round one very large central one, which the *Romans* called *Sol*, and we the *Sun*. This Collection of Bodies is therefore properly called the SOLAR SYSTEM, and sometimes the MUNDANE SYSTEM, from the *Latin* Word *Mundus*, the *World*.

3. That the Universe contains as many Solar Systems or Worlds as there are what we call *Fix'd Stars*, seems reasonable to infer from hence: That our Sun removed to the Distance of a Star would appear just as a Star does, and all the Bodies moving about it would disappear entirely. Now the Reason why they disappear is because they are opake Bodies, and too small to be seen at so great a Distance, without an intense Degree of Light; whereas theirs is the weakest that can be, as being first borrowed and then reflected to the Eye.

4. But the Sun, by reason of his immense Bulk and innate Light, which is the strongest possible, will be

visible

THE most celebrated Hypotheses or Systems of the World are three, *viz.* (1.) The

visible at an immense Distance; but the greater the Distance, the less bright it will appear, and of a lesser Magnitude: And therefore every Star of every Magnitude may probably be a Sun like our own, informing a System of Planets or moving Bodies, each of which may be inhabited like our Earth with various Kinds of Animals, and stored with vegetable and other Substances.

5. In this View of the Universe, an august Idea arises in the Mind, and worthy of the Infinite and Wise Author of Nature, who can never be supposed to have created so many glorious Orbs to answer so trifling a Purpose as the twinkling to Mortals by Night now and then; besides that the far greatest Part of the Stars are never seen by us at all, as will be farther shewn when we come to treat of those celestial Bodies.

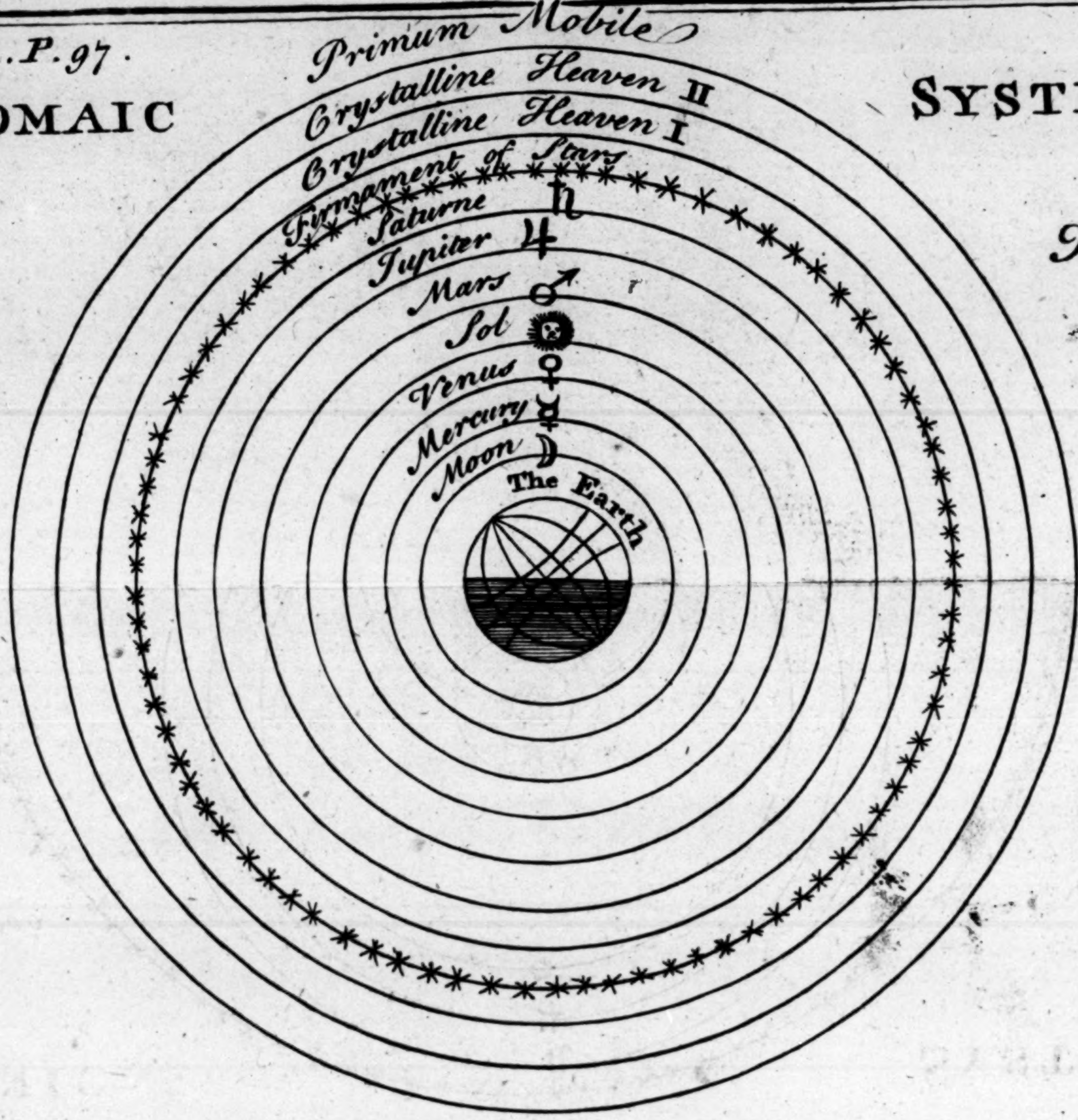
6. When therefore *Moses* tells us, that *in the Beginning God created the Heavens and the Earth*, it is to be understood in a limited Sense, and to mean only the *Making*, or rather *New-making* of our Terraqueous Globe; for 'tis expresly said that the Earth in its first State was a *Chaos*, (in *Hebrew* תהו ובהו *Shapeless and Void*) which probably might be only the Ruins of a pre-existent Globe inhabited by rational Creatures in the same Manner as since its Renovation. And though it be said, *God made two great Lights*, the *Sun* and the *Moon*, it does not follow they had no Existence before that Time, any more than it does that the Stars had not, which he is said to have *made also*.

7. Now if the Stars had no Existence before the *Mosaic* Creation, then were there no other Systems of Worlds before our own; then must all the Infinity of Space have been one eternal absolute *Inane* or *Empty Space* till that Time, and God who *made the Worlds* must be supposed to have made them all at once: Which Suppositions are too extravagant and unreasonable, and therefore cannot be the Sense of that Passage of Scripture; which I think

Vol. III. Plate LII. P. 97.

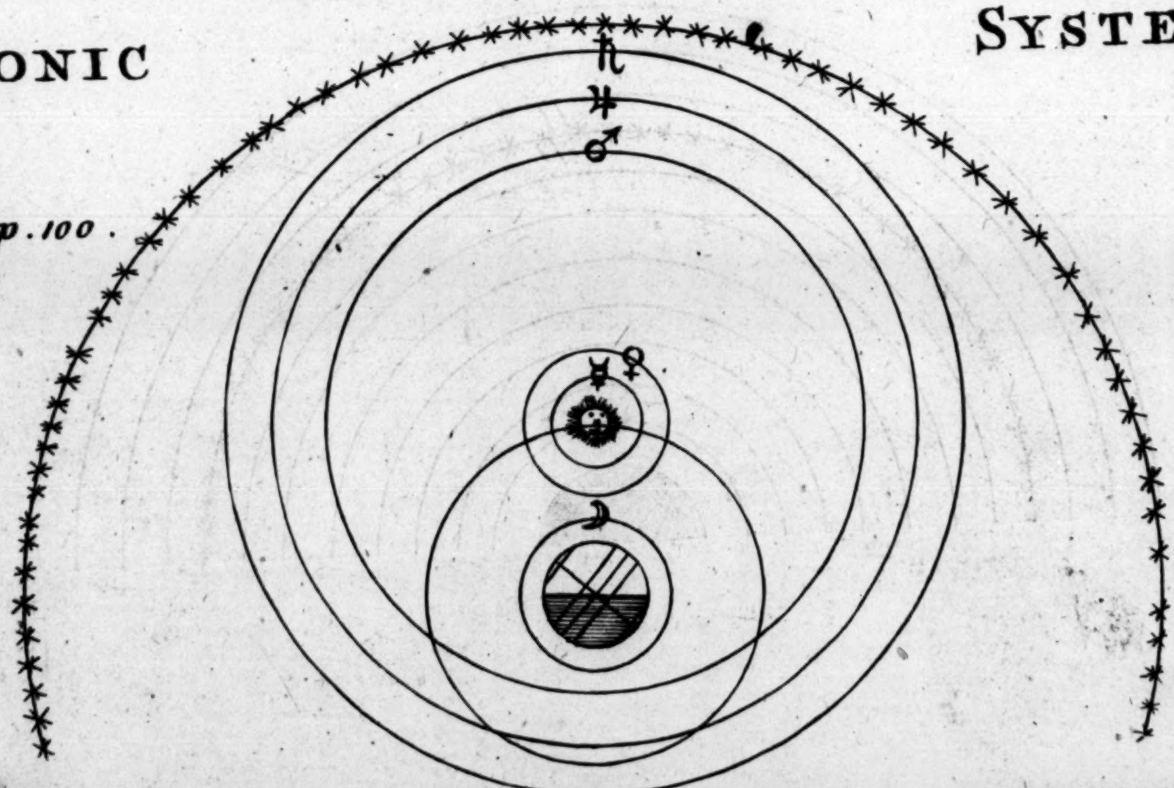
The PTOLOMAIC SYSTEM

Fig. 1. p. 97.



TYCHONIC SYSTEM

Fig. 2. p. 100.



The *Ptolomean*, invented by *Ptolomy*, an ancient *Egyptian* Philoſopher, which aſſigns ſuch Poſitions and Motions to the heavenly Bodies, as they appear to the Senſes to have. (2.) The *Tychonic Syſtem*, or that of the noble *Daniſh* Philoſopher, *Tycho Brahe*. (3.) The *Pythagorean*, *Copernican*, or *Solar Syſtem*, above-mentioned. Of all which in Order. (CXXXII.)

THE

I think can be no more than this, That when God had formed the Earth into an habitable Globe, he gave it ſuch a Poſition and Motion about the Sun, and about its own Axis, as ſhould cauſe an agreeable Variety in the Length of Days and Nights, and in the Temperature of the Seaſons of the Year: All which will be ſhewn to have their Exiſtence and Diſtinction reſulting from theſe Principles, and no other, in the Sequel of the Notes to this *Lecture*.

(CXXXII.) 1. I have thought it expedient to illuſtrate the Idea of the three remarkable Syſtems of the World above-mentioned by proper Diagrams; in the Firſt of which you view the Diſpoſition of the Heavenly Bodies according to the Hypotheſis of *Claudius Ptolomæus*, a famous Mathematician and Aſtronomer of *Peluſium* in *Egypt*, who lived in the firſt Part of the ſecond Century after *Chriſt*. Pl. LII. Fig. 1.

2. This was firſt invented and adhered to chiefly becauſe it ſeemed to correſpond with the ſenſible Appearances of the Celeſtial Motions. They took it for granted that the Motions which thoſe Bodies appeared to have were ſuch as they truly and really performed; and not dreaming of any Motion in the Earth, nor being apprized of the Diſtinction of *abſolute*, *relative*, or *apparent Motion*, they could not make a proper Judgment of ſuch Matters, but were under a neceſſity of being miſ-

PlateLII. Fig. 1. The *Ptolomean System* supposes the Earth immoveably fixed in the Centre, not of the *World*

led by their very Senses, for want of proper Assistance which After-Ages produced.

3. 'Tis easy to observe they had no Notion of any other System but our own, nor of any other World but the Earth on which we live. They thought nothing less than that all Things were made for the Use of Man; that all the Stars were contained in one concave Sphere, and therefore at an equal Distance from the Earth; and that the *Primum Mobile* was circumscribed by the *Cœlum Empireum* of a cubic Form, which they supposed to be the *Heavens*, or blissful Abode of departed Souls.

4. It would scarce have been worth while to have said so much about so absurd an Hypothesis, (as this is now well known to be) were it not that there are still numerous Retainers thereto, who endeavour very zealously to defend the same, and that for two Reasons principally, *viz.* because the Earth is apparently fixed in the Centre of the World, and the Sun and Stars move about it daily; and also because the Scripture asserts the Stability of the Earth, the Motion of the Sun, &c.

5. These two Arguments merit no particular Answer. It is sufficient, with respect to the first, to say, that we are assured Things may (yea must) appear to be, in many Cases, what they really are not, yea to have such Affections and Properties as are absolutely contrary to what they really possess. Thus a Person sitting in the Cabin of a Ship under Sail, will, by looking out at the Window, see an apparent Motion of the Houses, the Trees, &c. on the Strand the contrary Way, but will perceive no Motion at all in the Ship. Also a Person sitting in a Windmill, if the Mill be turned about, will see an apparent Motion of the upright Post the contrary Way, but will not perceive any in the Mill itself.

World only, but of the *Universe*; and that the *Sun*, the *Moon*, the *Planets*, and *Stars*, all

6. All those Cases are exactly parallel to that of the Earth, (the Reason of which has been shewn in the former Part of this Work, *Annot.* XX.) and it is as rational to assert the Ship and the Mill are really quiescent, and the other Bodies positively in Motion, as it is to insist on the Motion of the Sun, and the Earth's being at rest in the Centre.

7. As to the Scripture, as it was never intended for an Institution of Astronomy or Philosophy, so nothing is to be understood as strictly or positively asserted in relation thereto, but as spoken only agreeably to the common Phrase or Vulgar Notion of Things. And thus Sir *Isaac Newton* himself would always say, the *Sun rises*, and the *Sun sets*; and would have said with *Joshua*, *Sun stand thou still*, &c. though he well knew it was quite contrary in the Nature of the Thing.

8. How ridiculous a Thing does Popery appear to be to all rational Minds, or to those who are at liberty to *think*, by insisting on the literal Sense of Scripture so rigidly in the Expression, *This is my Body!* And is it not equally absurd to maintain that the *Earth stands upon Pillars*, only because we read so in the Bible? What an aukward Shift are those celebrated Mathematicians Mess. *Le Seur* and *Jacquier* obliged to make, in their Commentary on Sir *Isaac*'s *Principia!* The Editor, forsooth, is here the Commentator on all those Parts that relate to the Earth's Motion or *Copernican* System: And because their Declaration is something very singular in its Kind, I shall here give it in their own Words.

PP. LE SEUR & JACQUIER Declaratio.

Newtonus in hoc tertio libro Telluris motæ hypothesin assumit. Autoris Propositiones aliter explicari non poterant, nisi eâdem quoque factâ hypothesi. Hinc alienam coacti sumus gerere personam; cæterum latis à summis Pontificibus contra Telluris Motum Decretis nos obsequi profitemur.

all moved about it from *East* to *West* once in twenty-four Hours, in the Order following,

In *English* thus:

"*Newton* in this Third Book has assumed the Hypo-
"thesis of the Earth's Motion. The Author's Propo-
"sitions are not to be explain'd but by making the
"same Hypothesis also. Hence we are obliged to pro-
"ceed under a feigned Character; but in other Re-
"spects we profess ourselves obsequious to the Decrees
"of the Popes made against the Motion of the Earth."

9. By this it appears how well many People understand the Truth, who yet dare not to profess it. But to conclude this Head: There is no Authority equal to that of Truth; the common Opinion, the literal Expression of Scripture, the Decrees of Popes, and every thing else must give way to plain and evident Demonstration; of which we have abundantly sufficient for establishing the true System of the World against all Opposition.

10. The TYCHONIC SYSTEM is represented in the next Diagram. This had its Original from *Tycho Brahe*, a Nobleman of *Denmark*, who lived in the latter Part of the last Century; he built and made his Observations at *Uraniburg*, (i. e. *Celestial Tower*) in the Island *Weer* or *Huena*. This Philosopher, though he approved of the *Copernican* System, yet he could not reconcile himself to the Motion of the Earth; and being, on the other hand, convinced the *Ptolomean* Scheme in part could not be true, he contrived one different from either, which is represented by the next Diagram.

11. In this the Earth has no Motion allowed it, but the Annual and Diurnal Phænomena are solved by the Motion of the Sun about the Earth, as in the *Ptolomaic* Scheme; and those of *Mercury* and *Venus* are solved by this Contrivance, though not in the same Manner, so simply and naturally, as in the *Copernican* System; as is easy to observe in the *Figure*.

12. After

lowing, *viz.* the *Moon*, *Mercury*, *Venus*, the *Sun*, *Mars*, *Jupiter*, *Saturn*, the *Fixed Stars*; and, above all, the Figment of their *Primum Mobile*, or the Sphere which gave Motion to all the reſt. But this was too groſs and abſurd to be received by any learned Philoſopher, after the Diſcoveries by Obſervations and Inſtruments which acquaint us with divers Phænomena of the heavenly Bodies, altogether inconſiſtent with, and, in ſome Things, exactly contradictory to, ſuch an Hypotheſis; as will be ſhewn by the Arguments adduced to prove the Truth of the *Copernican Syſtem.*

THE *Tychonic Syſtem* ſuppoſed the Earth in the Centre of the World, that is, of the Firmament of Stars, and alſo of the Orbits of the Sun and Moon; but at the ſame time it made the Sun the Centre of the Planetary Motions, *viz.* of the Orbits of *Mercury*, *Venus*, *Mars*, *Jupiter*, and *Saturn*. Thus Pl. LII. Fig. 2.

12. After this Scheme had been propoſed ſome time, it received a Correction by allowing the Earth a Motion about its Axis, to account for the Diurnal Phænomena of the Heavens; and ſo this came to be called the *Semi-Tychonic Syſtem*. But this was ſtill wide of the Truth, and encumbered with ſuch Hypotheſes as the true Mathematician and genuine Philoſopher could never reliſh. Therefore both theſe Syſtems, and all others, at length gave way to the True Solar Syſtem, o be more fully deſcribed in the following Notes.

Thus the Sun, with all its Planets, was made to revolve about the Earth once a Year, to solve the *Phænomena* arising from the *annual Motion*; and the Earth about its Axis from West to East once in 24 Hours, to account for those of the *diurnal Motion*. But this *Hypothesis* is so monstrously absurd, and contrary to the great Simplicity of Nature, and in some respects even contradictory to Appearances, that it obtained but little Credit, and soon gave way to

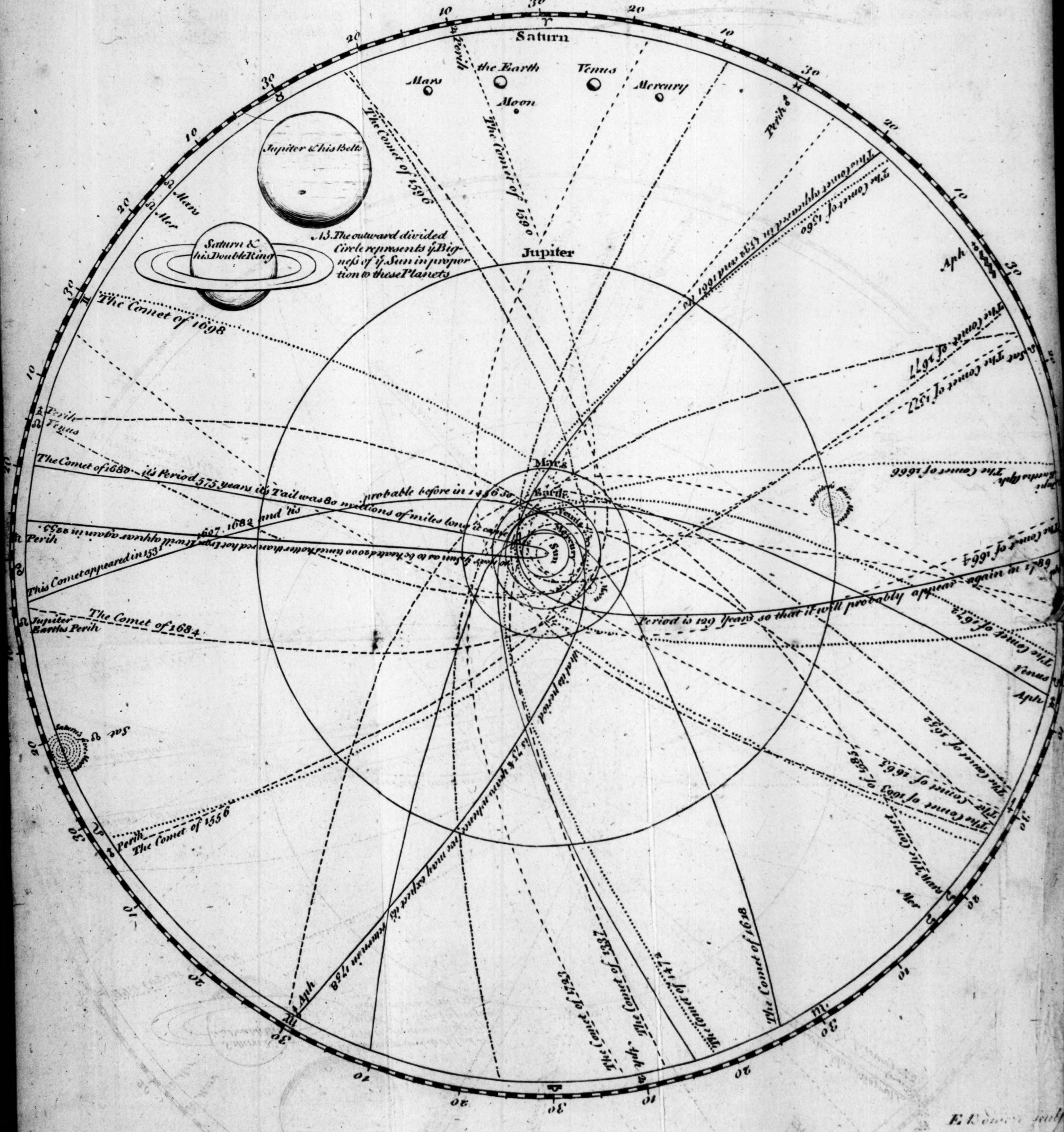
Pl. LIII. THE *Copernican System* of the World, which supposes the Sun to possess the central Part; and that about it revolve the *Planets* and Comets in different Periods of Time, and at different Distances therefrom, in the Order following, *viz.* (CXXXIII.)

I. MER-

(CXXXIII.) 1. The SOLAR SYSTEM, as it is now taught, was in some part invented by the Ancients, perhaps by *Pythagoras* himself; for though *Diogenes Laertius* in writing his Life says no more of him than *his asserting the Antipodes of the Earth*, yet *Aristotle* tells us that the Sect of the *Pythagoreans* taught *that the Earth was carried about the Centre*, (viz. the Sun) *among the Stars*, (i. e. the Planets) *and by turning about* (its Axis) *caused Day and Night*. Hence it came to be called the PYTHAGOREAN HYPOTHESIS OR SYSTEM *of the World*.

2. But some of these, 'tis said, allowed only one Motion of the Earth, *viz.* the *diurnal*; while others, as *Philolaus*, *Aristarchus* the *Samian*, *Plato* in his advanced Age, also *Seleucus* the Mathematician, and others, maintained

THE COPERNICAN OR SOLAR SYSTEM



E. Bowen sculp

1. MERCURY, at the Diſtance of about 32 Millions of Miles, revolves about the Sun

maintained the Earth had two Motions, the *diurnal* about its Axis, and the *annual Motion* about the Sun. Hence it is alſo called the PHILOLAIC SYSTEM.

3. But the Aſtronomy of theſe early Ages died in its Infancy, and was buried in Oblivion for many Ages after; till at length it began to be revived by Cardinal *Cuſa*, who wrote in Defence of it, but to no great Purpoſe; till after him it was eſpouſed by the celebrated *Nicholas Copernicus*, a Canon of *Thorn* in *Poliſh Pruſſia*, where he was born *A. D.* 1473. This Gentleman undertook to examine it thoroughly, and explained by it the Motions and Phænomena of the Heavenly Bodies ſo well to the Satisfaction of the Learned, that he was generally followed therein by the principal Aſtronomers of that and the following Age; as *Rheticus*, *Rothmannus*, *Lanſbergius*, *Schikardus*, *Kepplerus*, *Galileo*, and numberleſs others. From this Time it was called the COPERNICAN SYSTEM.

4. After this aroſe divers great Men, as *Gaſſendus*, *Hevelius*, *Bullialdus*, *Ricciolus*, the two *Caſſini's*, Mr. *Hugens*, *Horrox*, Biſhop *Ward*, Mr. *Flamſteed*, Dr. *Halley*, Dr. *Gregory*, Dr. *Keil*, and, above all, that ſuperlative Genius Sir *Iſaac Newton*; who all of them, with the greateſt Pains and Diligence, applied themſelves to make Obſervations, to invent Inſtruments, and to inveſtigate the Phyſical Cauſes of Celeſtial Phænomena; in which they ſo happily ſucceeded, eſpecially the laſt great Man, that the Nature, Extent, Order, and Conſtitution of all and every Part of the Solar Syſtem, both of Planets and Comets, became ſo well defined, ſtated and eſtabliſhed, as to admit of no Conteſt or Scruple, with any Man properly qualified to underſtand it; and which therefore ought for the future to be called the NEWTONIAN SYSTEM *of the World*.

5. This SYSTEM (no longer now to be called an *Hypotheſis*) is repreſented in a Plate by itſelf with the Orbits of all the Planets and Comets (hitherto deter- Pl. LIII.

Sun in the Space of 87 Days, 23 Hours, and 16 Minutes.

II. VENUS,

mined) and at their proper Distances from the Sun, represented by the central Point; it being impossible to represent, either by an Instrument or Diagram, the true Proportion both of Magnitude and Distances of the Sun and Planets, as will appear by what follows.

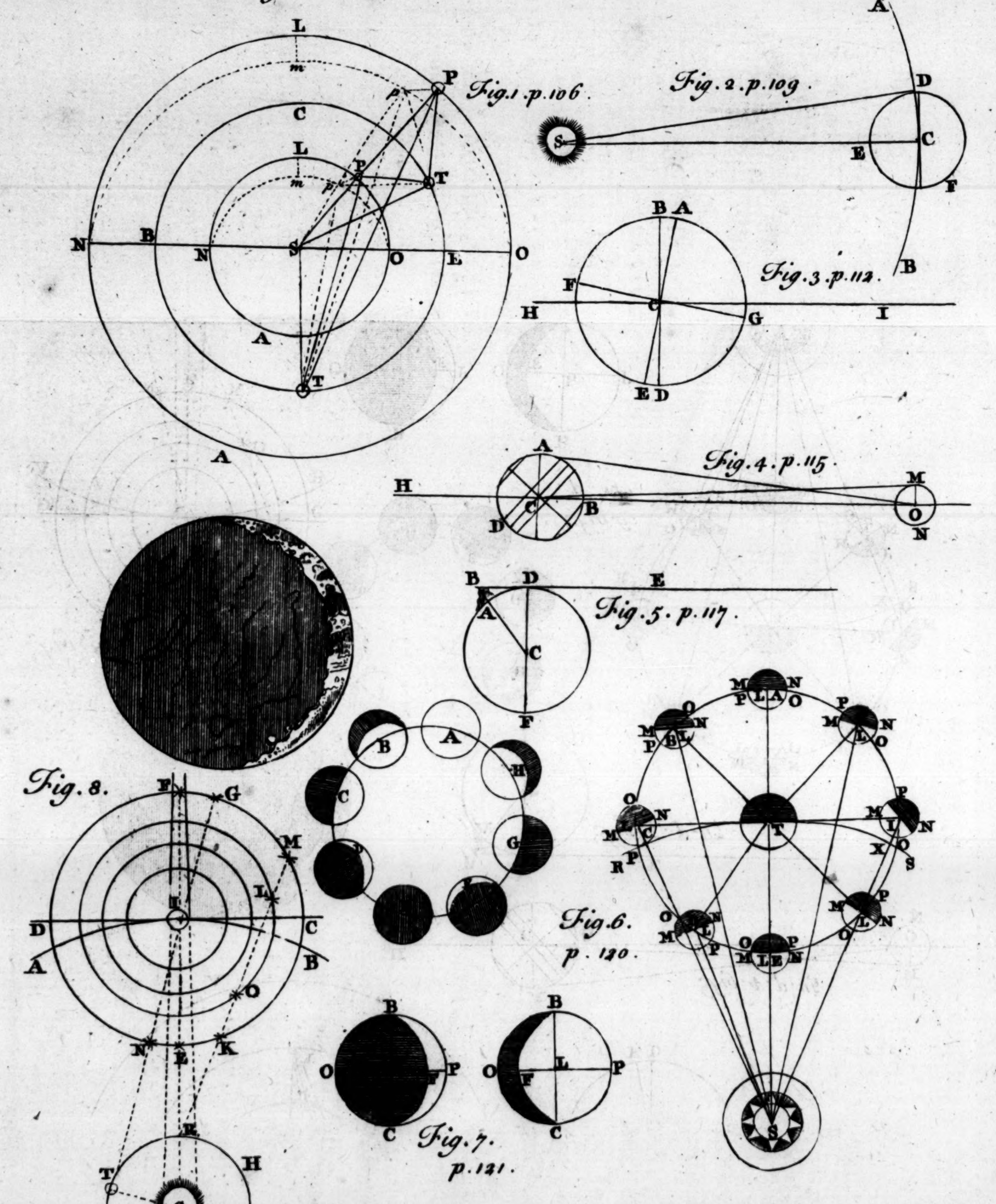
6. For it must be allowed, that to render any Machine or Delineation useful, the least Part ought to be visible; and one cannot well assign a less Bulk for the Globe of the Moon, than what is here represented in this Plate; which being fixed upon, the Magnitudes of the Planets *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn* and its Ring, must be such as are shewn under the respective Names in the Plate; and with respect to these the Sun's Bulk or Face will be represented by the exterior Circle of the Diagram, which here represents the Ecliptic in the Heavens, and is nearly 9 Inches in Diameter.

7. Now the Diameter of the Earth in this Scheme is $\frac{1}{10}$ of an Inch, its Semidiameter is therefore $\frac{1}{20}$; and the Distance of the Earth from the Sun's Centre is about 20000 Semidiameters. But $20000 \times \frac{1}{20} = 1000$ Inches $= 83\frac{1}{3}$ Feet; and since the Distance of *Saturn* is near ten times as great, it is evident *the Extent or Diameter of a Machine to exhibit the several Parts of the Solar System in their due Proportion of Distances and Magnitudes (though no bigger than those here assign'd) will be at least* 1600 *Feet, or more than a Quarter of a Mile: And consequently the Circumference of* Saturn's *Orbit will measure very near a Mile.*

8. In a much less Compass indeed the Distances might be represented very well in Proportion, but the respective Magnitudes can no otherwise be shewn than by such Globes or graphical Delineations as in the Plate of the Diagram under Consideration. Another Thing which cannot be properly represented in such a Plate is the Inclination of any Planetary Orbit to the Plane of the Ecliptic, especially the Orbits of the Comets, of

whose

Vol. III. *Plate* LIV. *P.* 105.



II. VENUS, at the Diſtance of 59 Millions of Miles, in 224 Days, 16 Hours, 49 Minutes.

III. THE EARTH, at the Diſtance of about 82 Millions of Miles, in 365 Days, 6 Hours, 9 Minutes, or *Sydereal* Year.

IV. MARS, at the Diſtance of 123 Millions of Miles, in 686 Days, 23 Hours, 27 Minutes, or 1 Year, 321 Days, 17 Hours, and 18 Minutes.

V. JUPITER, at the Diſtance of 424 Millions of Miles, in 4332 Days, 12 Hours, 20 Minutes, or almoſt 12 Years.

VI. SATURN, at the Diſtance of 777 Millions of Miles, in 10759 Days, 6 Hours, 36 Minutes, or nearly 30 Years.

VII. THE COMETS, in various and vaſtly eccentric Orbits, revolve about the Sun in different Situations and Periods of Time, as repreſented in the Scheme of Mr. *Whiſton*'s Solar Syſtem. (CXXXIV.)

whoſe Poſitions we can by no means this way get any Idea. The ſeveral Parts therefore of the *Solar Syſtem* muſt be explained and illuſtrated by diſtinct Theories, with proper Figures adapted to each: And this will be the Subject of the following Notes.

(CXXXIV.) 1. The Periodical Times of the primary Planets, Sir *Iſaac Newton* ſtated in Days and Decimal Parts of a Day, as follows;

☿	♀	⊕	♂	♃
87,9692.	224,6176.	365,2565.	686,9785.	4332,514.

♄
10759,275.

2. The

2. The mean Distances of the Planets from the Sun are thus stated by Sir *Isaac*:

		According to *Kepler*,			
☿	♀	⊕	♂	♃	♄
38806.	72400.	100000.	152350.	519650.	951000.
		According to *Bullialdus*,			
38585.	72398.	100000.	152350.	522520.	954198.
		According to the Periodical Times,			
38710.	72333.	100000.	152369.	520096.	954006.

Plate LIV. Fig. 1.

3. Before we can shew how the Periodical Times and Distances of the Planets are found, it will be necessary to premise the following Things, viz. *The Orbit of a Planet is not in the Plane of the Ecliptic.* Thus let ANLO be the Orbit of a Planet P, and let BCET be the Earth's Orbit, which is in the Plane of the Ecliptic; then will one-half of the Planet's Orbit lie above the Plane, as NLO, and the other Half NAO below it.

4. *The two Planes, therefore, of the Planet's Orbit and of the Ecliptic will intersect one another*, which Intersection will be a Right Line, as NO; and this is called the *Line of the Nodes*, the *Nodes* being the two Points N and O, in which the Planet descends below, or ascends above, the Plane of the Ecliptic: Whence O is called the *Ascending Node*, and N the *Descending Node.*

5. Let the Curve N *m* O be described in the Plane of the Ecliptic perpendicular under the Half Orbit NLO; then is the Curve N *m* O said to be the *Projection of the Planet's Orbit* NLO *on the Plane of the Ecliptic*, and *p* the *projected Place* of the Planet P, or its Place *reduced to the Ecliptic.*

6. The Angle LO*m* measures the *Inclination of the Plane of the Planet's Orbit to that of the Ecliptic*; which is also called the *Obliquity* thereof. The Perpendicular Distance P*p* is the *Latitude of the Planet* from the Plane of the Ecliptic; and L*m* is the greatest Latitude, if LO or LN be a Quarter of a Circle. Also the Distance of the Planet from the Node, *viz.* PO, is called the *Argument of Latitude.*

7. Draw SP, S*p*, and TP, T*p*, and join ST; then is the Angle PS*p*, the true Latitude seen from the Sun

at

at S, and therefore called the *Heliocentric Latitude*; and the Angle P T *p* is the apparent Latitude, or that which is ſeen from the Earth at T, and is therefore called the *Geocentric Latitude*.

8. The true Diſtance of the Planet from the Sun and Earth is meaſured by the Lines S P and P T; but S *p* and T *p* are called the *Curtate Diſtances*. Alſo in the Triangle S *p* T the Angle S T *p* is called the *Angle of Elongation*, or Diſtance of the Planet from the Sun. The Angle S *p* T is called the *Parallactic Angle*, as being that under which the Semidiameter of the Earth's Orbit is ſeen; and the Angle *p* S T at the Sun is uſually called the *Angle of Commutation*.

9. We may now proceed to ſhew the Methods of determining the *Periodical Time* of a Planet; which may be done either by the Conjunctions or Oppoſitions of the Planet to the Sun. Thus, for Example, obſerve well the Place of *Jupiter* in the Ecliptic at his Oppoſition to the Sun, and alſo when he comes to be in Oppoſition to the Sun again; and note well the Time that lapſed between. Then ſay, *As the Arch deſcribed between the two Oppoſitions is to the whole Circumference, ſo is the Time in which that Arch was deſcribed, to the Periodical Time*, very nearly; for it will not be exactly ſo, becauſe the Motion of a Planet is not quite uniform, as moving in an Ellipſis, and not in a Circle. In the ſame manner you proceed for an inferior Planet.

10. But a more accurate Method is by obſerving nicely the Time that elapſes between the Planet's being twice ſucceſſively in the ſame Node, (which may be eaſily known, becauſe in that Part of its Orbit the Planet has no Latitude) and that will be the Periodical Time of the Planet; for in one Revolution of a Planet, the Nodes (if they move at all) will not move ſenſibly, and may therefore be eſteemed as quieſcent.

11. In order to eſtimate the Diſtances of the Planets, we proceed for *Venus* and *Mercury* in the following Manner. Let the Place of the Planet in its greateſt Elongation from the Sun be duly obſerved, the Difference between that and the Sun's Place (as ſeen from the Earth) will be the Quantity of the greateſt Elongation, or of the Angle A T S, with reſpect to the Planet

Venus

Venus in her Orbit at A. And since the Orbit of *Venus* is nearly circular, the Line TA will touch the Orbit in the Point A, and so the Angle TAS will be a Right one. Suppose the Angle ATS = 47 Degrees by Observation; then if we put the Distance of the Earth ST = 100000, say, As Radius or Sine of 90° is to the Sine of 47°, so is TS = 100000 to SA = 73000, nearly the Distance of *Venus* from the Sun.

12. In like manner may the Distance of *Mercury* from the Sun be determined in the gross, but not so nearly as that of *Venus*, because the Orbit is much more excentric or elliptical, and therefore the Angle TRS will not be a Right one. Its Quantity therefore must be found from the Theory of the Motions of *Mercury* founded on Observations; and from thence the third Angle TSR will be known, and consequently the Side SR, which is the Distance of *Mercury* from the Sun.

13. In the Superior Planets this Matter is not quite so easy; however, there are divers Methods by which it may be done, by having the Theory of the Earth known, which gives the Side ST; and by Observation the Angle STP is known, which is the Difference of the Geocentric Place of the Sun and Planet; then there remains only the Angle SPT to be found, which Astronomers shew how to do several Ways; one of which is peculiar to *Jupiter*, being done by means of one of his *Satellites*, as will be shewn when we treat of them.

14. As I have in this Note mentioned the Inclination of the Planets Orbits to the Plane of the Ecliptic, I shall give the Quantity thereof for each Planet as follows:

		°	′	″
The Inclination of the Orbit of	*Mercury* is	6	59	20
	Venus —	3	23	5
	Mars —	1	52	0
	Jupiter —	1	20	0
	Saturn —	2	33	30

15. Also the Line of the Nodes in the several Planetary Orbits is determined; and the Place in the Ecliptic of the ascending Node for each Planet is as follows:

For

			°	′	″
For	*Mercury* in	♉	14	42	00
	Venus —	♊	14	25	54
	Mars —	♉	18	29	54
	Jupiter —	♋	7	19	54
	Saturn —	♋	21	49	54

16. The Distance of the Planets from the Sun as above determined are reducible to *English* Miles, by first finding the Earth's Distance in that Measure. And this is done by finding the Quantity of the Sun's Parallax, that is, of the Angle under which the Earth's Semidiameter appears at the Sun. Thus let S be the Centre of the Sun, and C the Centre of the Earth D E F in her Orbit A B; the Angle D S C is that which we speak of, as being that under which the Semidiameter C D of the Earth appears at the Sun. Pl. LIV. Fig. 2.

17. To find this Angle Astronomers have attempted Variety of Methods, but have as yet found none that will determine it exactly; however, by many repeated Observations of Dr. *Halley* it is found to be not greater than 12″, nor less than 9″. Wherefore $10\frac{1}{2}$″ (the Mean) has been fixed upon as near the Truth, which we must be contented with till *May* 26, 1761, when *Venus* will transit the Sun's Disk, by which means the same Gentleman has shewn the Sun's Parallax may be determined to a great Nicety, *viz.* to within a 500dth Part of the Whole. See *Phil. Trans.* N° 348, abridged by *Jones*, Vol. IV *.

18. Supposing therefore the Angle D S C = 10′ 30‴, and the Side D C = 1; then say,

As the Tangent of	D S C 10″ 30‴ =	5,706764
Is to Unity	D C = 1 =	0,000000
So is Radius	90° =	10,000000
To the Side	S C = 19657,8 =	4,293236

Then 19657,8 Semidiameters of the Earth multiplied by 4000 gives 78631200 *English* Miles for the Distance of the Sun.

19. Not the Distances only, but also the Diameters of the Planets are to be investigated, by measuring their apparent

* Since the above was wrote, both the Transits of Venus have passed; *viz.* that in 1761 and the last in 1769; but nothing certain relating to the Sun's Parallax has been determined as yet; the Observations on the last Transit having not been all received.

apparent Diameters with a Micrometer adapted to a good Telescope. Thus the Sun in his mean Distance will be found to subtend an Angle of $32' 12'' = 1932$, and the Earth at the Sun subtends an Angle of $21''$ (being double the Angle D S C.) Therefore the Sun's Diameter is to the Earth's Diameter as 1932 to 21, that is, as 10000 to 109.

20. Again: Mr. *Pound* (with the *Hugenian* Telescope of 123 Feet) found *Saturn* subtended an Angle of $16''$. Therefore if *Saturn* were brought to the mean Distance of the Earth from the Sun, his apparent Diameter would be increased in the Ratio of $\frac{954006}{100000}$ to 1; that is, its Diameter would be seen under an Angle equal to $\frac{954006}{100000} \times 16'' = 152,''64096$. Whence the Sun's Diameter is to *Saturn*'s as $1932'' : 152,''64096 :: 10000 : 790$.

21. The same Gentleman measured *Jupiter*'s apparent Diameter, and found it subtend an Angle of $37''$; wherefore *Jupiter* at the Distance of the Earth would subtend an Angle equal to $\frac{520096}{100000} \times 37'' = 192,''417$. Hence the Sun's real Diameter is to that of *Jupiter* as $1932'' : 192,''417 :: 10000 : 996$.

22. *Hugenius* measured the Diameter of *Mars* when nearest the Earth, and found it did not exceed $30''$; and that the Distance of *Mars* from the Earth was then to the Sun's mean Distance as 15 to 41. (See his *Systema Saturnium.*) Therefore *Mars* removed to the Distance of the Sun would subtend an Angle equal to $\frac{15}{41} \times 30'' = 10,''9756$. Whence the Diameter of the Sun is to that of *Mars* as $1932''$ to $10,''9756 :: 10000 : 57$.

23. Dr. *Halley* collected from the Appearance of *Venus* in the Sun's Disk, *May* 26, 1761, that *Venus* seen from the Sun at her mean Distance would appear under an Angle of $30''$; consequently, at the Sun's mean Distance she would appear under an Angle equal to $\frac{72333}{100000} \times 30'' = 21,6999$. Therefore the Sun's real

Diameter

Diameter is to that of *Venus* as 1932″ : 21,″6992 :: 10000 : 112 *.

24. The same learned Gentleman by the like means finds *Mercury* at his mean Distance subtend an Angle of 20″; and therefore at the Sun an Angle of $\frac{38710}{100000} \times 20'' = 7,''742$. Wherefore the Diameters of the Sun and *Mercury* are as 1932″ : 7,″742 :: 10000 : 40.

25. There are other Phænomena of the Planets to be observed, from whence several important Discoveries have been made in the Physical Part of Astronomy. Thus the Sun and some Planets, when viewed with a good Telescope, appear to have dark Spots on their Surface; by these Spots those Bodies are found to have a Motion about their Axis, and the Position of their Axis with respect to the Plane of the Ecliptic is by this means determined.

26. These Spots are most numerous and easily observed in the Sun. It is not uncommon to see them in various Forms, Magnitudes, and Numbers, moving over the Sun's Disk. They were first of all discovered by the lyncean Astronomer *Galileo*, in the Year 1610, soon after he had finished his new-invented Telescope.

27. That these Spots adhere to or float upon the Surface of the Sun, is evident for many Reasons. (1.) For many of them are observed to break out near the Middle of the Sun's Disk; others to decay and vanish there, or at some Distance from his Limb. (2.) Their apparent Velocities are always greatest over the Middle of the Disk, and gradually slower from thence on each Side towards the Limb. (3.) The Shape of the Spots varies according to their Position on the several Parts of the Disk; those which are round and broad in the Middle grow oblong and slender as they approach the Limb, according as they ought to appear by the Rules of *Optics*.

28. By comparing many Observations of the Intervals of Time in which the Spots made their Revolutions, by *Galileo*, *Cassini*, *Scheiner*, *Hevelius*, Dr. *Halley*, Dr. *Derham*, and others, it is found that 27 Days,

* The apparent Diameter of *Venus* (measured by a Micrometer) upon the Face of the Sun in both Transits, was 58″ at a Mean. (*See my Treatise on the Transits lately published*).

Days, 12 Hours, 20 Minutes is the Meaſure of one of them at a Mean: But in this Time the Earth deſcribes the angular Motion of 26° 22′ about the Sun's Centre; therefore ſay, As 360° + 26° 22′ is to 360°, ſo is 27 d. 12 h. 20′ to 25 d. 15 h. 16′; which therefore is the Time of the Sun's Revolution about its Axis.

29. Had the Spots moved over the Sun in right-lined Directions, it would have ſhewn the Sun's Axis to have been perpendicular to the Plane of the Ecliptic; but ſince they move in a curvilinear Path, it proves his Axis inclined to the Axis of the Ecliptic; and it is found by Obſervation, that that Angle is equal to 7° 30′; that is, if B D paſſing through the Centre of the Sun C be perpendicular to the Plane of the Earth's Equator H I, then will the Axis of the Sun's Motion A E contain with that Perpendicular the Angle A C B = 7° 30′ = G C I, the Angle which the Equator of the Sun G F makes with the Plane of the Ecliptic.

Plate LIV. Fig. 3.

30. And the Points in which a Plane paſſing through the Perpendicular B D and Axis A E cuts the Ecliptic are in the 8th Degree of *Piſces* on the Side next the Sun's North Pole A, and conſequently in the 8th Degree of *Virgo* on the other ſide next the South Pole E. *Schenier* had determined the Angle B C A to be 7 Degrees, and *Caſſini* made it 8 by his Obſervations; which is the Reaſon why 7° 30′ is choſen for a Mean.

31. As to the Magnitude of the Spots, it is very conſiderable, as will appear if we obſerve that ſome of them are ſo large as to be plainly viſible to the naked Eye. Thus *Galileo* ſaw one in the Year 1612, and I know two Gentlemen who have thus viewed them within 20 Years paſt: Theſe Spots therefore muſt ſubtend at leaſt an Angle of 1 Minute. Now the Diameter of the Earth, if removed to the Sun, would ſubtend an Angle of but 20″; hence the Diameter of a Spot juſt viſible to the naked Eye is to the Diameter of the Earth as 60 to 20, or as 3 to 1; and therefore the Surface of the Spot, if circular, to a Great Circle of the Earth as 9 to 1. But the Areas of 4 Great Circles are equal to the Earth's Superficies; whence the Surface of the Spot is to the Surface of the Earth as 9 to 4, or as $2\frac{1}{4}$ to 1.

32. *Gaſſendus* ſays, he ſaw a Spot whoſe Diameter was equal to $\frac{1}{20}$ of that of the Sun, and therefore ſubtended

tended an Angle at the Eye of 1′ 30″; its Surface was therefore above 5 times larger than the Surface of the whole Earth. What thoſe Spots are, I believe Nobody can tell; but they ſeem to be rather thin Surfaces than ſolid Bodies, becauſe they loſe the Appearance of Solidity in going off the Diſk of the Sun. They reſemble ſomething of the Nature of Scum or Scoria ſwimming on the Surface, which are generated and diſſolved by Cauſes little known to us.

33. But whatever the Solar Spots may be, 'tis certain they are produced from Cauſes very inconſtant and irregular: For *Scheiner* in his *Roſa Urſina*, which contains near 2000 Obſervations upon theſe Spots, ſays, he frequently ſaw 50 at once, but for 20 Years after, (*viz.* between the Year 1650 and 1670) ſcarce any appeared. And in this Century the Spots were frequent and numerous till the Year 1741, when for three Years ſucceſſively very few appeared. I ſaw but one in all that Time; and now ſince the Year 1744 they have appeared again as uſual.

34. Theſe *Maculæ* or dark Spots are not peculiar to the Sun; they have been obſerved alſo in the Planets. Thus *Venus* was obſerved to have ſeveral by Signior *Bianchini*, the Pope's Domeſtic Prelate, in the Year 1726; by which he determined her Revolution about her Axis to be performed in 24 Days and 8 Hours; and that her Axis is inclined to the Plane of the Ecliptic in an Angle of 15 Degrees; and laſtly, that the North Pole of this Planet faces the 20th Degree of *Aquarius*.

35. As in *Venus*, ſo in *Mars*, both dark and bright Spots have been obſerved by *Galileo* firſt, and afterwards by Signior *Caſſini*, Dr. *Hook*, *Miraldi*, Mr. *Roemer*, and others. By theſe Spots the diurnal Revolution of *Mars* about its Axis is determined to be 24 Hours and 40 Minutes; and that the Axis is nearly perpendicular to the Plane of its Orbit.

36. There ſeems to be good Reaſon to conclude *Mars* is encompaſſed with a large Atmoſphere; for *Caſſini* obſerved a Fixed Star, at the Diſtance of 6 Minutes from the Diſk of *Mars*, became ſo faint before

THESE are all the heavenly Bodies yet known to circulate about the Sun, as the Centre of their Motions; and among the Planets, there are three which are found to have their *secondary Planets*, *Satellites*, or *Moons*, revolving conſtantly about them, as

its Occultation, that it could not be ſeen with the naked Eye, nor with a Teleſcope of 3 Feet; though Stars of that Magnitude are plainly viſible even in Contact with the Moon, which for that Reaſon ſeems to have no Atmoſphere.

37. *Jupiter* has had his Spots obſervable ever ſince the Invention and Uſe of large Teleſcopes; and from repeated Obſervations they ſhew *Jupiter's* Revolution about its Axis is in 9 Hours and 56 Minutes. Beſides theſe Spots, *Jupiter* has the Appearance of three Zones or Belts encompaſſing his Body, ſometimes more, ſo that his Diſk ſeems clouded with them: What they are, Nobody yet can tell. The Axis of this Planet alſo is nearly perpendicular to the Plane of his Orbit.

38. Conſidering the large Magnitude of *Jupiter*, and his ſhort diurnal Rotation, the Equatorial Parts of his Surface muſt have a prodigious Velocity, which of Conſequence muſt cauſe him to be of a ſpheroidical Figure (as was ſhewn of the Earth). Accordingly *Caſſini* found the Axis of the Equator to be to that of the Poles as 14 to 15; but Mr. *Pound* afterwards more exactly determined them to be as 12 to 13, agreeable to Sir *Iſaac Newton's* Computation.

39. *Saturn* by reaſon of his great Diſtance on one hand, and *Mercury* by reaſon of his Smallneſs and Vicinity to the Sun on the other, have not as yet had any Spots diſcovered on their Surfaces; and conſequently nothing in relation to their diurnal Motions, and Inclinations of their Axis to the Planes of their Orbits, can be known.

as the Centres of their Motions, (CXXXV.) *viz.*

THE

(CXXXV.) 1. Of the six Primary Planets, we find but *three* that are certainly attended with Moons, *viz.* the *Earth*, *Jupiter*, and *Saturn*; for though Mr. *Short* has given an Account of a *Phænomenon* that he observed some Years ago, which seems extremely like a Moon about *Venus*, yet as it was never observed before nor since through the best of Telescopes, I can by no means think it was a real Moon: However, that the Reader may use his own Judgment, I refer him to the Account given of it in the *Philosophical Transactions*.

2. The Distance of our Moon from the Earth is determined by her horizontal Parallax, or the Angle which the Semidiameter of the Earth subtends at the Moon, *viz.* the Angle A O C, which is the Difference between the true Place of the Moon's Centre O when in the Horizon, and the apparent Place thereof as view'd from the Surface of the Earth at A. The former is known by Astronomical Tables, the latter by Observation: And the Quantity of this Difference or Angle at a Mean is $57' \, 12'' = AOC$. Pl. LIV. Fig. 4.

3. If therefore we say, As the Tangent of $57' \, 12''$ is to Radius, so is $AC = 1$ to $CO = 60{,}1$; this will be the mean Distance of the Moon in Semidiameters of the Earth. Therefore since one Semidiameter of the Earth contains 3982 Miles, we have $3982 \times 60{,}1 = 239318{,}2 = CO$ the mean Distance of the Moon.

4. The Moon's apparent Semidiameter M O measures (at her mean Distance) $15' \, 38'' = 938''$ by the Micrometer, which is the Quantity of the Angle M C O. The Earth's Diameter therefore is to the Moon's as $3432''$ to $938''$, that is, as 109 to 30, or as 3,63 to 1. Wherefore $\frac{30}{109} \times 7964 = 2192$ Miles is the Moon's Diameter.

5. Therefore the Face of the Earth, as it appears to the *Lunarians*, is to the Face of the Moon as it appears to us, as 109×109 to 30×30, *viz.* as 11881 to

 900,

THE EARTH, which has only *one Moon* revolving about it in 27 Days, 7 Hours, 43 Minutes,

9co, or as 13,2 to 1. And the real Bulk of the Earth is to that of the Moon as $109 \times 109 \times 109$ to $30 \times 30 \times 30$, *viz.* as 1295029 to 27000, that is, as 1295 to 27, or as 48 to 1 very nearly.

6. Sir *Iſaac Newton* mentions the Atmoſphere about the Moon, but other Aſtronomers think there is Reaſon (not to ſay a Demonſtration) for the contrary: For were there an Atmoſphere of Air like ours, it muſt neceſſarily obſcure the Fixed Stars in the Moon's Appulſe to them; but it has been obſerved that this never happens; on the contrary, they preſerve all their Splendor to the Moment of their Occultation, and then diſappear inſtantaneouſly; and in the ſame manner they recover their Light, when they appear again on the other Side. And this I am very certain of from the late remarkable Occultation of *Jupiter*, which I obſerved with a good reflecting Teleſcope from the Beginning to the End with all the Attention poſſible, becauſe I was very deſirous to be ſatisfied about that Matter; and all the Phænomena conſpired to convince me, there was nothing like an Atmoſphere about the Moon.

7. That the Surface of the Moon is not ſmooth or even, but diverſified with Hills and Vales, Continents and Seas, Lakes, *&c.* any one would imagine who views her Face through a large Teleſcope. That ſhe has Variety of Hills and Mountains is demonſtrable from the Line which bounds the light and dark Parts not being an even regular Curve, as it would be upon a ſmooth ſpherical Surface, but an irregular broken Line, full of Dents and Notches, as repreſented in the Figure: Alſo becauſe many ſmall (and ſome large) bright Spots appear in the dark Portion, ſtanding out at ſeveral ſmall Diſtances from the boundary Line; which Spots in a few Hours become larger, and at laſt unite with the enlightened Portion of the Diſk.

8. On the other hand, we obſerve many ſmall Spots interſperſed all over the bright Part, ſome of which have

Minutes, at the mean Distance of about 240,000 Miles.

JUPITER

have their dark Sides next the Sun, and their opposite Sides very bright and circular, which infallibly proves them to be deep, hollow, round Cavities; of which there are two very remarkable ones near together on the upper Part, and may be viewed exceeding plain when the Moon is about four or five Days old.

9. To measure the Height of a Lunar Mountain is a curious Problem, and at the same time very easy to effect in the following Manner. Let C be the Moon's Centre, E D B a Ray of the Sun touching the Moon's Surface in D, and the Top of a Mountain in B. Draw C B and C D; the Height of the Mountain A B is to be found. With a Micrometer in a Telescope find what Proportion the Distance of the Top of the Mountain B from the Circle of Illumination at D, bears to the Diameter of the Moon, that is, the Proportion of the Line D B to D F; and because D F is known in Miles, D B will be also known in that Measure. Plate LIV. Fig. 5.

10. Now admit that $DB : DC :: 1 : 8$, as in one of the Hills it will be; then $\overline{DC}^2 + \overline{DB}^2 = 64 + 1 = 65 = \overline{CB}^2$; whence $\sqrt{65} = 8{,}062 = BC$; wherefore $BC - AC = 8{,}062 - 8 = 0{,}062 = AB$, the Height of the Mountain required. Wherefore $AC : AB :: 8 : 0{,}062 :: 8000 : 62$. And since the Moon's Semidiameter $AC = 1096$ Miles, therefore $8000 : 62 :: 1096 : 8{,}5$ nearly. This Mountain then being $8\frac{1}{2}$ Miles high, is nearly three times higher than the highest Mountain on the Earth.

11. Again, the Cavities are proportionably large and deep. I have observed Cavities in the Moon more than the 100dth Part of the Moon's Diameter in Breadth, which is about 200 Miles upon the Moon's Surface; their Depths appear likewise proportional. The Lunar Cavities therefore prodigiously exceed the Height of the Mountains; and consequently the Surface of the Moon

JUPITER is observed with a Telescope to have four *Satellites*, which move about him

has but little Similitude to the Surface of the Earth in these Respects.

12. Since the Moon's Surface appears to be so very mountainous and irregular, it has been a Question, how it comes to pass that the bright circular Limb of the Disk does not appear jagged and irregular, as well as the Curve bounding the light and dark Parts? In Answer to this, it must be considered, that if the Surface of the Moon had but one Row of Mountains placed round the Limb of the Disk, the said bright Limb would then appear irregularly indented; but since the Surface is all over mountainous, and since the visible Limb is to be considered not as a single curve Line, but a large Zone, having many Mountains lying one behind another from the Observer's Eye, 'tis evident the Mountains in some Rows being opposite to the Vales in others, will fill up the Inequalities in the visible Limb in the remoter Parts, which diminish to the Sight and blend with each other, so as to constitute (like the Waves of the Sea) one uniform and even Horizon.

13. Whether there be Seas, Lakes, &c. in the Moon, has been a Question long debated, but now concluded in the Negative: For in those large darker Regions (which were thought to be Seas) we view through a good Telescope many permanent bright Spots, as also Caverns and empty Pits, whose Shadows fall within them, which can never be seen in Seas or any liquid Substance. Their dark and dusky Colour may proceed from a Kind of Matter or Soil which reflects Light less than that of the other Regions.

14. These Spots in the Moon have continued always the same unchangeably since they were first viewed with a Telescope; though less Alterations than what happen in the Earth in every Season of the Year, by Verdure, Snow, Inundations, and the like, would have caused a Change in their Appearance. But indeed, as there are no Seas nor Rivers in the Moon, and no Atmosphere,

so

him in the Times and Diſtances following, *viz.*

THE

ſo of courſe there can be no Clouds, Rain, Snow, or other Meteors, whence ſuch Changes might be expected.

15. Since (as we have ſhewn) the mean Diſtance of the Moon is about 60 Semidiameters of the Earth, at the Diſtance of the Moon one Degree of the Earth's Surface will ſubtend an Angle of one Minute, and will therefore be viſible; but ſuch a Degree is equal to 69$\frac{1}{2}$ Miles, therefore a Spot or Place 70 Miles in Diameter in the Moon will be juſt viſible to the naked Eye.

16. Hence a Teleſcope that magnifies about 100 times will juſt diſcover a Spot whoſe Diameter is $\frac{1}{100}$ of 70 Miles, or $\frac{7}{10}$ of a Mile, or 3698 Feet: And a Teleſcope that will magnify 1000 times will ſhew an Object that is but $\frac{7}{100}$ of a Mile, that is, whoſe Diameter is but 370 Feet, or little more than 120 Yards; and therefore will eaſily ſhew a ſmall Town or Village, or even a Gentleman's Seat, if any ſuch there be.

17. The Time which the Moon takes up in making one Revolution about the Earth, from a Fixed Star to the ſame again, is 27 d. 7 h. 43′, which is called the *Periodical Month*. But the Time that paſſes between two Conjunctions, that is, from one New Moon to another, is equal to 29 d. 12 h. 44′ 3″, which is called a *Synodical Month:* For after one Revolution is finiſhed, the Moon has a ſmall Arch to deſcribe to get between the Sun and the Earth, becauſe the Sun keeps advancing forwards in the Ecliptic. Now this Surplus of Motion takes up 2 d. 5 h. 1′ 3,″ which added to the Periodical Month makes the Synodical, according to the mean Motions.

18. The Moon moves about its own Axis in the ſame Time that it moves about the Earth, from whence it comes to paſs that ſhe always ſhews the ſame Face to us: for by this Motion about her Axis juſt ſo much of her Surface is turned towards us conſtantly, as by her Motion about the Earth would be turned from us.

THE *First* in 1 Day, 18 Hours, 27 Minutes, at the Distance of $5\frac{6}{10}$ Semidiameters

19. But since this Motion about the Axis is equable and uniform, and that about the Earth (or common Centre of Gravity) is unequal and irregular, as being performed in an Ellipsis, it must follow, that the same Part of the Moon's Surface precisely can never be shewn constantly to the Earth: And this is confirmed by the Telescope, through which we often observe a little Gore or Segment on the Eastern and Western Limb appear and disappear by Turns, as if her Body librated to and fro; which therefore occasioned this Phænomenon to be called the *Moon's Libration*.

20. The Orbit of the Moon is elliptical, more so than any of the Planets, and is perpetually changing or variable, both in respect of its Figure and Situation; of which we shall treat more largely in another Place. The Inclination of the Moon's Orbit to the Plane of the Ecliptic is also variable, from 5 Degrees to 5° 18′. The Line of Nodes likewise has a variable Motion from East to West, contrary to the Order of the Signs, and compleats an entire Revolution in a Space of Time a little less than 19 Years. Also the Line of the *Apsides*, or of the *Apogee* and *Perigee*, has a direct Motion from West to East, and finishes a Revolution in the Space of about 9 Years. All which will be more copiously treated of when we come to explain the Physical Causes thereof.

Plate LIV. Fig. 6.

21. The Phases of the Moon in every Part of the Orbit are easily accounted for from her different Situation with respect to the Earth and Sun: For though to an Eye placed in the Sun she will always exhibit a compleat illuminated Hemisphere, yet in respect to the Earth, where that Hemisphere is viewed in all Degrees of Obliquity, it will appear in every Degree from th greatest to the least; so that at E no Part at all of the enlightened Surface can be seen. At F a little Part oi it is turned towards the Earth, and from its Figure it is

ters of *Jupiter*'s Body from his Centre, as measured with a Micrometer.

THE

is then said to be *horned*. At G One-half of the enlightened Surface is turned to the Earth, and she is then said to be *dichotomized*, and in her first Quarter or *Quadrature*. At H a Part more than half is turned to the Earth, and then she is said to be *gibbous*. At A her whole illumined Hemisphere is seen, being then in *Opposition* to the Sun; and this is called the *Full Moon*. At B she is again *gibbous*, but on the other Part; at C she is again *dichotomized*, and in her last Quarter; at D she is *horned*, as before; and then becomes *new* again at E, where she is in *Conjunction* with the Sun.

22. If M N be drawn perpendicular to the Line S L joining the Centres of the Sun and Moon, and O P perpendicular to the Line T L joining the Centres of the Earth and Moon, 'tis evident the Angle O L M in the first Half of the Orbit, and P L N in the second, will be proportional to the Quantity of the illuminated Disk turned towards the Earth; and this Angle is every where equal to the Angle E T L, which is called the *Elongation of the Moon* from the Sun.

23. To find what Quantity of the Moon's visible Surface is illustrated for any given Time, we are to consider that the Circle of Illumination B F C is oblique to the View every where (but at G and A); and therefore by the Laws of the Orthographic Projection (which see in my *Elements of all Geometry*) it will be projected into an Ellipse whose longest Axis is the Diameter of the Moon B C, and the Semi-conjugate is F L = Cosine of the Angle of Elongation F B P. Hence F P = Versed Sine of the said Angle. But from the Nature of the Circle and Ellipse we have L P in a constant Ratio to F P, wherever the Line P O is drawn perpendicular to B; therefore also 2 L P = P O has a constant Ratio to F P. *But* (by *Euclid* V. 12.) *the Sum of all the Lines* O P = *Area of the Circle, is to the Sum of all the Lines* F P = *Area of the illuminated Part, as the*

Pl. LIV. Fig. 7.

THE *Second* in 3 Days, 13 Hours, 13 Minutes, at the Diſtance of 9 Semidiameters.

THE *Third* in 7 Days, 3 Hours, 42 Minutes, at the Diſtance of 14$\frac{3}{10}$ Semidiameters.

THE *Fourth* in 16 Days, 16 Hours, 32 Minutes,

the Diameter of the Circle O P *to the Verſed Sine of the Elongation* F P.

24. As the Moon illuminates the Earth by a reflex Light, ſo does the Earth the Moon; but the other Phænomena will be different for the moſt part. I ſhall recount them for the Reader's Curioſity as follows. (1.) The Earth will appear but to little more than One-half of the Lunar Inhabitants. (2.) To thoſe to whom the Earth is viſible, it appears fixed, or at leaſt to have no circular Motion, but only that which reſults from the Moon's *Libration*. (3.) Thoſe who live in the Middle of the Moon's viſible Hemiſphere ſee the Earth directly over their Heads. (4.) To thoſe who live in the Extremity of that Hemiſphere the Earth ſeems always nearly in the Horizon, but not exactly there, by reaſon of the *Libration*. (5.) The Earth in the Courſe of a Month would have all the ſame Phaſes as the Moon has. Thus the *Lunarians* when the Moon is at E, in the Middle of their Night, ſee the Earth at *Full*, or ſhining with a full Face; at G it is *dichotomized*, or half light and half dark; at A it is wholly dark, or *New*; and at the Parts between theſe it is *gibbous*. (6) The Earth appears variegated with Spots of different Magnitudes and Colours, ariſing from the Continents, Iſlands, Oceans, Seas, Clouds, &c. (7.) Theſe Spots will appear conſtantly revolving about the Earth's Axis, by which the *Lunarians* will determine the Earth's diurnal Rotation, in the ſame Manner as we do that of the Sun.

Minutes, at the Distance of $25\frac{3}{10}$ Semidiameters. (CXXXVI.)

SATURN

(CXXXVI.) 1. GALILEO first discovered the *Satellites* or *Moons* of *Jupiter*, in the Year 1610; and call'd them *Medicea Sidera*, or *Medicean Stars*, in Honour of the Family of the *Medici*, his Patrons. The famous Piece called *Sidereus Nuncius*, in which he particularly describes the Discovery of these Stars, he dedicated to COSMUS MEDICIS II. the fourth Great Duke of *Hetruria*.

2. The Orbits of *Jupiter*'s Moons lie nearly in the Plane of the Ecliptic, which is the Reason why their Motion is apparently in a right Line, and not circular, as it really is. To understand this, let S be the Sun, T the Earth in its Orbit T H, I the Planet *Jupiter* in his Orbit A I B, and in the Centre of the four Orbits of his Moons. Then, because the Plane of those Orbits does nearly pass through the Eye, the real Motion of the *Satellite* in the Periphery will be apparently in the Diameter of the Orbit, which is at Right Angles, to the Line joining the Centre of the Earth and *Jupiter*.

3. Thus supposing the Earth at R, if D C be drawn through the Centre of *Jupiter* perpendicular to R I, the Motion of each Moon and their Places will appear to be in that Line. Thus if the exterior Moon be at E or F, it will appear to be at I, either upon or behind the Centre of *Jupiter*; if the Moon move from E to K, it will appear to have moved from I to L; and when it moves from K to C, it will appear to move from L to C. Again, while the Satellite moves from C to M, it will appear to move from C to L; and as it goes from thence to F, it apparently moves from L to I. Thus also on the other Side the Orbit, while the Satellite describes the Quadrant F D, its apparent Motion will be from I to D; and then from D to I again, as it comes from D to E.

4. Whence, since this is the Case of each Satellite, it appears that while each Satellite describes the remote Half

SATURN has *five Moons*; and besides them a stupendous Ring surrounding his Body, whose

Half of its Orbit C F D, its apparent Motion will be direct, or from West to East along the Line C D; and while it describes the other Half D E C, its apparent Motion is retrograde, or from East to West back again along the Line from D to C. So that each Satellite traverses the Diameter of its Orbit twice in each Revolution.

5. The Moons of *Jupiter* severally shew the same Phases to him as ours does to us. They disappear from our Sight sometimes, so that 'tis very rare to have all the four in View at once; nor is it possible to know which Satellite in Order you see, but from the Knowledge of the Theory and Calculation; because the remotest Satellite may appear nearest to *Jupiter*, and the contrary, as is evident from a View of the Figure.

6. These Moons, like our own, suffer an Eclipse every time they come to the Shadow of *Jupiter*, as at F. Also, supposing the Earth at T, the Satellite at G will undergo an Occultation behind the Body of *Jupiter*, as is evident from the Scheme. Again, a Satellite will sometimes lose its Lustre as it passes over the enlighten'd Disk of its Primary; as when it is at E and N, and the Earth in R and T. Lastly, one Satellite at O may disappear behind another at K, or cause another to disappear behind it at M.

7. The Observations by Telescopes have been carried so far as to make it very probable, that all the Satellites do really revolve about their own Axis, by means of Spots which they have discovered to belong to them, and which by their Motion cause a great Variety in the Brightness of the Satellites, and sometimes do almost obscure them: For which see Mr. *Pound*'s *Observations in Jones*'s *Abridgement of the Philosophical Transactions*, Vol. IV. p. 307.

8. By means of *Jupiter*'s Satellites several noble Problems in Natural Philosophy have an easy and elegant Solution;

whose *Width* and *Distance* from *Saturn*'s Body are equal, and computed at upwards of 20,000 Miles. The Periodical Times and Distances of the *Saturnian* Moons in Semidiameters of the Ring are as follow:

THE *First*, or inmost, revolves about *Saturn* in 1 Day, 21 Hours, 18 Minutes, at

Solution; the First of which is, *to determine the Ratio of the Velocity of Light*. The Manner how this is done I have elsewhere shewn. See *Annot.* CXII. The Second is, *to determine the Longitude of a Place from any proposed Meridian*; which is easily done by the following Method. Let the Moment of Time in which the Satellite enters the Shadow of *Jupiter* be calculated for the given Meridian from Tables of its Motion; then let the Moment of Time be well observed when this Immersion happens at the proposed Place; the Difference of these two Moments turned into Motion will give the Longitude of the Place for that Meridian, allowing 15 Degrees for every Hour, 1 Degree for every 4 Minutes of Time, or 15 Minutes of a Degree for every Minute of Time.

9. The Third Problem is, *to find the Distance of* Jupiter *from the Sun*. This is done as follows: Let the middle Moment of the Occultation of a Satellite as at G be observed, and again the middle Moment of the following at F; this will give the Time in which the Arch GF is described. Then say, As the Time of the whole Revolution is to the Time now found, so is the whole Circle or 360 Degrees to the Degrees and Minutes contained in the Arch FG; which is therefore the Measure of the Angle FIG, or its Equal TIS, which is the Parallactic Angle at *Jupiter*; which being known, the Distance of *Jupiter* from the Sun IS is known, by what has been shewn in *Annot.* CXXXIV. Fig. 8.

(CXXXVII.) 1.

at the Distance of near 2 Semidiameters of the Ring.

THE *Second* in 2 Days, 17 Hours, 41 Minutes, at the Distance of $2\frac{2}{5}$ Semidiameters.

THE *Third* in 4 Days, 12 Hours, 25 Minutes, at the Distance of $3\frac{2}{3}$ Semidiameters.

THE *Fourth* in 15 Days, 22 Hours, 41 Minutes, at the Distance of 8 Semidiameters.

THE *Fifth* in 70 Days, 22 Hours, 4 Minutes, at the Distance of $23\frac{3}{10}$ Semidiameters. (CXXXVII.)

THESE

(CXXXVII.) 1. Though *Galileo*'s Telescope was sufficient to discover all *Jupiter*'s Moons, it would not reach *Saturn*'s, they being at too great a Distance. But yet this sagacious Observer found *Saturn*, by reason of his Ring, had a very odd Appearance; for his Glass was not good enough to exhibit the true Shape of the Ring, but only a confused Idea of that and *Saturn* together, which in the Year 1610 he advertised in the Letters of this Sentence transposed: *Altissimum Planetam tergeminum observavi*; i. e. *I have observed* Saturn *to have three Bodies.*

2. This odd Phænomenon perplexed the Astronomers very much, and various Hypotheses were formed to resolve it; all which seemed trifling to the happy *Hugenius*, who applied himself purposely to improve the Grinding of Glasses, and perfecting long Telescopes, to arrive at a more accurate Notion of this Planet and its Appendage. Accordingly in the Year 1655 he constructed a Telescope of 12 Feet, and viewing *Saturn* divers times, he discovered something like a Ring encompassing

THESE are the conſtituent Parts of the *Solar Syſtem*, which is now received and

compaſſing his Body; which afterwards with a Tube of 23 Feet he obſerved more diſtinctly, and alſo diſcovered a Satellite revolving about that Planet. This *Hugenian* Satellite is the fourth in Order from *Saturn*.

3. In the Year 1671 *Caſſini* diſcovered the third and fifth, and in the Year 1686 he hit upon the firſt and ſecond, with Tubes of 100 and 136 Feet; but could afterwards ſee all five with a Tube of 34 Feet. He called theſe Satellites *Sidera Lodoicea*, in Honour of *Louis le Grand*, in whoſe Reign and Obſervatory they were firſt diſcovered.

4. In the Year 1656 *Hugens* publiſhed his Diſcovery in relation to *Saturn*'s Ring in the Letters of this Sentence tranſpoſed, *viz. Annulo cingitur tenui, plano, nuſquam cohærente, ad Eclipticam inclinato*; that is, *Saturn is encompaſſed by a thin Plane or Ring, no where cohering to his Body, and inclined to the Plane of the Ecliptic.* This Inclination of the Ring to the Ecliptic is determined to be about 31 Degrees by *Hugens*, *Roemer*, *Picard*, *Campani*, &c. though by a Method not very definitive.

5. However, ſince the Plane of the Ring is inclined to the Plane of the Earth's Motion, it is evident when *Saturn* is ſo ſituated that the Plane of his Ring paſſes through the Earth, we can then ſee nothing of it; nor yet can we ſee it when the Plane paſſes between the Sun and the Earth, the dark Side being then turned to us, and only a dark Liſt appears upon the Planet, which is probably the Shadow of the Ring. In other Situations the Ring will appear elliptical more or leſs; when it is moſt ſo, the Heavens appear through the elliptic Space on each Side *Saturn* (which are called the *Anſæ*); yea, a Fixed Star was once obſerved by Dr. *Clarke*'s Father in one of them.

6. The Nodes of the Ring are in 19° 45′ of *Virgo* and *Piſces*. During *Saturn*'s Heliocentric Motion from

19°

and approved as the only *true System of the World*, for the following Reasons. (CXXXVIII.)

I. IT

$19^{\circ} 45'$ to the opposite Node, the Sun enlightens the Northern Plane of the Ring, and *vicissim*.

7. Since *Saturn* describes about one Degree in a Month, the Ring will be visible through a good Telescope till within about 15 or 20 Days before and after the Planet is in the $19^{\circ} 45'$ of *Virgo* or *Pisces*. The Time therefore may be found by an Ephemeris, in which *Saturn* seen from the Earth shall be in those Points of the Ecliptic; and likewise when he will be seen from the Earth in $19^{\circ} 45'$ of *Gemini* and *Sagittarius*, when the Ring will be most open, and in the best Position to be viewed,

8. There have been some Grounds to conjecture that *Saturn*'s Ring turns round an Axis, but that is not yet demonstrable. This wonderful Ring in some Situations does also appear double; for *Cassini* in the Year 1675 observed it to be bisected quite round by a dark elliptical Line, dividing it as it were into two Rings, of which the inner one appeared brighter than the outer. This was oftentimes observed afterwards with Tubes of 34 and 20 Feet, and more evidently in the Twilight or Moon-Light than in a darker Sky. See *Phil. Trans.* abridged, Vol. II. p. 221, 222.

(CXXXVIII.) 1. The sagacious *Kepler* was the first who discovered this great Law of Nature in all the Primary Planets, and afterwards the Astronomers observed that the Secondaries did likewise regulate their Motions by the same Law. I have already exhibited the Mathematical Theory thereof in *Annot.* XXXIV. 11. and given an Example in the Earth and *Venus*. And that the same Law holds in the System of *Jupiter*'s and *Saturn*'s Moons, will appear from the following Instances.

2. The first of *Jupiter*'s Moons is at the Distance of $2\frac{5}{6}$ of *Jupiter*'s Diameters from his Centre, and revolves

I. It is most simple, and agreeable to the Tenor of Nature in all her Actions; for

volves in 42 Hours. The outermost describes its Orbit in 402 Hours; therefore say, As 1764 (the Square of 42) is to 161604, (the Square of 402) so is $\frac{4913}{216}$ (the Cube of $2\frac{5}{6}$) to nearly $\frac{450000}{216}$ the Cube of $\frac{76}{6}$ or $12\frac{2}{3}$, the Distance of the fourth Satellite; which answers to Observations.

3. Or thus analytically by Logarithms, Let L = Logarithm of the Period of the first Satellite, L = Logarithm of any other Satellite's Period, and D and d the Logarithms of their Distances; then will it be $2\,L : 2\,l :: 3\,D : 3\,d$, and therefore $2\,L + 3\,d = 2\,l + 3\,D$; whence we have $d = D + \frac{2}{3}\,l - \frac{2}{3}\,L$. For Example; in the first and second Satellites of *Jupiter*, *Cassini* observed the Distance of the first in Semidiameters of *Jupiter* to be $5\frac{2}{3}$, whose Logarithm is 0,753353. The Periods of those Satellites give $\frac{2}{3}\,l = 2{,}32459$, and $\frac{2}{3}\,L = 2{,}122851$; from whence we get $d = 0{,}95509$, the Number corresponding to which is 9,07, the Distance of the second Satellite, agreeing wonderfully with Observation.

4. Now since the Moon turns round the Earth, if the Sun did likewise perform his Circuit about it, their Motions would undoubtedly be regulated by the same Law with all the rest. But the Period of the Moon is 27 Days, that of the Sun 365; the Distance of the Moon 60 Semidiameters of the Earth; therefore say, As 729 (the Square of 27) is to 133225 (the Square of 365); so is 216000 (the Cube of 60) to 39460356, the Cube Root whereof is 340, which ought to express the Sun's Distance in Semidiameters of the Earth. But we have shewn the Sun is really distant from the Earth near 20000. (See *Annot.* CXXXIV. 18.)

5. Admitting the Sun to be at the Distance of 20000 Semidiameters, his *Periodical Time* would then be more

 than

for by the Motions of the Earth all the *Phænomena* of the Heavens are reſolved, which by other *Hypotheſes* are inexplicable without a great Number of other Motions, contrary to philoſophical Reaſoning by Rule I.

II. It

than 450 Years, if its Motion were governed by *Kepler*'s Law, and compared with that of the Moon; for as 216000 $(=60^3)$ is to 8000000000000 $(=20000^3)$ ſo is 729 $(=27^2)$ to a Number, the Square Root of which is 164320 Days = 450 Years nearly, which is the Periodical Time of the Sun's Revolution at that Diſtance, and moving according to the Univerſal Law.

6. This beautiful and harmonious Syſtem, or Frame of the World, ſufficiently recommends itſelf from the Principles of right Reaſon only; ſuppoſing there were no ſuch Thing as abſolute Demonſtration attainable in the Caſe. It is therefore very ſurprizing to obſerve, how few among thoſe who are not mathematically learned, can be induced to believe, and acquieſce in this Doctrine of the Earth's Motion, and Stability of the Sun. *Copernicus*, above 200 Years ago, mentions the zealous Father *Lactantius*, as ridiculing thoſe who aſſerted the Spherical Figure of the Earth. Therefore, ſays he, it is not to be wondered at if ſuch Sort of People ſhould ridicule us. And whatever the Popes may have ſince decreed, 'tis certain, this Doctrine was ſo far from being then reputed heretical and damnable, that this great Man dedicated his Book to Pope *Paul* III. becauſe by his Holineſs' Authority, and Learning, he might be ſecured againſt the Calumnies of ignorant Gainſayers; yea, and appealed to his Holineſs at the ſame time for the Uſefulneſs of his Doctrine even to the Eccleſiaſtical Republick. His Words are, *Mathemata Mathematicis ſcribuntur, quibus & hi noſtri Labores, ſi me non fallit opinio, videbuntur etiam Reipublicæ Eccleſiaſticæ conducere aliquid, cujus Principatum tua Sanctitas nunc tenet.*

II. IT is more rational to ſuppoſe the Earth moves about the Sun, than that the huge Bodies of the Planets, the ſtupendous Body of the Sun, and the immenſe Firmament of Stars, ſhould all move round the inconſiderable Body of the Earth every twenty-four Hours.

III. THE Earth moving round the Sun is agreeable to that general Harmony, and univerſal Law, which all other moving Bodies of the Syſtem obſerve, viz. *That the Squares of the periodical Times are as the Cubes of the Diſtances*: But if the Sun move about the Earth, that Law is deſtroyed, and the general Order and Symmetry of Nature interrupted; ſince according to that Law the Sun would be ſo far from revolving about the Earth in 365 Days, that it would require no leſs than 5196 Years to accompliſh one Revolution.

IV. AGAIN: Did the Sun obſerve the univerſal Law, and yet revolve in 365 Days, his Diſtance ought not to be above 310 Semidiameters of the Earth; whereas it is eaſy to prove it is really above 20000 Semidiameters diſtant from us.

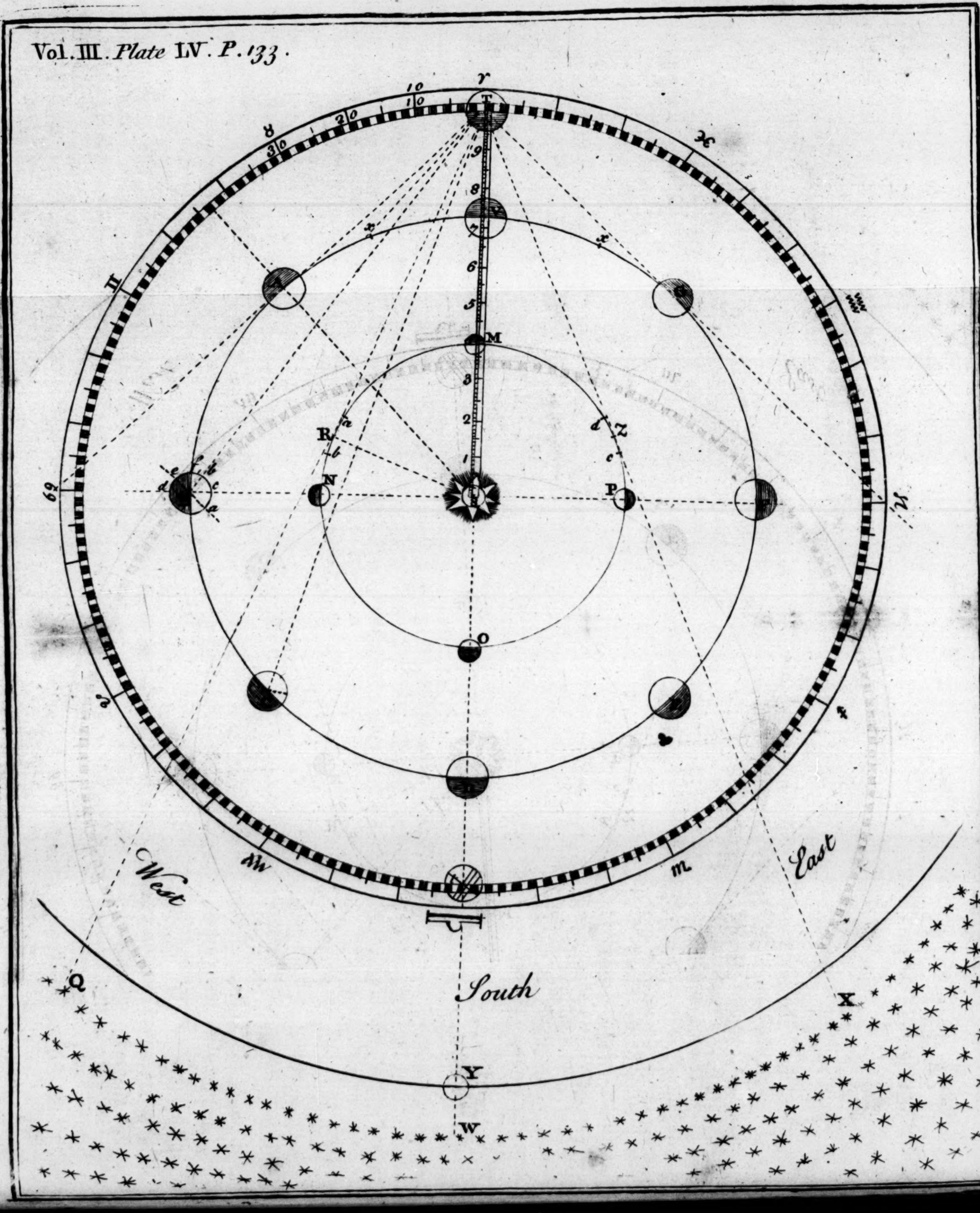
V. THE Sun is the Fountain of Light and Heat, which it irradiates through all the Syſtem; and therefore it ought to be

placed in the Centre, that ſo all the Planets may at all times have it in an uniform and equable Manner: For,

VI. If the Earth be in the Centre, and the Sun and Planets revolve about it, the Planets would then, like the Comets, be ſcorched with Heat when neareſt the Sun, and frozen with Cold in their *Aphelia*, or greateſt Diſtance; which is not to be ſuppoſed.

VII. If the Sun be placed in the Centre of the Syſtem, we have then the rational Hypotheſis of the Planets being all moved about the Sun by the univerſal Law or Power of Gravity ariſing from his vaſt Body; and every Thing will anſwer to the Laws of circular Motion, and central Forces: But otherwiſe we are wholly in the dark, and know nothing of the Laws and Operations of Nature.

VIII. But happily we are able to give not only *Reaſon*, but *demonſtrative Proofs*, that the Sun does poſſeſs the Centre of the Syſtem, and that the Planets move about it at the Diſtance and in the Order above aſſigned: The *firſt* of which is, That *Mercury* and *Venus* are ever obſerved to have *two Conjunctions* with the Sun, but no *Oppoſition*; which could not happen, unleſs the


Vol. III. Plate LV. P. 133.
T
M
N
P
O
R
Z
Q
X
Y
W
West
South
East

the Orbits of those Planets lay within the Orbit of the Earth. (CXXXIX.)

IX. THE

(CXXXIX.) 1. What relates to the Conjunctions and Oppositions of the Planets will be easily understood by a Diagram. Let S be the Sun, T the Earth, V *Venus*, and M *Mercury*, in their several Orbits. Now 'tis evident that when *Venus* and *Mercury* are at V and M, they will be seen from the Earth T in the same Part of the Heavens with the Sun, *viz.* at W, because they are all posited in one Right Line T W; and this is called the *Lower* or *Inferior Conjunction*. Pl. LV.

2. Again: When *Venus* and *Mercury* come to the Situations D and O, they are again in the same Right Line joining the Centres of the Earth and Sun, and are therefore again seen in the same Part of the Heavens with him; and this is called the *Upper* or *Superior Conjunction*. Here 'tis evident, those two Planets must appear twice in Conjunction with the Sun in each Revolution, to a Spectator on the Earth at T, which we at present will suppose to be at rest.

3. Hence we have an infallible Proof that the Orbits of *Venus* and *Mercury* lie both within the Orbit of the Earth. Also the Orbits of *Mars*, *Jupiter*, and *Saturn* must lie without the Orbit of the Earth; for otherwise they could not exhibit the Appearance they do of alternate Conjunctions and Oppositions. Thus let *Mars* be in his Orbit at Y, 'tis evident when the Earth is at T, that Planet will be seen in Conjunction with the Sun, and will be then at its greatest Distance from the Earth.

4. But when the Earth is at *t* between the Sun and *Mars*, 'tis plain they must appear in opposite Parts of the Heavens, because a Person at *t* viewing the Sun at S must look directly to the contrary Part to view the Planet at Y; and in this Opposition to the Sun *Mars* is nearest to the Earth: All which is so evident from the Scheme, and so exactly agreeable to the Phænomena of those Planets in the Heavens, that any Person must be strangely obstinate, and incapable of any Sort of Con-

 viction,

IX. THE *second* is, That *Mars*, *Jupiter*, and *Saturn*, have each their *Conjunctions* and

viction, who cannot see the Constitution of Nature, and the Disposition of the Planetary Orbits, are such as are above described.

5. But farther: If we divide the Distance of the Earth from the Sun, *viz.* the Line S T, into a hundred or a thousand equal Parts, and place the Orbits of *Venus* and *Mercury* at the Distance of S V = 724, and S M = 388, and then draw T A, T R, to touch those Orbits in the Points A and R; then 'tis plain the Angles A T S and R T S will measure the greatest Distance at which either of those Planets can be seen from the Sun; because the visual Ray passing to the Planet in any other Part of its Orbit will lie nearer to the Line T S W, and therefore shew the Planet nearer the Sun than when at A or R.

6. Now 'tis found by measuring those Angles geometrically in the Diagram, that the Angle A T S = 47 Degrees, and R T S = 20, very nearly; and this agrees exactly with their observed greatest Distances or Elongation from the Sun in the Heavens. Hence it is that *Mercury* is so rarely seen, and *Venus* but at certain Times of the Year; whereas if the Earth were at rest, and in the Centre of the Planetary Orbits, those Planets would be seen in all Positions and Distances from the Sun, in every respect like the Moon; and therefore 'tis perfectly surprizing, how any Man can resist such glaring Evidence of Truth on one hand, and Falshood on the other.

7. We have already shewn, that the apparent Magnitude and Brightness of an Object decreases as the Square Distance increases; therefore the Magnitude of *Venus* seen at V is to that as it appears from D in the Proportion of $\overline{TD}^2$ to $\overline{TV}^2$, that is, as 1724^2 to 276^2, or as 36 to 1 nearly. And when *Venus* is measured in both those Distances with a Micrometer in a Telescope, the Numbers shew the perfect Agreement of this System with Nature itself.

8. Thus

Vol III. *Plate* LVI. P. 135.

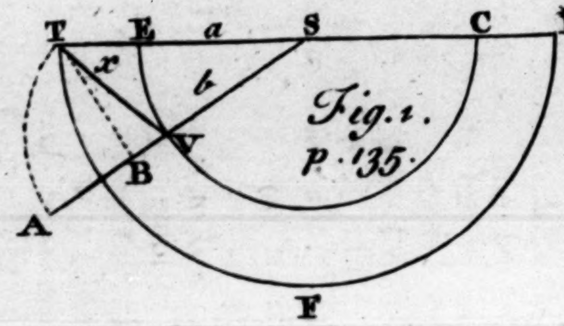


Fig. 1. p. 135.

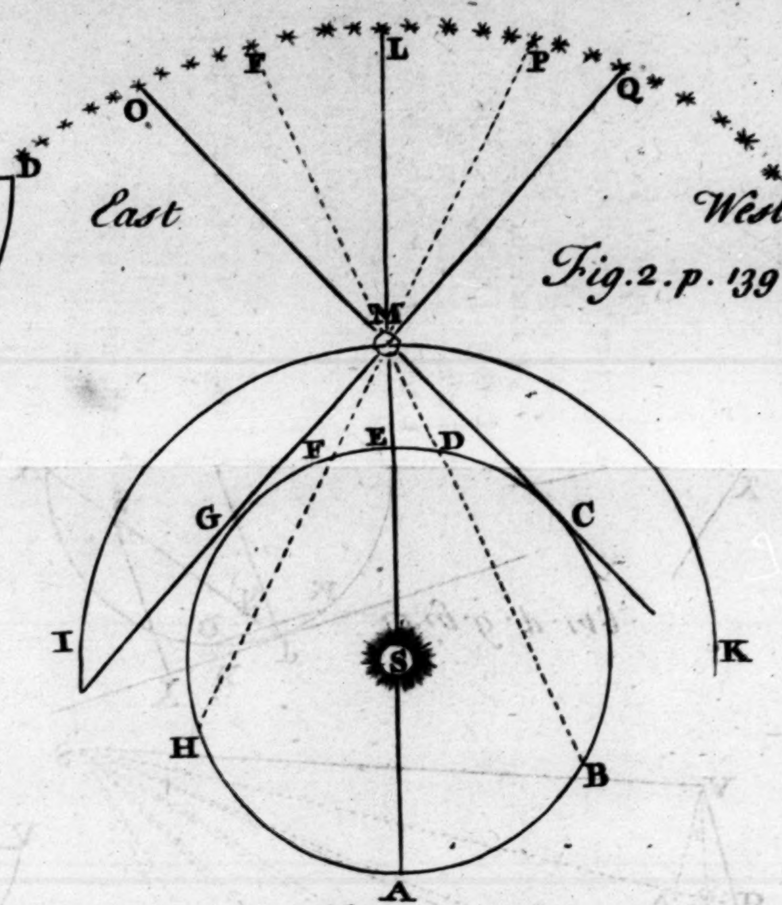


Fig. 2. p. 139.

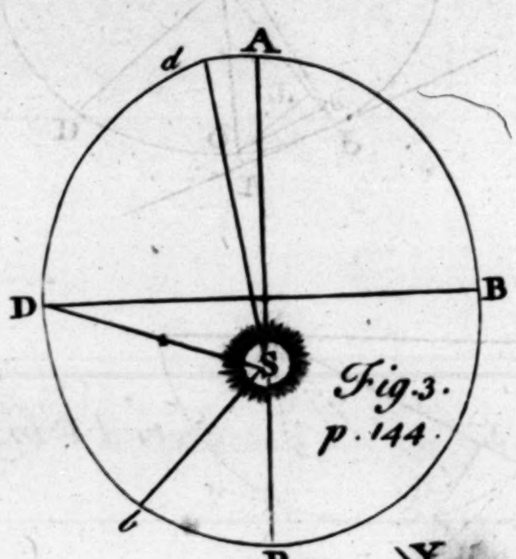


Fig. 3. p. 144.

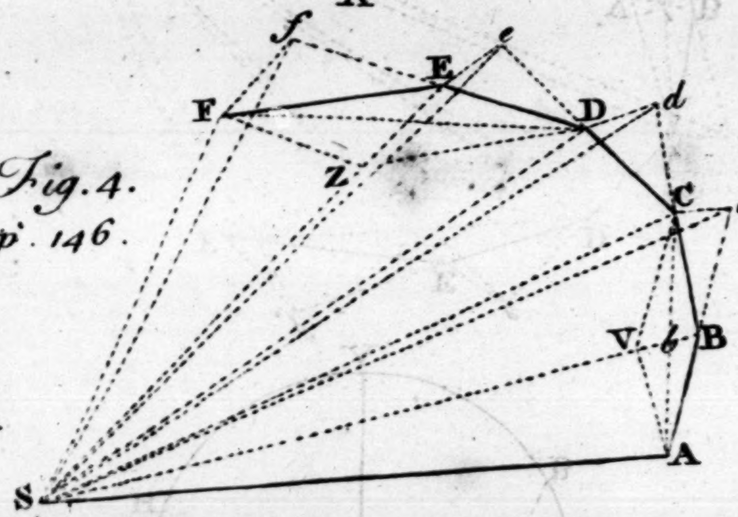


Fig. 4. p. 146.

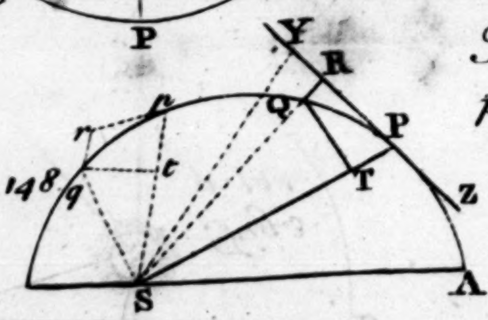


Fig. 5. p. 148.

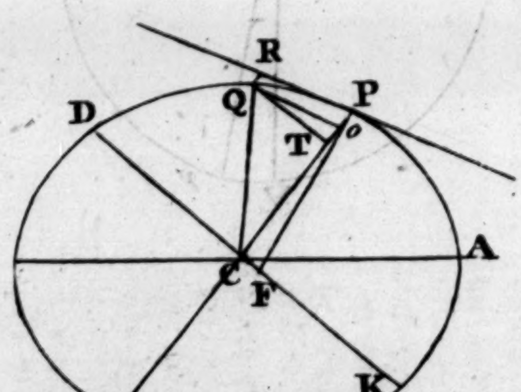


Fig. 8. p. 151.

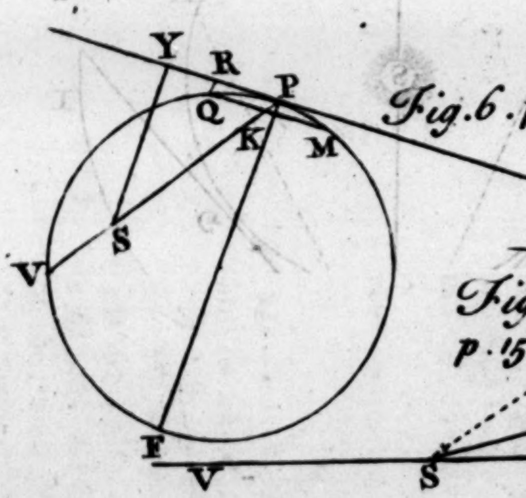


Fig. 6. p. 149.

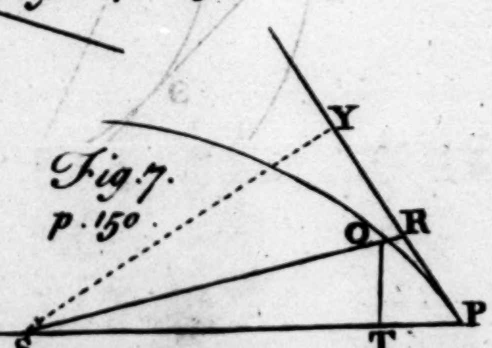


Fig. 7. p. 150.

and *Oppositions* to the Sun, alternate and successively; which could not be, unless their

8. Thus also the apparent Magnitude of *Mars*, when his Distance is t Y, is to that when his Distance is T Y, as $\overline{TY}^2$ to $\overline{tY}^2$; that is, as 2523^2 to 523^2, or very nearly as 25 to 1. And this we know is true in Fact, by measuring the Planet in both those Distances. It is likewise obvious to common Sense; for *Mars* in his nearest Distance appears so large that he has been often mistaken for *Jupiter*, whereas in his greatest Distance he appears so small as scarcely to be distinguished from a Fixed Star.

9. From what has been said of the Phases of the Moon, 'tis easy to understand that *Venus* and *Mercury* must have nearly the like Appearances. Thus when *Venus* is at V, all her illumined Hemisphere will be turned directly from the Earth, and she will then be *new*. As she passes from V to A she will appear *horned*. At A she will shew just half her enlightened Surface to the Earth, and appear *bisected*, or *dichotomised*. From A to D she will appear more and more *gibbous*; and at D would appear a *Full* enlightened Hemisphere, were it not that she is then lost in the Sun's Blaze, or hid behind his Body: All which Phases return again in the other half of the Orbit. The same Thing is obvious in *Mercury*, and *Mars* shews Part of those Phases; but *Jupiter* and *Saturn* appear always with a Full Face by reason of their very great Distance.

10. The Appearance of *Venus* in the Day-time for several Days together, in some certain Years, put the sagacious Dr. *Halley* on resolving the following Problem, viz. *To find the Situation of* Venus *in respect of the Earth, when the Area of the illuminated Part of her Disk is a Maximum.* I shall here give the Solution as he has proposed it in the *Philosophical Transactions*, N° 349; and also the Demonstration, which the Doctor omitted.

11. In order to this, let S be the Sun, V the Planet *Venus* in the Situation required, T the Earth, and T V her Distance sought. Put T S $= a$, S V $= b$, T V $= x$, and Pl. LVI. Fig. 1.

their Orbits were exterior to the Orbit of the Earth.

X. In

and on the Point V with the Radius VT describe the Quadrant TA; from T let fall the Perpendicular TB, and put $BV = d$; then $AB = x - d = v$, the Versed Sine of the Angle TVA. Now (by *Euclid* II. 12.) we have $a^2 = b^2 + x^2 + 2bd$, whence $a^2 - 2bd = b^2 + x^2$; and by adding $2bx$ on each Side, $a^2 + 2bx - 2bd = b^2 + 2bx + x^2$; that is, $a^2 + 2bv = b^2 + 2bx + xx = r$. Then $r - a^2 = 2bv = s$; and multiplying by $2x$ we have $4bxv = 2xs$, whence $4bx : s :: 2x : v$; that is, $4bx : b^2 + 2bx + xx - a^2 ::$ $2TV : AB ::$ the Diameter of a Circle to the versed Sine of the exterior Angle TVA.

12. But in any Situation B of the Planet *Venus* the Arch of Illumination af is equal to the Arch bd, which measures the exterior Angle bBd. And it has been shewn, that *the Area of the whole Disk of the Planet is to the Area of the enlightened Part as the Diameter of a Circle to the Versed Sine of the Arch of Illumination*, and therefore as $4bx$ to $b^2 + 2bx + x^2 - a^2$.

13. But the Area of the whole Disk is every where as $\frac{1}{xx}$; therefore, as $4bx : b^2 + 2bx + x^2 - a^2 :: \frac{1}{xx} : \frac{b^2 + 2bx + x^2 - a^2}{4bx^3}$, which in all Cases will be proportioned to the enlightened Area of the Disk. And to determine this a *Maximum*, its Fluxion must be $= o$, or the negative Parts thereof be equal to the affirmative, that is, that $\overline{2bx + 2xx} \times 4bx^3 = 12bx^2\dot{x} \times \overline{b^2 + 2bx + xx - a^2}$; and dividing all by $4bx^2\dot{x}$, the Equation becomes $2bx + 2x^2 = 3b^2 + 6bx + 3xx - 3a^2$. Consequently $\overline{3bb + 4bx + xx} = 3aa$; whence we get $x = \sqrt{\overline{3aa + bb}} - 2b = 427$.

14. If therefore we take 427 from the Scale of equal Parts ST, and set from T to the Orbit of *Venus*, it will intersect it in the Point x; and drawing Tx, it will give the

X. In the *third* Place, the *greatest Elongation* or *Distance* of *Mercury* from the Sun is

the Angle x T S = 40 Degrees nearly; which shews that when *Venus* is 40 Degrees distant from the Sun, before and after her *Inferior Conjunction* with him, she then shines with the greatest Lustre possible.

15. In this Position we see not much more than $\frac{1}{4}$ of her Disk enlightened, and yet she shines with so great a Lustre as to surpass the united Light of all the Fixed Stars that appear with her, and casts a very strong Shade on the horizontal Plane, and may be seen in the full Sun-shine of the Day; a Phænomenon very extraordinary, and which returns but once in eight Years.

16. The different Directions in which the Planets appear to move in the Heavens is an irrefragable Argument of the Truth of the Solar System; for in the *Ptolomean* System they would be seen to move with their true or real Motion, and in their Direction according to the Order of the Signs from West to East, in every Part of their Orbits, and that always in an equable Manner; whereas now we observe them move sometimes from *West* to *East*, when they are said to be *direct in Motion*; sometimes from *East* to *West*, when they are said to be *retrograde*, or to go backwards; and sometimes they appear not to move at all for a certain Time, when they are said to be *stationary*: And lastly, the Motion of a Planet when *direct* is always much slower than when it is *retrograde*.

17. Now all these Phænomena are not only explicable by, but necessarily follow from, the *Copernican* Theory. Thus with respect to the Planet *Mercury*, when at R he will appear at his greatest Distance from the Sun among the Stars at Q, being seen in the Line T Q; but as the Planet passes from R by N to O, the visual Line T Q will continually approach the Line T W, in which the Sun appears at W; and when the Planet is come to D it will be in Conjunction with the Sun, and will have apparently described the Arch Q W in the Heavens. After this, while the Planet moves from O to Z,

is but about 20 Degrees, and that of *Venus* but about 47; which answers exactly to

O to Z, it will appear to go in the Heavens from W to X, still the same Way as before; and because its apparent Motion agrees with the true, it is all this while *direct*.

18. But when the Planet moves from Z to M, the Ray T X will return, and describe the Arch X W back again; and as the Planet moves from M to R, the visual Ray will keep moving on from W to Q; and so in the Passage of the Planet through the Part of its Orbit Z M R it will appear to move in the Heavens through X W Q, the same Arch as before, but in a *retrograde* Direction.

19. Now because the Tangent Line or visual Ray T Q or T X coincides as to Sense with the Orbit of the Planet for a small Distance on each Side the Points R and Z, as from *a* to *b*, and from *c* to *d*; therefore the Planet when it arrives at *a* will appear to move in the Tangent from *a* to *b*, during which time it will be seen in the same Right Line T Q, and consequently in the same Point Q in the Heavens: So that in its Motion from *a* to *b* it must appear *stationary*, or without any Motion; and the same is to be observed in moving from *c* to *d*, when the Planet is in that Part of its Orbit.

20. Hence we observe, that in *Mercury* and *Venus*, the Places R, Z, and A, G, of their greatest Elongation are those in which they are *stationary*. It is in these two Points that we can at any time see *Mercury*; and it is in those Points that we see *Venus* such a glorious Morning-Star or *Phosphorus* at A, and such a splendid Evening-Star or *Hesperus* at G. Hence we observe, that from the Time *Venus* is a Morning-Star in her greatest Elongation at A, to the Time of her being an Evening-Star in her greatest Elongation at G, she is *direct* in Motion: Consequently, half the Time of her being a Morning or Evening-Star she is *direct*, and the other half *retrograde*.

21. Also

to their Distances in the System above assign'd: But in the *Ptolomean* System, they might

21. Also it is easy to observe, that since the same Arch Q X is described in Times very unequal, *viz.* in the Times the Planet describes the very unequal Parts of its Orbits R O Z and Z M R, the Velocity of the Motion in the former Case must be much less than that in the latter; that is, the Planet when *direct* moves apparently much slower than when it is *retrograde*.

22. If we consider the Dispositions of the Orbits of the superior Planets, we shall observe the same Phænomena of them also. Let S be the Sun, ACH the Earth's Orbit, I M K that of *Mars*, and O L Q the Firmament of Stars. Through *Mars* at M draw Q M G and O M C, to touch the Earth's Orbit in G and C. Then because the Earth and *Mars* do both move the same Way, but the Earth very quick in respect of *Mars*, all the Phænomena will be the very same if we suppose *Mars* to be at rest, and the Earth to move with the Difference of their Velocities. Plate LVI. Fig. 2.

23. Let *Mars* then be at rest in M, and the Earth begin her Motion from G. At G the Planet will be seen in the Line G Q, among the Stars at Q. When the Earth is at H, *Mars* will be seen in the Line H P, among the Stars at P. In the same Manner, at A, B, and C, the Planet will be projected to the Points L, N, O, in the Heavens Therefore while the Earth describes the Part of its Orbit G A C, *Mars* will appear to move through the Arch of the Heavens Q L O; which being from West to East is according to the Order of the Signs, and the Planet will be *direct in Motion*.

24. But as the Earth proceeds from C to D, *Mars* will appear to move from O to N; and as the Earth goes on through E, F, to G, *Mars* will appear to return by L, P, to Q, and so measure back again the same Arch as before: And thus during the Earth's Passage from C to G, this Planet will appear *retrograde*; which therefore must always be the Case when he is in Opposition

might and would ſometimes be ſeen 180 Degrees from the Sun, *viz.* in Oppoſition to him.

XI. FOURTHLY,

Oppoſition to the Sun and neareſt to the Earth, as in Conjunction he is always *direct in Motion*; and when the Earth is in G or C, the Planet muſt appear for ſome Time *ſtationary*, for the Reaſons mentioned in *Art.* 19. The ſame may be ſhewn of *Jupiter* and *Saturn*; but as the Earth has a much greater relative Velocity in reſpect to *Jupiter* than it has with reſpect to *Mars*, the Times of the *Conjunctions* and *Oppoſitions*, as alſo of the *progreſſive* and *regreſſive Motions*, will be more frequent in *Jupiter* than in *Mars*, and for the ſame Reaſon will happen oftener in *Saturn* than in *Jupiter*.

25. Again: Another Phænomenon, which infallibly proves the Truth of the *Copernican* Syſtem, is, that *Venus* and *Mercury* ſuffer an Occultation behind the Sun's Diſk, when they are in the remoteſt Parts of their Orbits, as at D and O; but this can never happen in the *Ptolomean* Hypotheſis, becauſe there the Orbit of the Sun is ſuppoſed exterior to the Orbits of thoſe two Planets.

26. All theſe Phænomena of the Planets plainly prove, that the Earth holds that Place in the Heavens which the preſent Philoſophy aſſigns her; but to ſhew moreover that ſhe has not only a Place among the Planets, but likewiſe that ſhe is carried in the ſame Manner with them about the Sun, we need only obſerve, that the Times in which theſe Phænomena happen to the Planets are no ways ſuch as they would be were the Earth at reſt, but ſuch as they muſt neceſſarily be ſuppoſing the Earth's Period about the Sun to be in 365¼ Days.

27. For Example: Suppoſe *Venus* at any Time in Conjunction with the Sun at V, then were the Earth at reſt at T, that very Conjunction would happen again when *Venus* had made juſt one Revolution, that is, in 225 Days; but every one knows this is contrary to Experience, for a much longer time than that lapſes between

XI. FOURTHLY, In the Disposition of the Planets they will all of them be sometimes much nearer to the Earth than at others; the Consequence of which is, that their Brightness and Splendor, and also their *apparent Diameters*, will be proportionally greater at one Time than another: And this we observe to be true every Day. Thus the

between two Conjunctions of the same Kind; as there evidently must, if we suppose the Earth to have a Motion towards the same Parts in the same Time; because then, 'tis plain, when *Venus* comes again to V, the Earth will have passed in that Time from T to some other Part of the Orbit, and from this keeps moving on till *Venus* gets again between it and the Sun.

28. What this Surplus of Time is, may be easily estimated, by supposing the Earth to be at rest in her Orbit, and *Venus* to move with the Difference of their mean Motions. Thus the daily mean Motion of the Earth is 59′ 8″, and the daily mean Motion of *Venus* is 1° 36′ 8″. The difference of these mean Motions is 37′; therefore say, As 37′ is to the whole Circle or 360° = 21600′, so is one Day to 583 Days, the Time between two Conjunctions as required, *viz.* 1 Year and 218 Days, in which Time *Venus* performs a little more than 2½ Revolutions. In the same Manner the Time may be found for any of the other Planetary Conjunctions, Oppositions, Stations, Retrogressions, &c.

27. These Arguments are plain, and easy to be understood; most of them require no more than common Observation, that is, in other Words, *common Sense*. To be ignorant of the Truths here specified, is to shew an unaccountable Inattention to the most obvious and glaring Phænomena of Nature: And if People are not convinced by these Proofs, it is not because they *cannot*, but because they *will not*; and therefore, *Si Populus vult decipi, decipiatur*.

the apparent Diameter of *Venus*, when greateſt, is near 66 Minutes, but when leaſt not more than 9 Minutes and a half; of *Mars*, when greateſt, it is 21 Minutes, but when leaſt no more than 2 Minutes and a half; whereas by the *Ptolomean* Hypotheſis they ought always to be equal.

XII. THE *fifth* is, That when the Planets are viewed with a good Teleſcope they appear with *different Phaſes*, or with different Parts of their Bodies enlightened. Thus *Venus* is ſometimes *new*, then *horned*, after that *dichotomiſed*, then *gibbous*, afterwards *full*; and ſo increaſes and decreaſes her Light, in the ſame Manner as the Moon, and as the *Copernican Syſtem* requires.

XIII. THE *ſixth* is, That the Planets all of them do ſometimes appear *direct* in Motion, ſometimes *retrograde*, and at other Times *ſtationary*. Thus *Venus*, as ſhe paſſes from her greateſt Elongation Weſtward to her greateſt Elongation Eaſtward, will appear *direct in Motion*, but *retrograde* as ſhe paſſes from the latter to the former; and when ſhe is in thoſe Points of greateſt Diſtance from the Sun, ſhe ſeems for ſome time *ſtationary*: All which is neceſſary upon the *Copernican Hypotheſis*, but cannot happen in any other.

XIV. THE

XIV. The *seventh* is, That the Bodies of *Mercury* and *Venus*, in their lower Conjunctions with the Sun, are *hid behind the Sun's Body*; and, in the upper Conjunctions, are seen to pass over the Sun's Body or Disk in Form of a *black round Spot*: Which is necessary in the *Copernican*, but impossible in the *Ptolomean* System.

XV. The *eighth* is, That the Times in which these *Conjunctions*, *Oppositions*, *Stations*, and *Retrogradations* of the Planets happen, are not such as they would be, were the Earth at rest in its Orbit; but precisely such as would happen, were the Earth to move, and all the Planets in the Periods above assigned them: And *therefore this, and no other, can be the true System of the World*; and it will stand the eternal Test of future Ages; for, MIGHTY IS THE FORCE OF TRUTH, AND SHALL PREVAIL.

But though the Planets all move round the Sun in Orbits commonly supposed *circular*, yet are they not exactly so, but *elliptical*, or in Form of an ELLIPSIS, which Figure is vulgarly called an *Oval*, as A B P D, described about two Centres S, F, called the *Foci*, or *Focal Points* of the Ellipse. The Point C is the Centre; A P the Axis, or longest Diameter; and B D the

the ſhorteſt Diameter: And in one of theſe Focus's, *viz.* S, the Sun is placed, about which the Planet moves in the Orbit ABPD. (CXL.)

HENCE,

(CXL.) 1. We have hitherto conſidered the Phænomena of the Heavenly Bodies without regard to the accurate Form of their Orbits, which is not *circular*, but *elliptical*; yet that it is very little ſo even in the moſt eccentric Orbit, as that of *Mercury*, will appear by comparing their Eccentricities with their mean Diſtances from the Sun. Thus ſuppoſe the mean Diſtance of the Earth from the Sun be divided into 1000 equal Parts, then in thoſe Parts we have,

In *Mercury*,	CS : DS ::	80 :	387 ::	1 :	4,84
Venus,	CS : DS ::	5 :	723 ::	1 :	144,6
Earth,	CS : DS ::	17 :	1000 ::	1 :	59
Mars,	CS : DS ::	141 :	1524 ::	1 :	10,8
Jupiter,	CS : DS ::	250 :	5201 ::	1 :	20,8
Saturn,	CS : DS ::	547 :	9538 ::	1 :	17,4

2. It is found by Experience that the Orbits of the Planets are quieſcent, or that the Line of the *Apſides* A P always keeps one and the ſame Poſition with reſpect to the Fixed Stars: And the *Aphelium*, or Point A, poſſeſſes different Points in the Ecliptic in the ſeveral Orbits as follows.

Pl. LVI. Fig. 3.

		°	′	″			°	′	″
In *Mercury*,	♐	12	44	00	In *Mars*,	♍	0	31	54
Venus,	♒	4	19	54	*Jupiter*,	♎	9	9	54
Earth,	♑	8	1	10	*Saturn*,	♐	27	49	54

3. That the Earth's Orbit is elliptical, is well known from common Experience; for were the Orbit circular, the Sun's apparent Diameter would always be the ſame; but we find it is not; for if it be meaſured with a Micrometer in Winter-time, it will be found conſiderably larger than in the Summer, and it will be greateſt of all when the Sun is in the 8° of ♑, (which ſhews that is

HENCE, when the Planet is in the Point P, it is nearest the Sun, which Point is, for

is the Place of the *Aphelium)* it being then 32′ 47″; whereas when the Sun is in the 8° of ♋, his Diameter is but 31′ 40″.

4. Hence it is evident that the Sun is really nearer to us in the Midst of Winter than in the Midst of Summer; but this seems a Paradox to many, who think the Sun must needs be hottest when it is nearest to us, and that the Sun is apparently more distant from us in *December* than in *June*. As to the Sun's being hotter, 'tis true it is so to all those Places which receive his Rays directly or perpendicularly; but we find his Heat abated on account of the Obliquity of the Rays, and his short Continuance above the Horizon at that Time. And as to his Distance, it is only with respect to the Zenith of the Place, not the Centre of the Earth; since it is plain, the Sun may approach the Centre of the Earth, at the same time that it recedes from the Zenith of any Place.

5. Agreeable to the Sun's nearer Distance in the Winter, we observe his apparent Motion is then quicker than in Summer; for in the 8° of ♑ it is about 61′ *per* Day, but in the 8° of ♋ his Motion is but 57′ *per* Day. Accordingly we find the Summer Half-Year 8 Days longer than the Winter Half-Year, as appears by the following Computation.

SUMMER *Half-Year includes*		WINTER *Half-Year includes*	
In *March*	$9\frac{1}{2}$ Days.	In *September*	6 Days.
April	30	*October*	31
May	31	*November*	30
June	30	*December*	31
July	31	*January*	31
August	31	*February*	28
September	24	*March*	$21\frac{1}{2}$
Summer-Half	$186\frac{1}{2}$		$178\frac{1}{2}$
Winter-Half	$178\frac{1}{2}$		
The Difference	8 Days.		

for that Reaſon called the *Perihelion*: Here, therefore, the Attraction of the Sun is ſtrongeſt,

6. For the Sun's attracting Force being one Part of the Cauſe of the Planet's Motion, and this Force always increaſing and decreaſing in the inverſe Ratio of the Squares of the Diſtances, 'tis evident the Velocity of the Planet will always be greater the nearer it is to the Sun, and *vice verſâ*. Hence the Motion of a Planet is every where unequable, being conſtantly accelerated as it paſſes from A by D to P, and in the other Half from P to A it is retarded.

7. Yet is this unequal Motion of a Planet regulated by a certain immutable Law, from which it never varies, which is, *That a Line drawn from the Centre of the Sun to the Centre of the Planet does ſo move with the Planet about the Sun, that it deſcribes elliptic Areas always proportional to the Times.* That is, if when the Planet moves ſloweſt it deſcribes the Arch A *a* in a given Time, and when it moves quickeſt it deſcribes the Arch *b* P in the ſame Time, then will the trilineal Area A S *a* be equal to the other trilineal Area *b* S P.

Pl. LVI. Fig. 4. 8. To demonſtrate this, let the Time in which the Planet moves through the Periphery of its Orbit be divided into equal Parts, and ſuppoſe that in the firſt Part it deſcribed any Right Line A B, by the Projectile Force in any Direction and the Centripetal Force conjointly; then in the ſecond Part of Time it would proceed in the ſame Right Line to *c*, if nothing prevented; ſo that B *c* = A B, as is manifeſt from the firſt *Law of Motion*.

9. Draw the Right Lines S B, S *c*, and the Triangles A B S and B *c* S will be equal, as having equal Baſes A B, B *c*, and the ſame Altitude of the Vertex S. But when the Body comes to B, let the centripetal Force act with a new Impulſe either equal to the former or unequal, and let it cauſe the Body to decline from the Right Line B *c*, and deſcribe the Right Line B C; draw C *c* parallel to B S, meeting B C in C; and at the

ſtrongeſt, his Light and Heat greateſt, and his apparent Diameter largeſt; and in this Point

the End of the ſecond Part of Time the Body will be at C, and in the ſame Plane with the Triangle A S B. Join S C, and becauſe of the Parallels S B, C *c*, the Triangle S B C will be equal to the Triangle S B *c*, and therefore equal to the Triangle S A B. By the ſame Way of Reaſoning, if the centripetal Force act ſucceſſively in the Points C, D, E, cauſing the Body in each equal Part of Time to deſcribe the Right Lines C D, D E, E F, *&c.* the Triangles S C D, S D E, S E F, *&c.* will be equal, and all in the ſame Plane.

10. In equal Times, therefore, equal Areas are deſcribed; and by Compoſition of Ratios, any Sums of Areas S A D S, S A F S, are to each other as the Times in which they are deſcribed. Let now the Number of Triangles be increaſed, and their Breadth be diminiſhed *in infinitum*; then will their Perimeter A D F be ultimately a Curve: And therefore the centripetal Force, by which the Body is drawn perpetually from the Tangent of this Curve, acts inceſſantly; and the Areas deſcribed are alſo in this Caſe proportional to the Times of their Deſcription.

11. Hence the Velocity of the revolving Body or Planet is every where inverſely as the Perpendicular let fall from the Centre S to the Tangent of the Orbit in the Place of the Planet. For the Velocities in the Points A, B, C, *&c.* are as the Baſes of the Triangles A B, B C, C D, *&c.* as being the Spaces deſcribed in the ſame Time; and the Baſes of equal Triangles are reciprocally as their perpendicular Altitudes; and therefore ſince in the evaneſcent Triangles A S B, A S C, *&c.* the Right Lines A *c*, B *d*, C *e*, *&c.* become Tangents to the Curve in the Points A, B, C, *&c.* 'tis manifeſt the Velocity in thoſe Points will be inverſely as a Perpendicular from S let fall upon thoſe Tangent Lines produced.

12. Hence alſo it follows, that the Times in which equal Arches are deſcribed in any Planetary Orbit are

Point the Planet must consequently move with the greatest Velocity. But in the Point

directly as those Perpendiculars, because they are inversely as the Velocities.

13. If two Chords of very small Arches described in the same Time A B, B C, and D E, E F, be compleated into the Parallelograms A B C V and F E D Z, and the Diagonals B V and E Z be drawn; then will those Lines tend to the Sun or Centre S, and be proportional to the centripetal Force: For the Motion B C and E F is compounded of B V, B c, and E Z, E f; but B V $=$ C c, and E Z $=$ F f; but C c and F f were generated by the Impulses of the centripetal Force in B and E, and are therefore proportional to them; and consequently so are B V and E Z.

14. Draw the Diagonal A C, and it will bisect the Line B V in b; consequently the *Sagitta* B b is as the centripetal Force by which the Arch A B C is described, whose Chord is A C.

Pl. LVI. Fig. 5. 15. Hence if a Body revolve in any Curve A P q about an immoveable Centre S, the Force in any Point P will be to that in any other Point p as $\frac{QR}{SP^2 \times QT^2}$ to $\frac{qr}{Sp^2 \times qt^2}$; for the *Sagittæ* Q R, $q\,r$, (which call S, s,) are as the centripetal Forces (F, f,) in P and p, when the Times (T, t,) are given, (by the last) that is, S : s :: F : f. But when the Forces are given, the *Sagittæ* will be as the Squares of the Times, *viz.* S : s :: T T : $t\,t$. Therefore when neither the Times nor the Forces are the same, it will be S : s :: F $\times$ T^2 : $f \times t^2$; and so $\frac{S}{T^2} : \frac{s}{t^2} :: F : f$. And because the elliptic Areas S Q P and S $q\,p$ are as the Times in which they are described, therefore when the Arches P Q and $p\,q$ are indefinitely small, we have T : t :: $\frac{1}{2}$ S P $\times$ Q T : $\frac{1}{2}$ S p $\times$ $q\,t$ :: S P $\times$ Q T : S p $\times$ $q\,t$. Consequently we have, as

$$F : f :: \frac{QR}{SP^2 \times QT^2} : \frac{qr}{Sp^2 \times qt^2}.$$

16. Let

Point A, where the Planet is fartheſt diſtant from the Sun, (for that Reaſon call'd the

16. Let S Y be a Perpendicular let fall from S upon the Tangent P R produced; then will the centripetal Force be as $\frac{QR}{SY^2 \times QP^2}$, becauſe the Rectangle $SY \times QP = SP \times QT$; for the evaneſcent Arch Q P is coincident with the Tangent P R, and may therefore be eſteemed as the Baſe of the Triangle S P Q, whoſe Area is either $\frac{1}{2} SP \times QT$, or $\frac{1}{2} QP \times SY$; therefore $SP \times QT = QP \times SY$. Which was to be ſhewn.

17. If the Orbit were a Circle, as P Q V F, and P V a Chord drawn through the Centre of Force S; then drawing the Chord Q M in ſuch Manner as it may be biſected in K by the Chord P V, we have $QK^2 = VK \times PK$, (by *Euclid*, III. 35.) but in the vaniſhing State of P K it will be $VK = VP$, and $QR = PK$ (by *Art.* 13.) alſo $QK = QP$, therefore $QP^2 = VP \times QR$, and $PV = \frac{QP^2}{QR}$; whence, in this Caſe, the central Force will be inverſely as $SY^2 \times PV$. Pl. LVI. Fig. 6.

18. Wherefore, ſince the Velocity is as $\frac{1}{SY}$, we have SY^2 as the Square of the Velocity inverſely; therefore the centripetal Force is as the Square of the Velocity directly, and the Chord P V inverſely.

19. Hence if the curvilinear Figure A P Q be given, and any Point S to which the centripetal Force is continually directed, the Law of the centripetal Force may be found, by which any Body P perpetually drawn from a right-lin'd Courſe ſhall be detain'd in the Perimeter of that Figure, and by revolving ſhall deſcribe it, *viz.* by computing the Value of the Expreſſion $\frac{QR}{SP^2 \times QT}$, or of $SY^2 \times PV$.

20. For Example: *Let a Body* P *revolve in the Circumference of a Circle, 'tis required to find the Law of the centripetal*

the *Aphelion)* every thing is juſt the reverſe: And in the Points B or D it is in its mean Diſtance from the Sun.

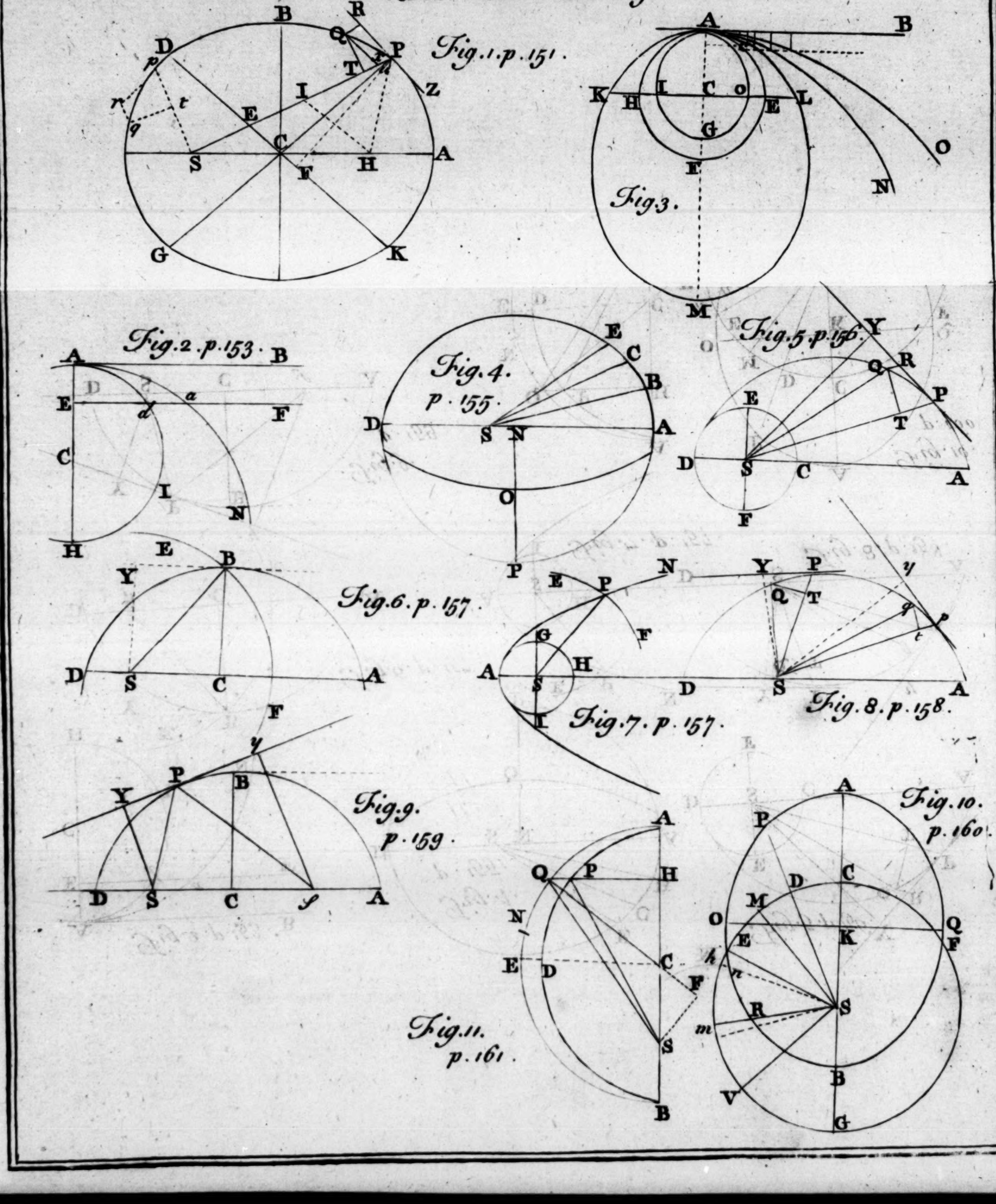
Now

Pl. LVI. Fig. 6. *centripetal Force tending to any given Point* S. Let P Y be a Tangent in the Point P, and S Y the Perpendicular, and V P the Chord paſſing through S. Let V A be the Diameter of the Circle, and join A P. Then is the Triangle S Y P ſimilar to the Triangle V A P; as may be ſhewn from *Eucl.* III. 32. Therefore A V : P V :: S P : S Y; conſequently $\frac{SP \times PV}{AV} = SY$, and ſo $\frac{SP^2 \times PV^3}{AV^2} = SY^2 \times PV$, which therefore is as the centripetal Force inverſely; but becauſe AV^2 is a given Quantity, we have the ſaid Force reciprocally as $SP^2 \times PV^3$.

21. Again: *Let it be required to find the Law of the centripetal Force by which a Body is moved, ſo as to deſcribe the equiangular Spiral* P Q S *about the Centre* S. In this Caſe all the Angles are given in every trilineal
Fig. 7. Area S Q P, and therefore alſo the Ratio of all the Sides in the Figure S P R Q T; therefore the Ratio of $\frac{QT}{QR}$ is given, whence $\frac{QT}{QR} \times QT$ is as Q T; that is, (becauſe of the given Ratio of Q T to P S) $\frac{QT^2}{QR}$ is as S P. And this Ratio will be conſtant, let the Angle P S Q be changed in any Manner whatſoever: For let $QR = a$, when the Angle P S Q is conſtant, and $QT = b$; but when it is variable, let $QR = x$, and $QT = y$; then (by *Lem.* 11. of *Princip.)* it will be $a : x :: b^2 : y^2$, whence $\frac{b^2}{a} = \frac{y^2}{x} = \frac{QT^2}{QR}$; which ſhews that $\frac{QT^2}{QR}$ will always remain the ſame as at firſt, *viz* as S P. Therefore $\frac{QT^2 \times SP^2}{QR}$ will become SP^3; conſequently the centripetal Force Q R will be inverſely as SP^3.

22. *Let*

Vol. III. Plate LVII. P. 151.



Now though the Planetary Orbits are really *elliptical*, yet is the *Eccentricity* CS, in

22. *Let a Body revolve in an Ellipsis* APQ, *by a Force every where directed to the Centre* C; *it is required to find the Law of that Force.* Let Qv be drawn parallel to the Tangent PR, and PF perpendicular to KC; and parallel to PF join CQ; the rest as before. The right-angled Triangles QTV and PFC are similar; for the Angle $QvC = PCF$, (by *Euclid*, XXIX. 1.) therefore $QT : Qv :: PF : PC$; and $QT \times PC = Qv \times PF$. But $QT \times PC$ is equal to twice the Triangle PQC, which is a constant Quantity, as being proportional to the constant Particle of Time in which it is described. Also in the Ellipsis $DK \times PF$ is a constant Quantity *(per Conics)*. Therefore $DC \times PF$ is to $QT \times PC$, or $Qv \times PF$, that is, DK to Qv in a given Ratio, wherever the Point P is taken in the Ellipsis. Hence also the Ratio of DK^2 to Qv^2 is a constant one: But in the Ellipsis $DK^2 : Qv^2 :: PG^2 : Pv \times vG$ *(per Conics.)* Now because $Qv = QR$, and the Difference between Gv and GP is infinitely small, therefore $Pv \times vG = QR \times PG$; whence PG^2 is in a constant Ratio to $PG \times QR$, that is, QR *or the centripetal Force is every where in a constant Ratio to* PG, *or to* PC, *the Distance from the Centre.* Pl. LVI. Fig. 8.

23. Hence if the Centre C of the Ellipsis were to go off to an infinite Distance, the Ellipsis would be changed into a *Parabola*, in which the Body would move, and the Force now tending to a Centre at an infinite Distance would become equable, or the same with *Gravity*, according to the Theory of *Galileo*. And if the *Parabola* should be changed into an *Hyperbola*, the Body would move in that Curve by the same Law of the Force now changed from a centripetal to a centrifugal one, because now it causes the Body to recede from the Centre.

24. Lastly: *Let it be required to find the Law of the Force tending to one of the Foci of an Ellipsis.* Draw SP to the Focus S, and PH to the Focus H, and HI parallel Plate LVII. Fig. 1.

in most of them, so extremely small, as to be almost insensible; and therefore their Motions

rallel to DK. Now because $CS = CH$, we have $SE = EI$; and because the Angle $HPZ = SPR$, (*per Conics*) and HI parallel to PR, therefore the alternate Angle $PHI = PIH$, and so $PI = PH$; consequently $EP = \frac{PS + PH}{2} = AC$, from the Genesis of an Ellipsis. Let the *Latus Rectum* of the Ellipsis be $L = \frac{2BC^2}{AC}$, (because $2AC : 2BC :: 2BC : L$) and Qv intersect PS in x. Then because $QR = Px$, and the Triangle Pxv similar to the Triangle PEC, we have $Px : Pv :: PE (= AC) : PC$; therefore $QR : Pv :: AC : PC :: L \times QR : L \times Pv$. (*Theorem* I.) Again, $L \times Pv : Gv \times vP :: L : Gv$. (*Theorem* II.) Also, $Gv \times vP : Qv^2 :: PC^2 : DC^2$ (*per Conics.*) *Theorem* III. Again, $Qx^2 : QT^2 :: PE^2 : PF^2$; but when the Points P and Q coincide, it is $Qx^2 = Qv^2$, and $PE^2 = CA^2$; wherefore then $Qv^2 : QT^2 :: CA^2 : PF^2$. Now because $PF \times CD = AC \times BC$, (*per Conics*) therefore $PF^2 \times CD^2 = AC^2 \times BC^2$, and so $AC^2 : PF^2 :: CD^2 : BC^2$; consequently $Qv^2 : QT^2 :: CD^2 : CB^2$. (*Theorem* IV.)

25. These four Theorems set separately as below.

THEOREM I. $L \times QR : L \times Pv :: AC : PC$.

II. $L \times Pv : Gv \times vP :: L : Gv$.

III. $Gv \times vP : Qv^2 :: PC^2 : CD^2$.

IV. $Qv^2 : QT^2 :: CD^2 : CB^2$.

It is evident, by joining all the Ratios we have $L \times QR : QT^2 :: AC \times L \times PC^2 \times CD^2 : PC \times Gv \times CD^2 \times CB^2$; but because $AC \times L = 2BC^2$, we have $L \times QR : QT^2 :: 2PC : Gv$. Now when P and Q coincide, $2PC = Gv$, and then $L \times QR = QT^2$; and multiplying each Side by $\frac{SP^2}{QR}$, we shall have $L \times SP^2 =$

SP²

Motions may be looked upon as *circular*, and as ſuch repreſented in Orreries and Diagrams, without any ſenſible Error.

THE

$\frac{SP^2 \times QT^2}{QR}$. Therefore the centripetal Force is as $L \times SP$ inverſely; or, becauſe L is a given Quantity, it will be directly as $\frac{1}{SP^2}$.

26. I ſhall now ſhew what Ratio the projectile Force which cauſes a Body to deſcribe a *Circle* has to that which *(cæteris paribus)* cauſes the Body to deſcribe any *Conic Section*. Let us aſſume this Ratio to be that of n to 1; and putting $2a$ and $2b$ for the tranſverſe and conjugate Diameters of the Conic Section A N, the Circle being A I H, ſuppoſe the right Line E F to move parallel to itſelf, and the Points a and d therein ſo as to deſcribe the Curves A I and A N; and let the Diſtance of that Line from A B be call'd x, *viz.* $AE = x$; and let $2d = AH$ the Diameter of the Circle. Plate LVII. Fig. 2.

27. Now $\sqrt{2dx - xx} = Ed$ in the Circle, and $\frac{b}{a} \times \sqrt{2ax \mp x^2} = Ea$ in the Conic Section. The Fluxions of the Ordinates E d and E a, *viz.* $\frac{\overline{d - x} \times \dot{x}}{\sqrt{2dx - xx}}$ and $\frac{a}{b} \times \frac{\overline{a \mp x} \times \dot{x}}{\sqrt{2ax \mp xx}}$, will be as the Velocities in every Point of the Curves in the Direction E F or A B. But theſe Fluxions are as $\frac{d - x}{\sqrt{2d - x}}$ and $\frac{b}{a} \times \frac{a \mp x}{\sqrt{2a \mp x}}$, $\left(\text{dividing by } \frac{\dot{x}}{\sqrt{x}}\right)$ and therefore when E F arrives to A B, or $x = o$, the Ratio of thoſe Fluxions of Velocities will become that of $\frac{d}{\sqrt{2d}}$ to $\frac{b}{a} \times \frac{a}{\sqrt{2a}}$, or as $\sqrt{d}$

to

THE ORRERY is, therefore, an adequate Representation of the TRUE SOLAR SYSTEM,

to $\frac{b}{\sqrt{a}}$ in the Point A. Wherefore $\sqrt{d} : \frac{b}{\sqrt{a}} :: 1 : n$; whence we have $nnad = bb$.

28. And when $x = d =$ A C, the Distance of the Centre of Force, we have $\frac{b}{a}\sqrt{2ax \mp xx} = p = \frac{bb}{a}$ become $2ad \mp dd = bb = nnad$. Whence we get $a = \frac{\pm d}{2 - n^2}$ and $b = \frac{\pm nd}{\sqrt{2 - n^2}}$. Having therefore the Diameters $2a$ and $2b$, the Conic Section is given in Specie.

29. Now because Unity, or 1, represents the projectile Force to describe a Circle, the Force n may be any other Number greater or less to describe a Conic Section. And first let $n^2 = 2$; then will $a = \frac{\pm d}{2 - n^2} = \frac{\pm d}{0} =$ Infinite, or the Centre of the Curve will be at an infinite Distance from A, and consequently be the *Parabola* A N.

30. If the Value of n^2 be between 1 and 2, or if n be any Number between 1 and $\sqrt{2}$, then will the Conic Section be an Ellipse between the Circle A E F H and the Parabola A N, having the Centre of Force C in the upper Focus next A, as the Ellipse A L M K.

31. But if n be any Number less than 1, the Curve will still be an Ellipse, but within the Circle, having the Centre of Force C in the lower or remote Focus, as the Ellipsis A I G O.

32. Again; if n^2 be greater than 2, or n greater than $\sqrt{2}$, then will a be negative; consequently the Curve will be an *Hyperbola*, as A O.

33. Lastly; if $n^2 = 0$, then $b = \frac{\pm nd}{\sqrt{2 - n^2}} = 0$, and $a = \frac{1}{2}d$; that is, if the projectile Velocity be diminished

ad

TEM, and gives a just Idea of the *Number*, *Motions*, *Order*, and *Positions* of the heavenly Bodies:

ad infinitum, then the Curve or Trajectory will become the Right Line A C; or the Projectile will descend directly to the Centre of Force C.

34. Let A = the Area of any Ellipse, S, *S*, *s*, the Areas of the Sectors A S B, B S C, C S D, &c. and T, *T*, *t*, the Times in which they are described; then we have S : *S* :: T : *T*, and S : *s* :: T : *t*, and so on for every Sector through the whole Area. Therefore S : T :: S + *S* + *s* : T + *T* + *t* :: Sum of all the Sectors : Sum of all the Times in which they are described; so is the whole Area A to the periodical Time P of a whole Revolution. Consequently, S × P = T × A, and $P = \frac{A \times T}{S}$; and in a given Particle of Time T, we have P as $\frac{A}{S}$. Plate LVII. Fig. 4.

35. By *Art.* 25. we have the principal *Latus Rectum* $L = \frac{QT^2}{QR}$, but in a given Time the centripetal Force QR is as $\frac{1}{SP^2}$; wherefore in a given Time $L : QT^2 \times SP^2$; and so $L^{\frac{1}{2}} : QT \times SP : S$, the Sector A S B described in a given Time. Whence $P : \frac{A}{L^{\frac{1}{2}}}$; therefore $A : P \times L^{\frac{1}{2}}$, that is, *The Area of an Ellipse is in the Subduplicate Ratio of the* Latus Rectum *and Periodical Time conjointly*.

36. Now let *a* = Transverse Axis, and *b* = Conjugate; then (by *Conics*) $a : b :: b : L$, and so $b^2 = a\,L$, and $b = a^{\frac{1}{2}} \times L^{\frac{1}{2}}$; whence $a\,b = a^{\frac{3}{2}} \times L^{\frac{1}{2}}$. But the Rectangle $a \times b$: A, the Area of the Ellipse (by *Conics*) therefore $a^{\frac{3}{2}} \times L^{\frac{1}{2}} : A : P \times L^{\frac{1}{2}}$ (by *Article* 35.) that is, $a^{\frac{3}{2}} : P$; or, *The Periodical Time is in the Sesquiplicate*

Bodies: But the Proportion of *Magnitude* and *Distances* of the Planets is not to be expected

plicate Ratio of the Transverse or greater Axis of the Ellipse.

37. Hence the Periodical Time will be the same in all the Species of an Ellipsis from a Right Line to a Circle described upon the same transverse Diameter; or, more particularly, the Time of describing the Semi-Ellipse A E D will be the same as that of the Semi-Ellipse A O D; and the same also as the Time of describing the Semi-Circle A P D, which is only one Species of an Ellipsis, where the Foci coincide with the Centre N, and the Semi-Conjugate N O becomes the Semi-Diameter N P. Lastly, when the Semi-Ellipse A O D degenerates into a Right Line A D by diminishing the Semi-Conjugate N O *in infinitum*, and the Focus receding to the End of the Axis at D, it is plain the Time of describing the Line A D is still the same.

Plate LVII. Fig. 5.

38. The Velocity of the revolving Body P is as $\frac{SY}{L}$, S Y being a Perpendicular let fall on the Tangent P Y from the Centre of Force S; for the Velocity is ever as the small Arch Q P described in a given Time. But Q P = P R, in its evanescent State: And because of the Right Angles at T and Y, and the Angle Q P T = Y P S when the Points Q, P, coincide, the evanescent Triangle Q P T will be similar to P S Y; and therefore give Q P (= P R) : Q T :: P S : S Y; whence P R = $\frac{SP \times QT}{SY}$. But S P × Q T : $L^{\frac{1}{2}}$; therefore P R : $\frac{L^{\frac{1}{2}}}{SY}$. That is, *The Velocity is in the Subduplicate Ratio of the* Latus Rectum *directly, and the Perpendicular inversely.*

39. Hence the Velocities in the greatest and least Distances A and D are in the Ratio compounded of the Distances

pected from the Orrery, but by Delineation, as in Mr. *Whiston*'s *Solar System*; where the several

Distances S A and S D inversely, in the same Figure where L is a given Quantity; because in that Case the Distances are the Perpendiculars.

40. Therefore if a Circle D E C F be described at the same Distance S D, because the Circle is that Species of Ellipsis whose *Latus Rectum* is equal to the Diameter 2 D S, and since in this Point D the perpendicular Distance is the same in both, the Velocity of the Body in the Ellipsis at the Point D is to that of a Body describing the Circle in the Subduplicate Ratio of L to 2 D S, or as $\sqrt{L}$ to $\sqrt{2\,D\,S}$; and the same may be shewn with respect to the Velocities at the other Point A.

41. To compare the Velocity in the Ellipse at the mean Distance B with that of a Body describing a Circle E F at the same Distance C B from the common Focus S, let R = Radius of the Circle = A C = C D = S P, and let B = lesser Semi-Axis B C, which is here equal to the Perpendicular S Y to the Tangent in the Point B. Let the Velocity in the Ellipse be V, and in the Circle v; and as L = *Latus Rectum* of the Ellipse, so 2 A is that of the Circle; therefore (*Art.* 38.) $V : v :: \frac{L^{\frac{1}{2}}}{B} : \frac{\overline{2A}^{\frac{1}{2}}}{A}$, or $V^2 : v^2 :: \frac{L}{B} : \frac{2A}{A^2} :: L \times A : 2B$. (Plate LVII. Fig. 6.)

But because (by *Conics*) A : B :: 2 B : L, therefore $2B^2 = A \times L$: consequently $V^2 = v^2$, and so $V = v$. That is, *The Velocity of the Body in the Ellipse in the Point* B *is equal to that in the Circle* E F *described with the mean Distance* S B.

42. It has been already shewn (*Art.* 29.) that the Velocity of a Body in the Vertex of a *Parabola* is to that in a *Circle* at the same Distance from the Focus, as $\sqrt{2}$ to 1. And because every thing that has been shewn relating to the Motion in an Ellipse may be demonstrated also of the *Parabola* and *Hyperbola*, (See *Princip.* (Fig. 7.)

Lib.

ſeveral Orbits of the Planets are laid down in their proportional Diſtances from the Sun;

Lib. I. Prop. XII. XIII.) therefore in the *Parabola* the Velocity will be every where at P as a Perpendicular S Y let fall upon the Tangent P Y reciprocally. And (by *Conics*) $SY^2 : SP$, and ſo $SY : \sqrt{PS}$; therefore, *The Velocity in the Parabola will be every where as* $\sqrt{\frac{1}{SP}}$, *or in the Subduplicate Ratio of the Diſtance inverſely.*

43. We have alſo ſhewn (*Annot.* XXXIV. 13.) in a Circle whoſe Radius is a, P = Periodical Time, V = Velocity, that $VP = a$, and $V = \frac{a}{P}$, and therefore $V^2 = \frac{a^2}{P^2}$; but alſo $P^2 : a^3$, (*ibid.* 11.) whence $V^2 : \frac{a^2}{a^3} : \frac{1}{a}$; therefore $V : \sqrt{\frac{1}{a}}$. Therefore the Velocity (V) in the Circle A G H I is to the Velocity in the Circle E P F deſcribed with the Radius S P, as $\sqrt{\frac{1}{AS}}$ to $\sqrt{\frac{1}{SP}}$; or $V : v :: \sqrt{SP} : \sqrt{AS} = \sqrt{\frac{1}{4}L}$. But the Velocities in the Points A and P in the *Parabola* alſo are in the ſame Ratio of $\sqrt{SP}$ to $\sqrt{\frac{1}{4}L}$ (by 42.); conſequently, *The Velocity in the Parabola at the Vertex* A *is to the Velocity in the Circle in the ſame Diſtance* A S, *as the Velocity in the Parabola at* P *is to the Velocity in the Circle deſcribed at the ſame Diſtance* S P; *that is, in the Ratio every where of* $\sqrt{2}$ to 1.

44. Again; the Velocity in the Circle whoſe Radius is $\frac{1}{2}$ S P is to the Velocity in a Circle whoſe Radius is S P, as $\sqrt{SP}$ to $\sqrt{\frac{1}{2}SP}$, or as $\sqrt{2}$ to 1; conſequently, *The Velocity in the Parabola at* P *is equal to the Velocity in a Circle whoſe Radius is* $\frac{1}{2}$ S P.

Plate LVII. Fig. 8.

45. The angular Velocity of a Body P revolving in any Orbit, that is, the Angle which is made at the Centre

Sun; and their Magnitudes comparatively with each other, and with that of the Sun, expreſs'd

Centre S, *viz.* PSQ, by the *Radius Vector* SP deſcribing in a given Time the Arch PQ, is as QT directly, and as SP inverſely; that is, the Angle PSQ : $pSq :: \frac{QT}{PS} : \frac{qt}{pS}$. This is eaſy to underſtand when we conſider that any Angle is greater as the Arch PQ or pq, deſcribed in a given Time, is ſo; and leſs in Proportion to the Diſtance SP and sp, becauſe the Velocities with which thoſe Arches are deſcribed are inverſely as the Perpendiculars SY, Sy, to the Tangents in thoſe Points; and when the Arches QP and qp are indefinitely ſmall, we may eſteem them equal to the Lines QT and qt. Whence the Propoſition is evident.

45. Hence the angular Velocity at P and p is as $\frac{1}{SP^2}$ and $\frac{1}{Sp^2}$; for the Sectors PSQ and pSq, being deſcribed in the ſame Time are equal; whence $QT \times SP = qt \times Sp$. Therefore $QT : qt :: Sp : SP$; and hence $\frac{QT}{SP} : \frac{qt}{Sp} :: \frac{Sp}{SP} : \frac{SP}{Sp} :: Sp^2 : SP^2 :: \frac{1}{SP^2} : \frac{1}{Sp^2}$.

46. From the Foci S, s, of the Ellipſe ABD let fall the Perpendiculars SY, sy, to the Tangent Yy in the Point P; let the centripetal Force tend to the Focus S; and let CB be the leſſer Semi-Axis. Then will the Velocity (v) in B be to the Velocity (V) in P, in the Ratio of $\sqrt{sP}$ to $\sqrt{SP}$. For $V : v :: CB : SY$ (*Art.* 11.) whence $V^2 : v^2 :: CB^2 : SY^2$. But (by *Conics*) $BC^2 = SY \times sy$; therefore $V^2 : v^2 :: SY \times sy : SY^2 :: sy : SY$. But becauſe of the ſimilar Triangles SPY and sPy, it is $sy : SY :: sP : SP$; wherefore $V^2 : v^2 :: sP : SP$; conſequently $V : v :: \sqrt{sP} : \sqrt{SP}$. Plate LVII. Fig. 9.

47. From what has been ſaid it appears, that the Motion of a Planet in its Orbit is very unequal and anomalous;

anomalous; and this Anomaly or Irregularity of the Planet's Motion is in itself very irregular also, being sometimes more, and sometimes less than at others. And in order to explain this, it will be requisite to compare it with an equal and uniform Motion of a Body moving in a Circle. Let therefore the Ellipse AEBF be the Orbit of a Planet whose Focus is S, its greater Axis AB, and lesser OQ. On the Centre S, and with the Distance SE (which is a mean Proportional between AK and OK, the two Semi-Axes) describe the Circle CEGF. The Area of this Circle will be equal to the Area of the Ellipse, as I have shewn in my *Elements of Geometry*.

Plate LVII. Fig. 10.

48. In this Circle let us suppose a Point to move with an uniform or equal Motion through the Periphery CEGF, in the same Time that the Planet describes the Ellipse; and when the Planet is in its *Aphelium* A, let the circulating Point be in C, and the Motion of this Point will represent the equal or mean Motion of the Planet; and the Point will describe round S Areas proportional to the Times, and equal to the elliptic Areas the Planet at the same time describes.

49. Let now the equal Motion or angular Velocity in the Circle be CSM; and take the Area ASP equal to the Sector CSM; and then the Place of the Planet in its Orbit will be P; and the Angle MSD, the Difference between the true Motion of the Planet and its mean Motion, is the Equation, and is call'd the *Prosthaphæresis*, from its being *added to* or *taken from* the mean Motion, to obtain the true or equated Anomaly.

50. Hence the Area ACDP will be equal to the Sector DSM, and therefore proportional to the *Prosthaphæresis*; and consequently where this Area is biggest, there the *Prosthaphæresis* or Equation will be greatest, or a *Maximum*; which evidently happens when the Planet arrives at E, where the Ellipse and the Circle cut each other. For when the Planet descends farther to R, the Equation becomes proportional to the Difference of the Areas ACE and *m*ER, or to the Area GBR*m*; for when the Planet is at R, let the Point be at V, and the Sector CSV will be equal to the elliptic Area ASR, that is, ACE + CERS = CERS + *m*ER

$+ mER + mSV$; consequently $ACE - mER = mSV = mRBG$.

51. In the *Perihelion* the equal Motion and the true Motion of the Planet coincide, because the Semicircle CEG and Semi-ellipse AEB are equal, and are described in the same Time. As the Planet descended from the *Aphelium* A to the *Perihelium* B, its Motion was slower, or less than the mean Motion; in which Case the Equation or *Prosthaphæresis* is to be *subtracted* from the mean Motion, to get the true Motion and Place of the Planet.

52. But during the Ascent of the Planet from the *Perihelium* B to the *Aphelium* A, its Motion will be quicker than the mean Motion, as might be shewn in the same Manner as above. In A the Velocity is least of all, and in B greatest, as we have shewn; and in E it is equal to the mean Velocity in the Circle. For when the Planet is in E, let the Point be in m, and let the Area ESn and Sector mSi be described in the same infinitely small Particle of Time, and therefore equal to each other; for $Eb \times Sn = (Eb \times SE =)\ mi \times mS$; but $SE = mS$, therefore $Eb = mi$; therefore the angular Velocity ESb at E is equal to the angular Velocity mSi, which is the mean Velocity.

53. In order therefore to find the equated or true Anomaly from the mean, we are to find the Position of a Line SP that shall cut off the elliptic Area ASP, to which the whole Area of the Ellipse has the same Proportion as the whole Periodical Time of the Planet has to the Time given in which the elliptic Sector was described. Or if AQB be a Semicircle described on the longer Axis of the Ellipse, we must draw from S the Line SQ, which shall cut off the Area ASQ, to which the Area of the whole Circle is in the abovementioned Ratio; for then a perpendicular QH will cut the Ellipse in P, so that the Line PS being drawn, the elliptic Area ASP will be to the Sector ASQ as the whole Area of the Ellipse to that of the Circle, as is shewn. Pl.LVII. Fig. 12

54. To cut an Ellipse or Circle in this Proportion was the famous Problem long since proposed by *Kepler*, which is solved as follows. Upon QC, produced if

required, let fall the Perpendicular S F; the Area A S Q is equal to the Sector A C Q and the Triangle Q S C, that is, equal to $\frac{1}{2}$ Q C × A Q + $\frac{1}{2}$ Q C × S F; and because $\frac{1}{2}$ Q C is a constant Quantity, the Area A S Q will be proportional to A Q + S F. Hence if we take the Arch Q N = S F, we have the Arch A N proportional to the Time or mean Anomaly of the Planet; which we can easily find by having the true Anomaly given.

55. For Example; in the Orbit of *Mars* we have Q C : S C :: 152369 : 14100; and because the Length of an Arch equal to Radius is 57°,29578, say,

As the Radius	Q C = 152369	= 5.182985
Is to the Eccentricity	S C = 14100	= 4.149229
So is the Length of the Arch	57°,29578	= 1.758078
To the Length of an Arch B,	5°,302	= 0.724312

Then say,

As Radius S C	90°00	= 10.00000
Is to the Sine S F of the Angle S C F = A C Q, which suppose	30°00	= 9.698970
So is the Length of the Arch B =	5°,302	= 0.724312
To that of the Arch Q N = S F =	2°,651	= 0 423282

56. Therefore A Q + Q N = 30° + 2°,651 = 32° 39′ 3″. Thus from the eccentric Anomaly A C Q we gain the mean Anomaly A Q + Q N = A N, which is proportional to the Time; and the Reverse of this, *viz.* from the mean Anomaly A N given, to find the eccentric Anomaly A C Q, is to be done by the Method of *Infinite Series*, as follows. Let the Arch N Q = y, the Sine of the Arch A N be = e, the Cô-sine = f, and the Eccentricity S C = g. The Sine of the Arch A Q is equal to the Sine of the Arch A N − N Q, equal to the Sine of the Arch A N − y, which Sine is thus expressed by a Conv. Series, $e - \frac{fy}{1} - \frac{ey^2}{1.2} + \frac{fy^3}{1.2.3} - \frac{ey^4}{1.2.3.4}$, &c. as Dr. *Keill* has shewn in his *Trigonometry*.

57. Call

57. Call that Series s, then Radius (1) : Sine of A Q (s) :: SC (g) : SF = (y) N Q; therefore $y = gs = ge - \frac{gfy}{1} - \frac{gey^2}{1.2} + \frac{gfy^3}{1.2.3} + \frac{gey^4}{1.2.3.4}$, &c. Consequently we have $ge = y + \frac{gfy}{1} + \frac{gey^2}{1.2} - \frac{gfy^3}{1.2.3} - \frac{gey^4}{1.2.3.4}$, &c. Let $ge = z$, $\overline{1+fg} = a$, $\frac{ge}{2} = b$, $\frac{gf}{1.2.3} = c$, $\frac{ge}{1.2.3.4} = d$, and the Equation will become $z = ay + by^2 - cy^3 - dy^4$, &c. which reverted gives $y = \frac{1}{a}z - \frac{b}{a^3}z^2 + \frac{2b^2 + ac}{a^5}z^3 - \frac{5abc - 5b^3 + a^2d}{a^7}z^4$, &c. Or, by substituting the Values of b and d, $y = \frac{1}{a}z - \frac{1}{2a^3}z^3 + \frac{c}{a^4}z^3 - \frac{5c}{2a^6}z^5$, &c.

58. But if the Arch A N be greater than 90 Degrees, and less than 270, then $ge = z = y - \frac{gfy}{1} + \frac{gey^2}{2} + \frac{gfy^3}{6} - \frac{gey^4}{24}$, &c. And then $a = 1 - fg$, and $y = \frac{z}{a} - \frac{z^3}{2a^3} + \frac{cz^3}{a^4}$, &c. This Series expresses the Arch Q N in Parts, whereof the Radius contains 100,000; but to have it in Degrees and Parts of a Degree, say, As Radius (1) is to this Series (s), so is the Radial Arch 57°,29578 (R) to Q V = y in Degrees; that is, $y = sR = \frac{R}{a}z - \frac{R}{2a^3}z^3 + \frac{Rc}{a^4}z^3$, &c.

59. Now the very first Term of this Series $\frac{R}{a}z$ is sufficient to determine the Anomaly of the Eccentricity in almost all the Planets nearly enough; for in the Earth's Orbit, where C Q : C S :: 1 : 0,01691, the Error is only a 10,000 Part of a Degree. For Example, Let the Arch A Q = 30°;

 Then

$$\text{Then}\begin{cases}\text{The Log. of the Eccentricity CS} = g = .8.228144\\ \text{The Log. of the Sine of AN} = e = 30^\circ = 9.698970\\ \text{The Log. of Radial Arch R} = 57^\circ,295 = 1.758122\end{cases}$$

The Sum is the Log. $g\,e \times R$, or R z $= 9.685236$
Subduct the Log. of $a = 1 + fg$ $= 0.006314$

There remains the Log. of $\frac{Rz}{a} = y = 0,4774 = 9.678922$

But 0,4774 Parts of a Degree are equal to $28'\ 38''$; therefore $AN - NQ = 30^\circ - 28'\ 38'' = 29^\circ\ 31'\ 22''$ $=$ AQ or Angle ACQ, the eccentric Anomaly. In the Triangle QCS, having two Sides QC and CS, and the included Angle given, we find the Angle $CSQ = 29^\circ\ 3'\ 7''$.

60. Now making $2\,CS = SH$ Radius, we have $QH : PH\ (:: CE\ (= AC) : CD)$ :: Tangent of ASQ : Tangent of $ASP = 29^\circ\ 2'\ 54''$, the equated or coequated Anomaly required. And that this is sufficiently near the Truth, let us see the Value of the second Term of the Series, *viz.* $\frac{Rz^3}{2a^3}$.

Thus, the Logarithm of $\frac{z}{a}$	$= .7.920800$
Multiply by	2
The Product is the Logarithm of $\frac{z^2}{a^2}$	$= .5.841600$
Add the Logarithm of $\frac{Rz}{a}$	$= 9.678922$
The Sum is the Logarithm of $\frac{Rz^3}{a}$	$= .5.520522$
Subduct the Logarithm of 2	$= 0.301030$
The Logarithm of the second Term $\frac{Rz^3}{2a^3}$	$.5.219492$

To which Logarithm answers the Number 0,000016, or the $\frac{1}{100000}$ Part of a Degree; too small to be regarded.

expreſs'd by the outmoſt Circle of the Scheme. (CXLI.)

THE

garded. And in the Orbit of *Mars* and *Mercury* the two firſt Terms $\frac{Rz}{a} - \frac{Rz^3}{2a^3}$ will determine the Value of y to more than any neceſſary Degree of Exactneſs.

(CXLI.) 1. The ORRERY (though a modern Name) has ſomewhat of Obſcurity in reſpect of its Origin, or Etymology; ſome Perſons deriving it from a *Greek* Word which imports to *ſee* or *view*, becauſe in it the Motions of the heavenly Bodies are all repreſented to the View, or made evident by Inſpection: But others ſay that Sir *Richard Steele* firſt gave this Name to an Inſtrument of this Sort which was made by Mr. *Rowley* for the late Earl of *Orrery*, and ſhew'd only the Movement of one or two of the Heavenly Bodies. From hence many People have imagined that this Machine owed its Invention to that Noble Lord.

2. But the Invention of ſuch Machines as we now call ORRERIES, and PLANETARIUMS, is of a much earlier Date. The firſt we have any mention of is that of *Archimedes*, generally called *Archimedes*'s SPHERE; though it was more than what we now-a-days call a SPHERE, which is an Inſtrument conſiſting only of large and ſmall Circles artfully put together. But this famous Machine of *Archimedes* was of a more complex Nature, and conſiſted of a Sphere, not of Circles, but of an hollow globular Surface of Glaſs, within which was a Piece of Mechaniſm to exhibit the Motions of the Moon, the Sun, and the Five Planets. This *Cicero* aſſerts in his *Tuſculan Queſtions*.

3. But the moſt copious and accurate Deſcription of this Sphere is that of *Claudian*, in *Latin Verſe*. Thus the Poet ſings:

Jupiter in parvo cum cerneret æthera vitreo,
Riſit, & ad Superos talia dicta dedit.
Huccine mortalis progreſſa potentia curæ?
Jam meus in fragili luditur orbe labor.

 Jura

The principal Use of the Orrery is to render the Theory of the *Earth* and the *Moon*

Jura poli, rerumque fidem, legesque Deorum,
Ecce Syracusius transtulit arte senex.
Inclusus variis famulatur spiritus astris,
Et vivum certis motibus urget opus.
Percurrit proprium mentitus Signifer annum,
Et simulata novo Cynthia mense redit.
Jamque suum volvens audax industria mundum
Gaudet, & humanâ sidera mente regit.
Quid falso insontem tonitru Salmonea miror?
Æmula Naturæ parva reperta manus.

4. This Machine appears from hence to have been sufficiently grand and universal, as comprehending all the Heavenly Bodies, and exhibiting all their proper Motions; which is all that can be said of our common modern Orreries. 'Tis true, this Orrery of *Archimedes* was contrived to represent the *Ptolemaic* System; but the Mechanism and Nature of the Instrument is the same, whether the System of *Ptolomy* or *Copernicus*, or any other be represented by it.

5. The next Orrery we have any mention of is that of *Posidonius* the *Stoic*, in *Cicero*'s Time, 80 Years before our Saviour's Birth: Concerning which the Orator, in his Book *De Nat. Deorum*, has the following Passage.—*Quid si in Scythiam, aut in Britanniam, Sphæram aliquis tulerit hanc, quam nuper familiaris noster effecit Posidonius, cujus singulæ conversiones idem efficiunt in Sole, & in Luna, & in quinque Stellis errantibus, quod efficitur in Cælo singulis diebus & noctibus; quis in illa barbarie dubitet, quin ea Sphæra sit perfecta Ratione?* That is, "If any Man should carry this Sphere of *Posidonius* "into *Scythia* or *Britain*, in every Revolution of which "the Motions of the Sun, Moon, and Five Planets "were the same as in the Heavens each Day and Night, "who in those barbarous Countries could doubt of its "being finished (not to say actuated) by perfect Rea-"son?" What can be a more genuine Account of a compleat Orrery than this? And, by the way, what would *Cicero* say, were he now to rise from the Grave, and

Moon easy and intelligible; and to evidence to our Senses how all those Appearances happen,

and see his *Barbarous Britain* abounding in Orreries of various Kinds and Sizes?

6. From this Time we hear no more of Orreries and Spheres, till about 510 Years after *Christ*, when the famous *Severinus Boethius*, the *Christian* (though *Roman)* Philosopher, is said to have contrived one which *Theodoric* King of the *Goths* wrote to him about, and desired it for his Brother-in-law *Gundibald* King of *Burgundy*; in which Letter he calls it *Machinam Mundo gravidam*,—*Cœlum gestabile*,—*Rerum Compendium*; that is, *a Machine pregnant with the Universe*,—*a portable Heaven*,—*a Compendium of all Things*. What more can be said of our Orreries?

7. After this succeeded a long Interval of Barbarism and Ignorance, which so deluged the Literary World, that we find no Instances of Mechanism of any Note till the Sixteenth Century, when the Sciences began again to revive, and the Mechanical Arts to flourish. Accordingly we meet with many Pieces of curious Workmanship about this Time; and in the Astronomical Way particularly is the stately Clock in his Majesty's Palace at *Hampton-Court*, made in *Henry* the Eighth's Time, *A. D.* 1540, by one *N. O.* This shews not only the Hour of the Day, but the Motion of the Sun and the Moon through all the Signs of the Zodiac, with other Matters depending thereon; and is therefore to be esteemed a Piece of Orrery-Work.

8. Such another is mentioned by *Heylin* at the Cathedral of *Lunden* in *Denmark*; but the most considerable at this Time is that Piece of Clock-work in the Cathedral of *Strasburg* in *Alsace*; in which, besides the Clock-Part, is the Celestial Globe or Sphere, with the Motions of the Sun, Moon, Planets, and Fix'd Stars, *&c.* It was finished in the Year 1574, and is much superior to that pompous Clock at *Lyons*, which also contains an Orrery-Part.

9. About the Beginning of the Seventeenth Century this Sort of Mechanism began to be greatly in vogue,

 and

happen, which depend on the *annual* or *diurnal Rotation* of the *Earth*, and the *monthly Revolutions*

and Spheres and Orreries were now no uncommon Things; though Orreries bore an excessive Price till very lately. The first large one made in *London* by Mr. *Rowley* was purchased by King *George* I. at the Price of 1000 Guineas; nor has any of that large Sort, which contains all the Movements of Primaries and Secondaries, been sold for less than 300 *l.* at any Time since.

10. There have been various Forms invented for this noble Instrument, two of which have principally obtain'd, *viz.* the *Hemispherical Orrery*, and the *Whole Sphere*; though the Orrery at first was made without any Sphere, and with only the Sun and the Earth and Moon revolving about it; but this was too imperfect a State, and they soon began to invest it, some with a *Half-Sphere*, some with a *Whole* or *Compleat Sphere*; for otherwise it could not be an adequate Representation of the Solar System.

11. The Hemispherical Orrery has been made in greater Numbers than any other, on account of their being made much cheaper and easier than those in a Sphere of the same Size; there being a vast Difference between *placing an Hemisphere on the Box of an Orrery*, and *disposing an Orrery in a large moveable Sphere*. But then the Idea given us by the former is very unnatural and imperfect; and 'tis surprising to think they should have had such a Run as they had, Mr. *Wright* having made between forty and fifty of that Sort since the Death of Mr. *Rowley* his Master. And though I incline to think few more of that Form will be made, yet as they have had so great a Name, I have thought proper to give the Reader a View of one in a Print.

12. This ill-judged and erroneous Form of an Orrery had this Effect with those who knew the Nature of such Machines very well, that some applied themselves to construct Orreries in a Compleat Sphere, others invented such Instruments as served to exhibit the Motions of the

Revolutions of the Moon: As, the Variety of Seasons, the Vicissitudes and various Lengths

the Heavenly Bodies separately, which they accordingly call'd PLANETARIUMS, LUNARIUMS, &c. and others declared against all Orreries in general, as giving false Ideas of the System of the World, especially as the Magnitudes and Distances of the Heavenly Bodies could not be represented by them in their proper Proportions.

13. But they must be supposed to reason very weakly, who object an inconsiderable Deficiency in any Instrument, against its most important Uses. No one ever decried an Air-Pump, because an absolute *Vacuum* was impossible by it; or the Use of a Telescope, because we cannot see the Inhabitants of the Planets. And on the other hand, to represent the Solar System by Parts, or in a piece-meal Manner, is little less than mangling one of the most noble and uniform Parts of Natural Philosophy. The *Planetarian Scheme* therefore of Dr. *Desaguliers* soon became extinct.

14. An Orrery then, adapted to an Armillary Sphere, is the only Machine that can exhibit a just Idea of the true System of the World, with the diurnal and annual Motions of the Heavenly Bodies; and none but an Orrery of this Kind can do that: Yea furthermore, such an Orrery is capable of exhibiting the *third* Motion of the Earth, by Means of the Armillary Sphere; I mean that Motion of the Earth by which the Poles of the World revolve about the Poles of the Ecliptic, and occasions what is commonly called the *Precession of the Equinoxes*, or more properly the *Retrogression of the Earth's Nodes*.

15. That the Reader may have a proper Idea of this *third* Motion of the Earth and its Axis, I shall explain it as follows. Let D C H be a Part of the Earth's Orbit, C its Centre, E C the Axis of the Ecliptic, E its Pole, C P the Axis of the Earth, P its Pole, through the Points E and P draw the great Circle E P A, meeting the Ecliptic A L in A; the Arch P A measures the Inclination

Plate LXIV. Fig 1.

Lengths of Days and Nights, the Manner of Solar and Lunar Eclipses, the various Phases of the Moon, &c.

IN

Inclination of the Axis of the Earth to the Plane of the Ecliptic, *viz.* the Angle P C H, which is found by Observation to be about 66° 30′, and therefore its Complemental Arc E P or the Angle P C E = 23° 30′.

16. Through the Pole P from the Point E describe a lesser Circle P F G, which will be parallel to the Ecliptic; then if the Axis of the Earth be directed at any particular time to P, it is found by Observations of many Years, that it will not constantly be directed to the Point P in the Heavens, but will in 72 Years time be directed to some other Point Q, so that the Arch P Q = 1 Degree; and therefore in the Space of 360 × 72 = 25920 Years, the Point P or Pole of the World will describe the Circle P F G about the Pole of the Ecliptic E, which Revolution is call'd the *Great Year*.

17. The Cause of this *Conical Motion* of the Earth's Axis was unknown to all the Astronomers and Philosophers before Sir *Isaac Newton*'s Time, none of them being able to guess from whence it could proceed: But this divine Geometer soon investigated the Cause thereof, and demonstrated it to result from the Laws of Motion and Gravity, that is, from the *Spheroidical Figure of the Earth*; for were the Earth a perfect Globe, its Axis would always remain Parallel to itself, and have no such Motion. See the *Principia*.

18. From this Motion of the Earth's Axis follow several remarkable *Phænomena*; as *First*, a *constant Change of the Pole-Star*; for 'tis evident, if any Star should chance to coincide with the Pole P at any time, it will after 72 Years be left at the Distance Q P, or 1 Degree Westward, and the Star at Q becomes then the North Pole-Star.

19. *Secondly*, The present Polar Star will in Time be on the South Part of our Meridian; that is, the Star, which suppose at present at P, will after 12960 Years be at G, which being 47 Degrees (in the Arch of a great

In my *Orrery*, which is of a peculiar and most elegant Structure, the *Earth* in its

great Circle) distant from P, will be on the South Part of the Meridian of *London*, which suppose on the Earth's Surface at *h*. For if T R be the Equator, then the Latitude of *London* T $h = 51^{\circ}\ 30'$, and its Complement $hp = 38^{\circ}\ 30'$; therefore $gp - hp = 47^{\circ} - 38^{\circ} 30' = 8^{\circ} 30' = g\,h$, the Distance of the present North Star towards the South at that time.

20. *Thirdly*, The Circle E P A passing through both the Pole of the Ecliptic and Equator, will be the *Solstitial Colure*, and A the *Solstitial Point*, when the Axis of the Earth points to P; but after 72 Years, when it points to Q, then the great Circle E Q B will be the *Solstitial Colure*, and B the *Solstice*, for the same Reason. And hence also the *Equinoctial Points* (which are always 90 Degrees distant from the *Solstices*) must move in the same Time through the same Arch, the same Way, *viz.* Westward.

21. *Fourthly*, Hence 'tis evident, all the Points of the Ecliptic do move backwards, or Westwards, through one Degree every 72 Years; which Motion is said to be *in Antecedentia*, and is contrary to the Order of the Signs: As the other Motion, by which the Planets are carried round the Sun, is said to be *in Consequentia*, or according to the Order of the Signs, *viz.* from *Aries* ♈ to *Taurus* ♉, *Gemini* ♊, *&c.* And this retrograde Motion of the Equinoctial Points is called the *Recession of the Equinoxes*.

22. *Fifthly*, This Recession of the Equinoctial Points, and indeed of the whole Ecliptic, is the Cause of the slow apparent Motion of the Fixed Stars forwards; for since the several Circles of Longitude by which they are referr'd to the Ecliptic are continually shifting backwards, the Stars, which are immoveable, must with respect to those Circles have their Distance, that is, their Longitude, constantly increasing from the first Point of *Aries*. Thus all the Constellations do continually change their Places at the Rate aforesaid: The bright

its annual Motion passes round by a Circle, on which is engraved a *Calendar*, and the *Ecliptic*; and the Plate which carries the Earth about has an Index on the opposite Part from the Earth, to shew the apparent Place of the Sun in the *Ecliptic*, for every Day of the Year; and one Turn of the Winch carries the Earth once round its Axis, and the said Index over the Space of one Day in the Calendar: So that by this means the true Place of the Earth, and the apparent Place of the Sun, also the Place and Phases of the Moon, may be readily shewn for any Day required.

THE *Orrery-Part*, containing the *Wheel-Work*, is placed within a large and most beautiful ARMILLARY SPHERE, which turns about upon its Axis, with a fairly-engraved and silver'd Horizon, which is also moveable every Way upon a most elegant Brass Supporter, with four Legs richly wrought; at the Bottom of which is a noble large silver'd Plate, with a Box and NEEDLE, and

bright Star of *Aries*, for Instance, which in *Hipparchus's* Time was near the Vernal Equinox, is now removed near a whole Sign of 30° Eastward, and is in the Beginning of *Taurus* ♉, and *Taurus* is got into *Gemini* ♊; and thus all the Constellations of the Zodiac have changed their Places, and possess different Signs from what they formerly did.

and COMPASS, with the Names of all the *Points* finely engraven in Words at Length. The Circles of the Sphere are as follow.

THE EQUINOCTIAL, which divides the Sphere into two Parts, *viz.* the *Northern* and the *Southern Hemiſphere*; and is ſo call'd, becauſe when the Sun comes to paſs over it, (as it does twice every Year) the *Days and Nights are then equal.* This Circle is divided into 360 Degrees, call'd the *Right Aſcenſion* of the Sun or Stars.

THE ECLIPTIC is that great Circle which repreſents the apparent annual Path of the Sun through the Heavens. It is divided into 12 equal Parts call'd *Signs*, conſiſting of 30 Degrees each; whoſe *Names* and *Characters* are as follow: 1. *Aries*, the Ram, ♈; 2. *Taurus*, the Bull, ♉; 3. *Gemini*, the Twins, ♊; 4. *Cancer*, the Crab, ♋; 5. *Leo*, the Lion, ♌; 6. *Virgo*, the Virgin, ♍; 7. *Libra*, the Scales, ♎; 8. *Scorpio*, the Scorpion, ♏; 9. *Sagittarius*, the Bowman, ♐; 10. *Capricorn*, the horned Goat, ♑; 11. *Aquarius*, the Waterer, ♒; 12. *Piſces*, the Fiſhes, ♓. The *Ecliptic* interſects the *Equinoctial* in the Beginning of *Aries* and *Libra*, in an Angle of 23 Degrees 29 Minutes. In this Circle the Longitude of the

the heavenly Bodies is reckoned. The *Ecliptic* is the Middle of

THE ZODIAC, which is a broad ſilver'd *Zone*, encompaſſing the Sphere to five Degrees on each Side the *Ecliptic*; ſo called from the Figures of the ſeveral *Animals*, or *Conſtellations of the Signs*, with which it is adorned and embelliſhed. This Zone comprehends within it the *Paths* or *Orbits* of all the *Planets*.

THE MERIDIAN is a great Circle paſſing through the *Poles*, and cutting the *Equinoctial* at Right Angles; ſo call'd, becauſe when the Sun is upon any Meridian, it makes the *Meridies*, Mid-Day, or Noon, to all Places under it. Of theſe there is one called

THE GENERAL MERIDIAN, within which the whole Sphere turns, and upon which are engraven the *Degrees of Latitude*, beginning and proceeding each Way from the Equinoctial to the Poles. To this Circle the Sphere is ſuſpended; and being moveable within the Horizon, the Sphere may be *elevated or rectified for the Latitude of any Place*.

THE HORIZON is that broad ſilver'd Frame, or Circle, which contains the whole Machine, moveable every Way within it. It

It is ſo called becauſe it bounds our Sight in the Heavens, and divides the Sphere into *the upper and lower Hemiſphere.* Upon this Circle are curiouſly engraven the *Ecliptic Signs* and the *Calendar*, for readily finding the Sun's Place for any given Day or Time. On this Circle is alſo reckon'd the *Amplitude of the Sun*, &c.

THE Points where the Ecliptic interſects the Equinoctial are call'd the *Equinoctial Points*, or EQUINOXES, becauſe when the Sun is in them, *the Days and Nights are equal.* As the Sun is in one of them in the *Spring*, it is called the *Vernal Equinox*; and in the other at *Autumn*, it is call'd the *Autumnal Equinox.*

THE Beginning of *Cancer* and *Capricorn* are call'd the *Solſtitial Points*, or the SOLSTICES; which is as much as to ſay, *the Stations of the Sun*, becauſe when the Sun is in thoſe Points, he ſeems *ſtationary*, or *not* to *move* for ſome Days: The firſt is the *Summer*, the other the *Winter Solſtice.*

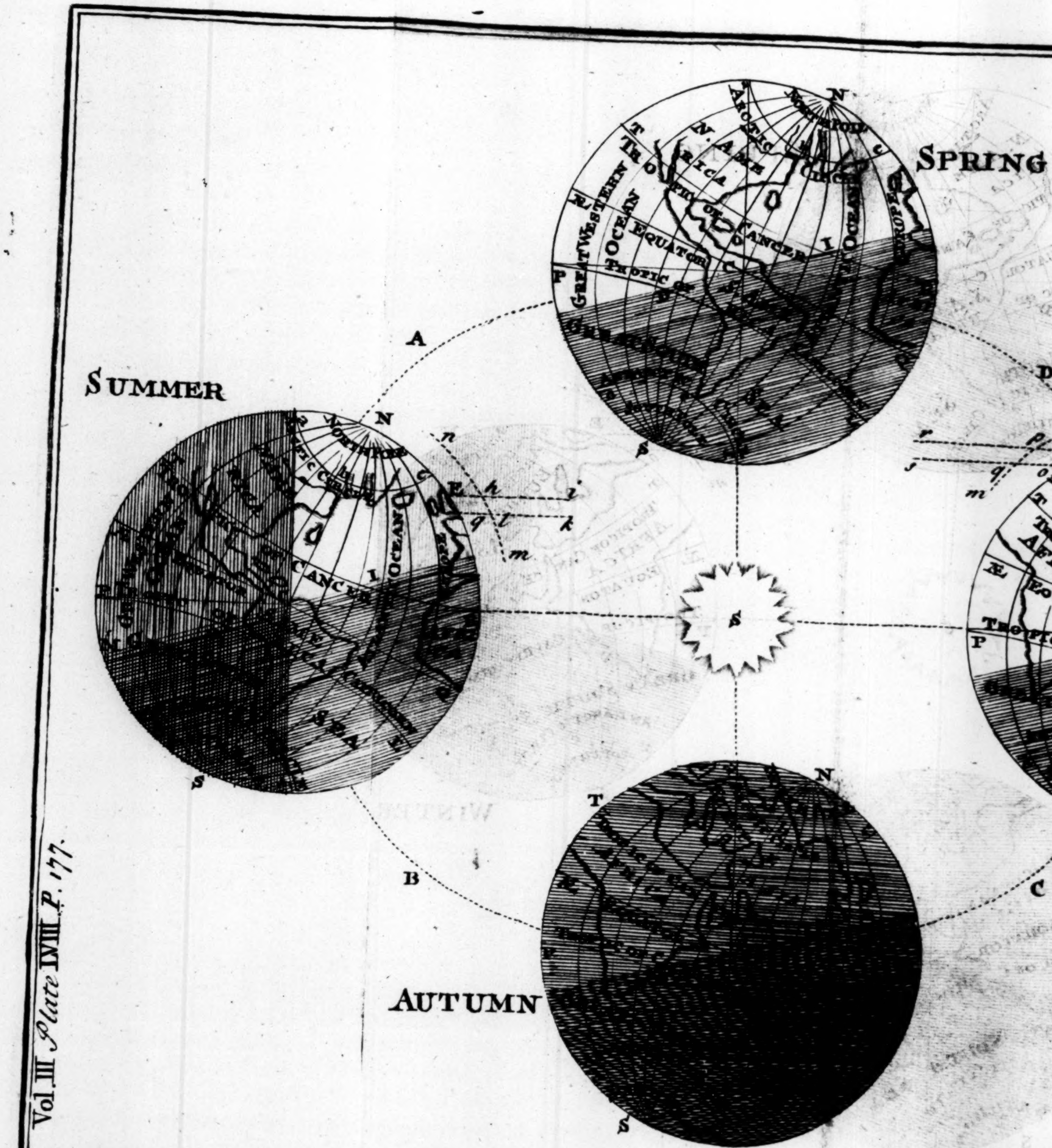
THE Meridians which paſs through the *Points* above-mention'd are call'd the *Equinoctial* and *Solſtitial* COLURES reſpectively. They divide the Sphere into *four Quarters*, in the Middle of the *four Seaſons of the Year.*

THE

THE Leſſer Circles of the Sphere are the TROPICS and POLAR CIRCLES; which are all parallel to the Equinoctial, and are two on either Side. The *Northern Tropic* is that of *Cancer*; the *Southern* that of *Capricorn*; as paſſing through the Beginning of thoſe Signs. They are diſtant from the Equinoctial 23 Degrees 29 Minutes, and include that Space or Part of the Sphere which is call'd the *Torrid Zone* on the Terreſtrial Globe, becauſe the Sun is at one Time or other perpendicular over every Part, and extremely torrifies or heats it.

WITHIN 23 Deg. 29 Min. of each Pole lie the *Polar Circles*; of which that about the North Pole is call'd the *Arctic Circle*, becauſe of the Conſtellation of the *Bear* in that Part; and the other about the South Pole, the *Antarctic Circle*. They include thoſe Spaces which are call'd the *Frigid Zones*, by reaſon of the intenſe Cold which reigns in thoſe Regions the greateſt Part of the Year. Thoſe Spaces which lie between the Tropics and Polar Circles, on either Side, are call'd the *Temperate Zones*, as enjoying a mean or moderate Degree of Heat and Cold.

THE Circles above are eſſential to the Sphere; beſides which there is the *Quadrant*


SPRING
SUMMER
WINTER
AUTUMN
A
B
C
D
N
S
Vol. III Plate LVIII P. 177.

drant of Altitude, for ſhewing the Height of any Luminary above the Horizon; and a large and moſt beautiful *Horary Circle and Index*, ſhewing the Time correſponding to the Motion of the Sphere: Alſo the *Solar Label*, for fixing the Sun to its proper Place in the *Ecliptic*.

It is eaſy to conceive, that the Sun will always enlighten *One-half of the Earth*; and that when the Sun is in the Equinoctial, the Circle which terminates *the enlightened and darkened Hemiſpheres* (which is called *the Circle of Illumination)* will paſs through the *Poles of the Earth*, and alſo divide all the *Parallels of Latitude* into two equal Parts. But ſince the Earth moves not in the Plane of the *Equinoctial*, but that of the *Ecliptic*, the Axis of the Earth will be inclined to that of the *Ecliptic* in an Angle of 23 Degrees 29 Minutes; and therefore the *Circle of Illumination* will, at all other Times, divide the *Parallels of Latitude* into two unequal Parts.

Now ſince any Parallel is the Path or Tract which any Place therein deſcribes in one Revolution of the Earth, or 24 Hours; therefore that Part of the Parallel which lies in *the enlightened Hemiſphere* will repreſent *the Diurnal Arch*, or *Length of the*

Day; and that Part in *the dark Hemisphere* will be the *Nocturnal Arch*, or *Length of the Night*, in that Parallel of Latitude.

HENCE, when the Orrery is put into Motion, the Earth moving with its Axis always *parallel to itself*, yet always *inclined to the Plane of the Ecliptic*, will sometimes have the *Northern* Parts turned more directly to the Sun, and most enlightened; and at other Times the *Southern* Parts will be so. Hence various Alterations of *Heat and Cold*, and *Length of Days and Nights*, will ensue in the Course of the Revolution of the Earth about the Sun, which will constitute all *the Variety of Seasons*, as will most naturally and evidently be shewn in the Orrery, as follows (CXLII.)

WE

(CXLII.) 1. Though these Things are plain to a Person who has his Eye on an Orrery, while he hears or reads this Account of the Nature and Manner of the Seasons, and the Variety of Day and Night, yet Ideas of this Sort are not so easy to be obtained by mere Reading and Cogitation only, unless assisted by a proper Diagram or Representation; which therefore I shall here subjoin and explain.

Plate LVIII. 2. Let S be the Sun, A B C D the *Orbis Magnus*, or annual Path of the Earth about the Sun. In this Orbit the Earth is represented in *four* several Positions, in the midst of the four Seasons respectively. On the Earth are drawn the several Circles and Lines as follow.

Æ C Q The Equator.
T O R The Tropic of *Cancer*.
P M L The Tropic of *Capricorn*.

a b c

Vol. III. Plate LIX. P. 179.

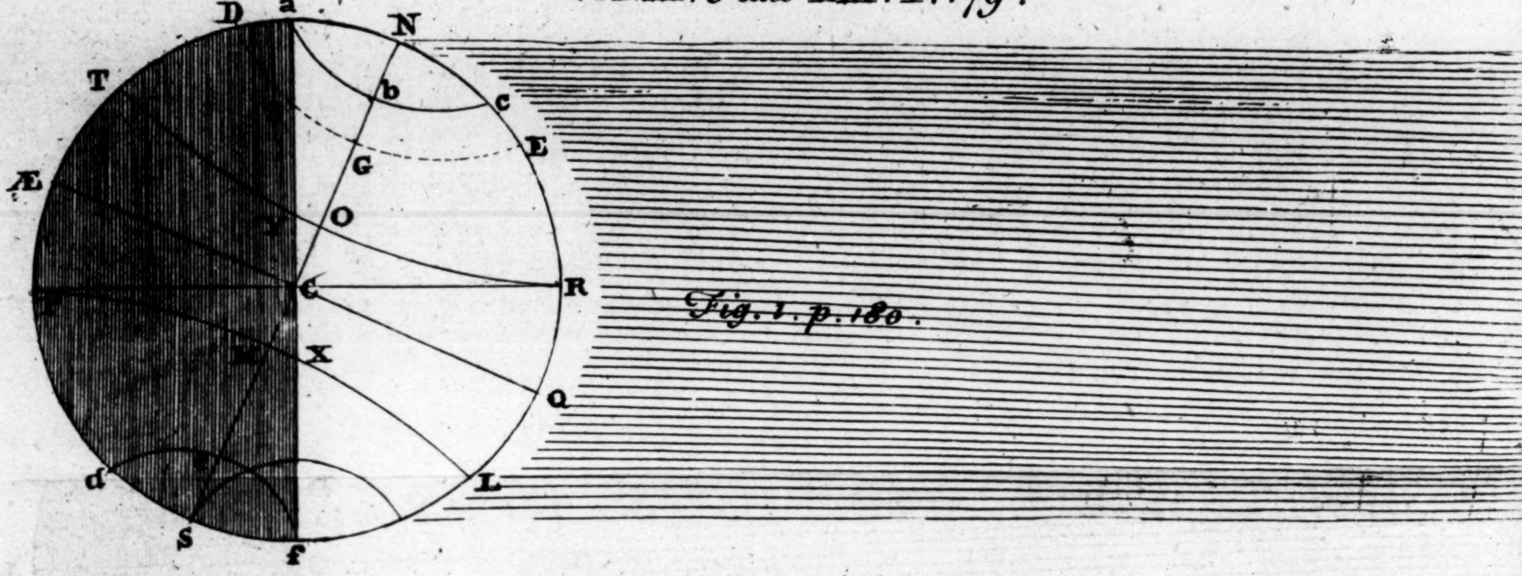


Fig. 1. p. 180.

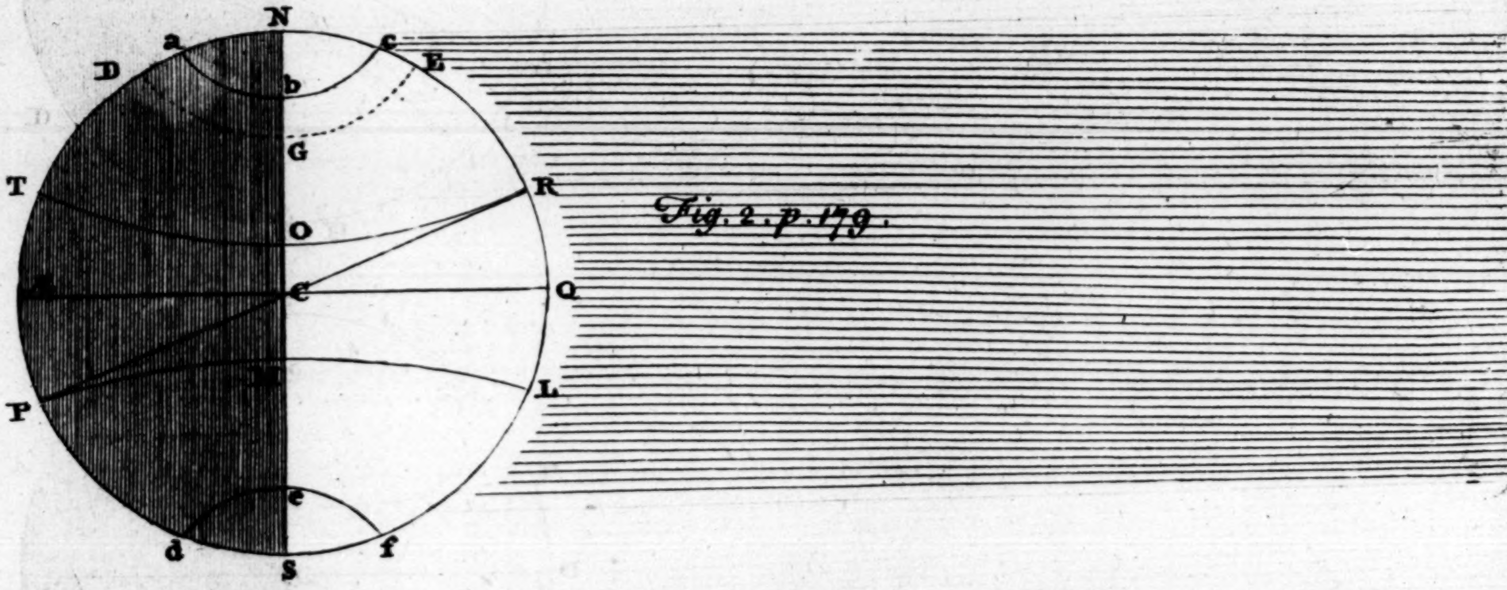


Fig. 2. p. 179.

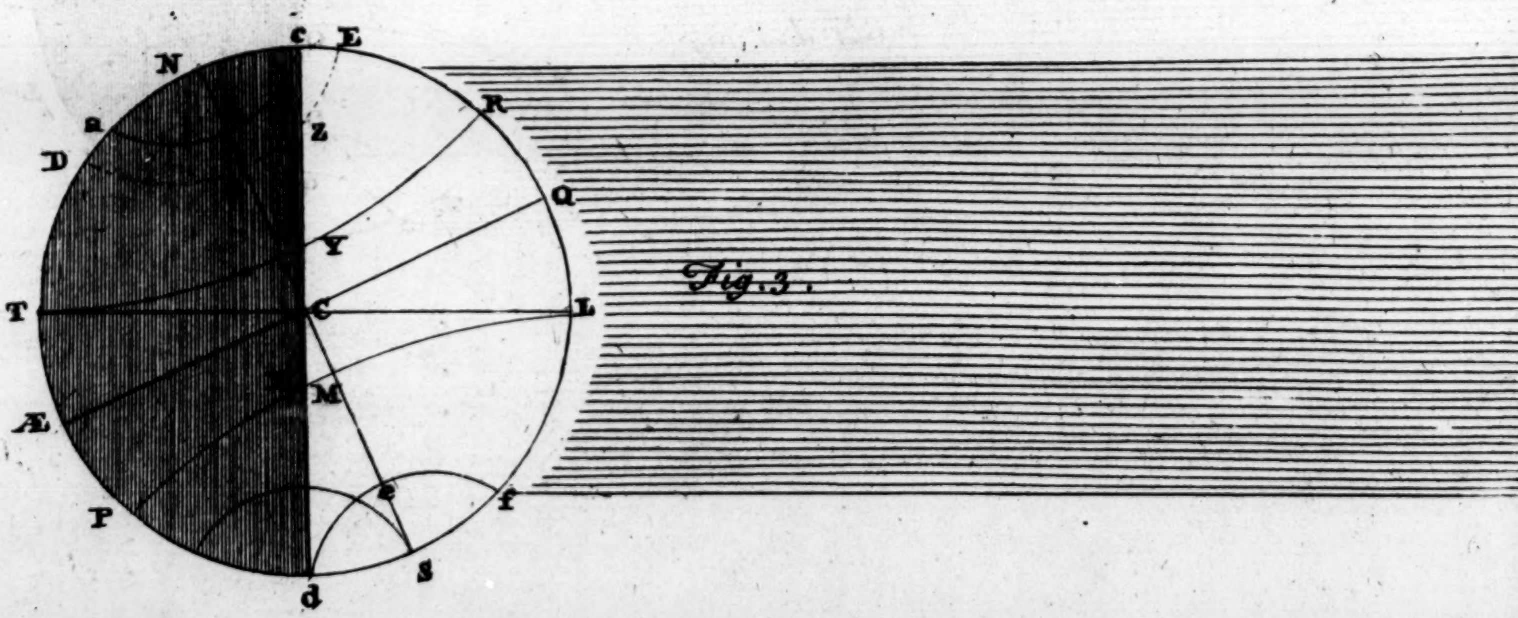


Fig. 3.

We will first give the Earth Motion in the first Point of *Libra*; the Sun will then appear

a b c	The North Polar or *Arctic* Circle.
d e f	The South Polar or *Antarctic* Circle.
E G D	The Parallel of *London*.
N C S	The Earth's Axis.
a C f	The Axis of the Ecliptic Plane.

3. As the Sun is supposed to be at so great a Distance, that the Rays coming from it do arrive at the Earth nearly *parallel*, they will therefore illuminate very nearly One-half of the Globe of the Earth, abstracting from the Refraction of the Air. And if we are supposed to view the Earth circulating about the Sun at a very great Distance in the Positions represented in the Scheme, we shall have all the enlightened Part turn'd to the Eye on the Equinoctial Day in the Spring, but on that in the Autumn we see only the dark Part; as on the Summer and Winter Solstices we see only Half the light and dark Hemispheres respectively: And accordingly the Earth is thus represented in the Figure.

4. But (as I find by Experience) the best Way to convey an Idea of the Seasons, and Day and Night, is to represent the Earth also in Positions exhibiting the visible Hemisphere equally divided into the light and dark Parts, or semicircular Areas, as in the next Plate; and to compare these both together in the Description. Plate To begin therefore with the Situation of the Earth in LIX. the Spring and Autumn.

5. In either of these Cases, 'tis evident the Sun is in the Plane of the Equator Æ Q, and therefore equally distant from each Pole of the World; consequently the *Circle of Illumination* will pass through both the Poles, N, S; and therefore every Place at an equal Distance Fig. 2. on either Side will have an equal Degree of the Sun's Light and Heat. And as the Earth revolves upon its Axis, every Place must describe a Circle parallel to the Equator, One-half of which will be in the *light*, the other Half in the *dark* Hemisphere; and as Parts of the Circle measure the Day and Night, it is plain they must

 then

appear to enter *Aries*, and this will be the *Vernal Equinox*; for now, *the Sun being in*

then be equal. Thus in the Equator, the Diurnal Arch QC is equal to the Nocturnal Arch C Æ; in the Tropics R O and L M are equal to O T and M P; in the Latitude of *England* the Day E G is equal to the Night G D; and so in all other Parts.

6. Hence, by the way, we may observe, that had the Sun always moved in the Equator, there could have been no Diversity of Day and Night, and but *one Season* of the Year for ever to all the Inhabitants of the Earth, No Alteration of Heat or Cold, so agreeable now both to the Torrid and the Frozen Zones; but the same uniform eternal Round of unvariable Suns had been our uncomfortable Lot, every way contrary to that Disposition we find all Mankind formed with, of being delighted and charmed with Variety to an extreme Degree. The Obliquity of the Ecliptic is therefore not to be looked upon as a Matter of Chance or Indifferency, but an Instance of Wisdom and Design in the adorable Author of Nature, who does Nothing in vain.

7. If we consider the Earth moving on its Orbit, with its Axis N S always parallel to itself, till it comes into the Summer Situation, we shall there see, that by this Parallelism of the Axis all the Northern Parts of the Earth will be brought towards the Sun, which will in this Case be in the Plane of the Northern Tropic, and his Rays perpendicular upon it, as at R. The Circle of Illumination a C f will now be in such a Site, as to include the North Pole and all about it to the Distance N A = 23° 30′; and on the contrary to exclude the South Pole S, and Southern Regions to the same Distance S f. The Northern Climates must therefore now have their *Summer*, and the Southern Climates their *Winter*; as will appear more particularly if we consider,

Plate LVIII. Pl. LIX. Fig. 1.

8. *First*, The Sun-Beams fall more perpendicularly upon any Northern Parallel than upon the same Southern Parallel

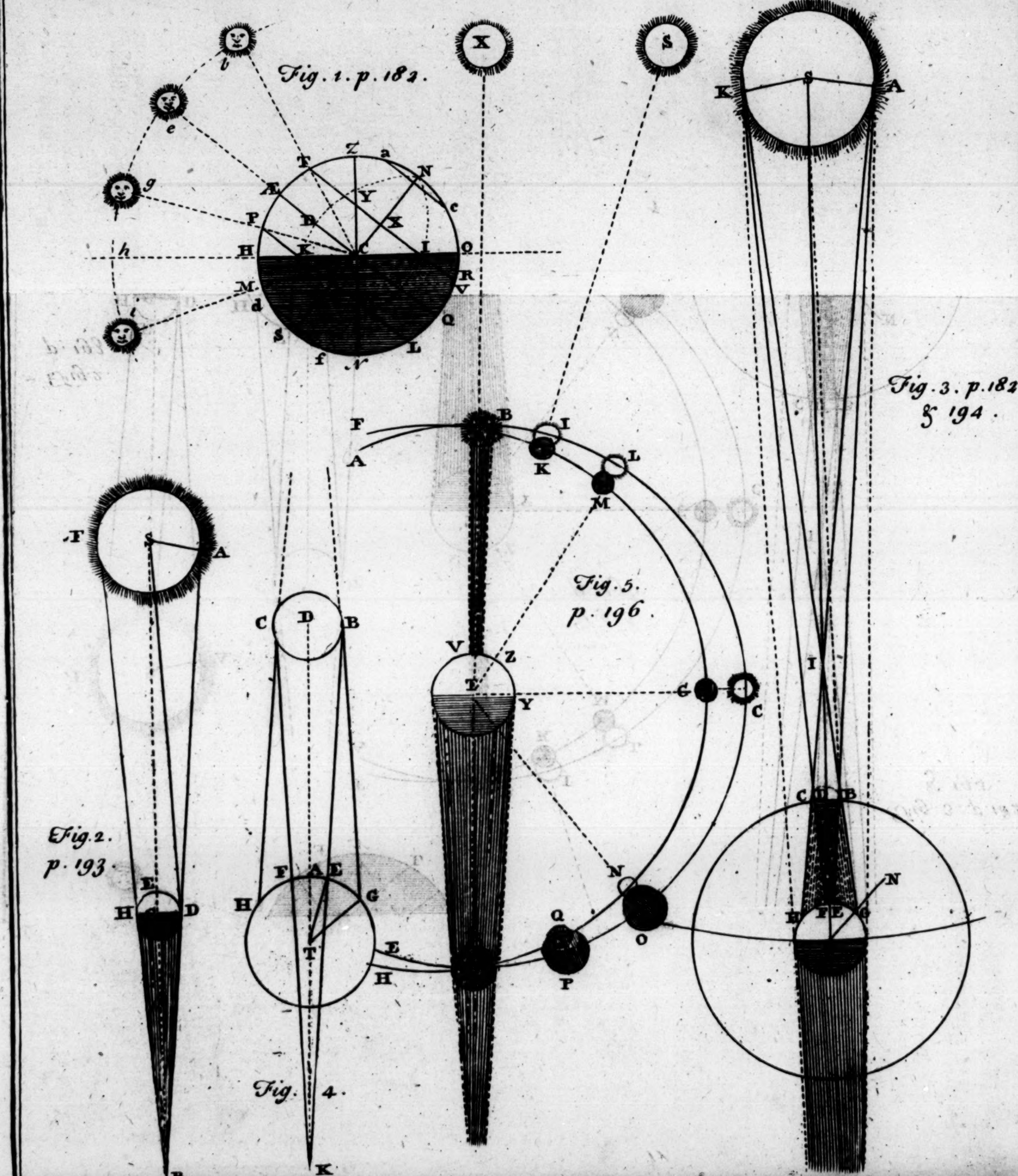


Fig. 1. p. 182.
Fig. 2. p. 193.
Fig. 3. p. 182. & 194.
Fig. 4.
Fig. 5. p. 196.

in the Equinoctial, all Parts of the Earth will be equally enlightened from Pole to Pole,

Parallel, and have therefore a ſhorter Paſſage through the Atmoſphere. Thus, for Inſtance, in the Parallel of *England* E, let the Rays *i* E, *k g*, be incident on the Atmoſphere *m n* in *h* and *l*; then will their Paſſage *h* E, *l g*, be ſhorter than it would be in the ſame Latitude Southwards, and therefore will not be ſo much refracted, blended, and abſorb'd; and conſequently their Effect will be more conſiderable and ſenſible. Again, as Rays are more perpendicular, they will ſtrike with a greater Force; alſo the more will fall on a given Space; on both which Accounts their Effect, in reſpect of Light and Heat, will be greater.

9. *Secondly*, As the Earth revolves about its Axis, every Place in North Latitude will deſcribe a greater Part of its Parallel in the enlighten'd than in the dark Hemiſphere; or, in other Words, the Day will be longer than the Night. Thus in the Northern Tropic the Diurnal Arch is R Y, the Nocturnal Y T, which is leſs than the other by the Difference Y O. Again, in the Parallel of *London* the Length of Day is ſhewn by the Arch E Z, of the Night by Z D, which is ſhorter than the Day by the Difference G Z. And laſtly, at the Solar Circle c b a it is all Day, no Part of that Parallel lying within the dark Hemiſphere a Æ f. On which Account it is evident the Light and Heat of the Sun is greater in any Place of North Latitude now than at any other Time of the Year. It is therefore now the Midſt of the Summer Seaſon in all the Northern Climates.

10. In the Southern Part of the World it is *Winter*, for the ſame Reaſons reverſed, *viz.* becauſe the Sun's Rays fall more obliquely there; they therefore paſs through a greater Quantity of the Atmoſphere, on which Account they are more refracted, blunted, and ſtifled, and their Effect weaken'd. Alſo a leſs Quantity of the Solar Rays will fall on a given Space, and each Ray ſtrike with a leſs Force; and laſtly, the Duration of

Pole, and all the Parallels of Latitude divided into two equal Parts by the *Circle of Illumination.*

their Preſence will be ſhorter than that of their Abſence, or the Day will be ſhorter than the Night; as in the Southern Tropic the Day is L X, but the Night X P, longer by the Difference M X; which Difference is ſtill greater the farther you go, till you come to the *Antarctic* Circle d e f; where there is no Day at all, and all within to the South Pole S is involved in Night, of greater or leſs Duration.

11. For the ſame Reaſons, when the Earth arrives to the oppoſite Part of its Orbit, it will be SUMMER to all the Southern Climates, and WINTER in the Northern. It is evident this muſt neceſſarily happen by the Paralleliſm of the Earth's Axis, and the Change of her Place in the Orbit: By which Means the Sun now illumines that very Half of the Globe which in the other Poſition was dark; and whence it follows, that in all North Latitudes the Length of the Days *now* are equal to the Length of the Nights *then*, and *vice verſâ* in South Latitudes. Thus the Day (in the Parallel of *England*) E Z = D Z, the Night in the Summer Seaſon; and the Night now, *viz.* Z D = Z E, the Day at that Time. All which Things are too plain from the Schemes to want farther Explication.

Plate LVIII. Pl. LX. Fig. 3.

12. Thus the Viciſſitudes and Variety of the Seaſons, and of the Day and Night, appear in general; but to exhibit the ſame in an eſpecial Manner for any particular Place, as *London*, another Scheme is neceſſary, wherein the Sphere ſhall have the ſame Poſition with reſpect to that Place, as the Earth itſelf has. Thus let Æ N Q S be the Earth; Z will be the higheſt Point, or Place of *London*; H O the Horizon, and N the loweſt Point or *Antipodes*; and Æ Q the Equator, T R and P L the two Tropics, a c and d f the two Polar Circles, as before.

Pl. LX. Fig. 1.

13. Then when the Sun is in the Plane of the Equator at *e*, the *Semi-diurnal Arch*, or Half the Length of the Day, will be repreſented by Æ C; and that of the Night by C Q, which is equal to the former. In this Caſe

Illumination. Hence the Days and Nights will be equal, and the Sun's Heat is now at

Case the Angle *e* C *h*, which measures the Altitude of the Sun above the Horizon *h* O, is $38° 30' = h\,e$.

14. Again: When the Sun is in the Tropic T R, and consequently nearest to the Zenith of *London*, the Semi-diurnal Arch is then T I, which is longer than the former in the Proportion of the Right Angle Æ N C = 6 Hours to the obtuse Angle Æ N F = 8 Hours 16 Minutes; N E S being an Hour-Circle drawn through the Point I, and intersecting the Equator in E. The Semi-nocturnal Arch is I R, and equal in Time to the Angle E N Q = 3 Hours 44 Minutes, the Complement of the other to 12 Hours.

15. Lastly: When the Sun appears in the Southern Tropic at P, and most remote from the Zenith of *London*, the Semi-diurnal Arch is then P K, equal to the Angle Æ N D = 3 Hours 44 Minutes nearly, equal to the Night when the Sun was in the other Tropic; and the Semi-nocturnal Arch K L at this Time is evidently equal to the Semi-diurnal Arch T I at the opposite Time of the Year.

16. Whenever the Sun comes upon the Line N S, representing the Hour-Circle of Six, it is then *Six o'Clock*, as at X in the Summer Tropic, before Sun-set at I; and at B in the Winter Tropic, after Sun-set at K. Also when the Sun comes upon the Line Z C N (which represents the *Prime Vertical*, or *Azimuth* of *East and West*) it is then due *East* and *West*, which happens at Y in the Northern Tropic, after Six in the Morning, and before Six in the Afternoon, and *vice versâ* at W in the Southern Tropic.

17. It is found by Observation, that the Air is not absolutely dark, till the Sun is depress'd about 18 Degrees below the Horizon, *viz.* at *i*, that is, till the Angle $h\,C\,i = 18° =$ H M; and drawing M V parallel to the Horizon H O, it will represent the Circle at which the *Crepusculum*, or Twilight, begins and ends, in the several Points where it cuts the Parallels of the Sun's Declination, as at G in the Tropic P L, and at F in

 the

at a Mean between the greateſt and the leaſt: All which Particulars conſtitute that agreeable

the Equator. But ſince R O = P H = 15 Degrees, the Arch O R is leſs than O V, and ſo the Tropic T R will not touch the Circle M V at all; which ſhews that for ſome Time in the Middle of Summer there is *no dark Night*: And this happens between *May* 23 and *July* 22. See my SYNOPIS SCIENTIÆ COELESTIS, on a large Imperial Sheet, New-Stile.

18. Moreover it is evident that C F = K G, becauſe P L is parallel to Æ Q; the Time, however, of deſcribing C F and K G will not be the ſame; from whence it appears there is a certain Parallel in which the Twilight will be the *leaſt of all*, and another in which it will be a *Maximum* or *greateſt*. The former is when the Sun has 6° 7′ South Declination, *viz.* in *Libra* ♎, or *Piſces* ♓ 17° 30′, which happens *March* 7, and *October* 9, in the preſent Age: And 'tis plain the Twilight is greateſt of all in the Parallel which touches the Point V, on *May* 23 and *July* 22, as aforeſaid. *Note*, How the Time of the leaſt Duration of Twilight is inveſtigated, may be ſeen in the beſt Manner in Dr. *Gregory*'s *Elements of Aſtronomy*; and I would have given it here, but that it is very tedious, and in itſelf a Matter of little Importance.

19. It is a Problem of much greater Conſequence and Curioſity, to determine the Ratio or Proportion of Heat which any Place receives from the Sun in any Day of the Year. In Order to this it muſt be conſidered, that *the Quantity of Heat will be as the Time*, if we ſuppoſe the Sun to have the ſame Altitude; and as the *Sine of the Altitude*, if the Time be the ſame. Therefore if neither the Time nor Sine of the Altitude be given, the Quantity of Heat will be as the Rectangle or Product of both.

20. Therefore let a = Sine of the Latitude Æ Z; its Co-ſine (or Sine of Z N) = b; the Sine of N S = c, and of its Complement (or Declination) S D = d; the Sine of the Hour from Noon (or Angle Æ N D) = x, its

agreeable Seaſon we call the SPRING; the Middle of which is ſhewn by the Index to be the 11th of *March*, as this Machine was made for the Old-Stile.

As

its Arch ÆD $= z$, and Radius $= 1$; then is $\sqrt{1-xx}$ Plate
$=$ Co-ſine of the Angle ZNS (*viz.* Sine of the Angle LXIV.
DNC) and (*per Spherics*) we have $bc\sqrt{1-xx} \pm ad$ Fig. 6.
$=$ Sine of the Sun's Altitude SB; which multiplied by the Fluxion of the Arch of Time $= \dot{z}$ will produce the Fluxion of the Sun's Heat, *viz.* $\dot{z} + \overline{bc\sqrt{1-x^2} \pm ad}$. Or, putting $bc = g$, $\sqrt{1-x^2} = h$, $ad = f$, we have the Fluxion of the Heat $= \dot{z} \times \overline{gh \pm f}$.

21. Now to find the Value of $\dot{z}$, let AB $= z$, BE Plate
its Sine, EC the Co-ſine, and Radius CB; and ſuppoſe LXIV.
FG drawn infinitely near to EB, and BD, parallel to Fig. 7.
AC; then 'tis evident from the ſimilar Triangles EBC and BDG, that EC : BC :: DG : GB, or $h : 1 :: \dot{x} : \dot{z}$, whence $\dot{z} = \frac{\dot{x}}{h}$; wherefore $\frac{\dot{x}}{h} \times \overline{gh \pm f} = \dot{x}g \pm \frac{\dot{x}f}{h} =$ Fluxion of the Heat, whoſe Fluent is $xg \pm fz$, which therefore is as the Quantity of Heat from Noon to the given Time, as required.

22. From this Theorem we may calculate the Heat of any Day in the Year in any given Latitude required; of which I ſhall give the ſeveral following uſeful Examples. Let it be required *to expreſs the Heat of an Equinoctial Day under the Equator*. In this Caſe the Latitude of the Place is Nothing, therefore $a = 0$; conſequently $fz = adz = 0$. In the other remaining Part $xg = xbc$, $b = 1$, $c = 1$; therefore the Heat will be as x; and ſince the Semi-diurnal Arch is 90 Degrees, the Heat of the half Day will be as $x = 1$, and of the whole Day the Heat is as 2.

23. *Let the Heat of an Equinoctial Day be required for the Latitude of* 51° 30′; then becauſe in this Caſe there is no Declination of the Sun, $d = 0$, and ſo $adz = 0$.

And

As the Earth passes on from West to East, through *Libra*, *Scorpio*, and *Sagittarius*,

And since N S = 90°, we have $c = 1$; and for the Semi-diurnal Arch = 90°, $x = 1$ also; therefore the Heat is as $b = 0{,}6225$ = Co-sine of the Latitude, which for the whole Day is 1,245, and which is to that under the Equinoctial as $1\frac{1}{4}$ to 2 nearly. At the Pole $b = 0$, therefore the Heat of an Equinoctial Day at the Poles is Nothing. Lastly, in the Latitude of 60° the Heat of such a Day is half that under the Equator, or 1; because then $b = \frac{1}{2}$ Radius, or 0,5.

24. In the next Place, let us *calculate the Heat of the Summer Tropical Day*. Here we have the Time of $\frac{1}{2}$ the Day 8 Hours 12 Minutes nearly; therefore the Arch of the Equator which passes the Meridian in that Time is $123° = z$. And because when Radius is 1 the Circumference is 6,28318, therefore say, As $360 : 6{,}28318 :: z : \frac{6{,}28318\,z}{360} = 0{,}01745329\,z$, the Length of the Arch z in the Measure of the Radius. Therefore we have

The Logarithm of the Sine $z = 123° = 2.089905$
The Logarithm ——— $0{,}01745 = 8.241795$
The Logarithm ——— $a = 51°\ 30' = 9.893544$
The Logarithm ——— $d = 23°\ 30' = 9.600700$

Total, the Logarithm of $a\,d\,z = 0{,}6698 = 9.824944$

25. Then for the other Part of the Theorem, *viz.* $x\,b\,c$, we have

The Logarithm of the Sine $x = 57°\ 00' = 9.923591$
The Logarithm ——— $b = 38°\ 30' = 9.794149$
The Logarithm ——— $c = 66°\ 30' = 9.962398$

Total, the Logarithm of $x\,b\,c = 0{,}4788 = 9.680138$

Therefore the Heat of half the Day is $0{,}6698 + 0.4788 = 1{,}1486$; and of the whole Day it is 2,2972, almost twice as great as that of the Equinoctial Day with us, and greater than the Heat of such a Day to those who live under the Equator.

26. To

rius, to the Beginning of *Capricorn*, the Sun will appear from the Earth to move through the opposite Signs of the Ecliptic, viz. *Aries*, *Taurus*, *Gemini*, to the Beginning

26. To find the Expression of the Winter Tropical Day, we have the Semi-diurnal Arch $z = 57°$, and the rest the same as before. Therefore

The Logarithm of Sine ——	$z = 57° = 1.755875$
The Logarithm of ——	$0,01745 = 8.241795$
The Logarithm of ——	$a\,d = 9.494244$
Total, the Logarithm of $a\,d\,z$	$-0,3104 = 9.491914$

Then $x\,b\,c - a\,d\,z \times 0,4788 - 0,3104 = 0,1684$, and so $2 \times 0,1684 = 0,3368$ = Heat of the whole Day, which is almost 7 Times less than that of the Summer Tropic.

27. The Sum of the Heat of the two Tropical Days is $2,2972 + 0,3368 = 2,634$; which is greater than the Heat of two Equinoctial Days with us, which is but 2,49. Hence by means of the Obliquity of the Ecliptic, we who live beyond the Tropic have much more of the Sun's Heat than we could have enjoyed had the Sun moved always in the Equinoctial. And on the other hand, it will be found by Calculation, that for those who live between the Tropics and the Equator, the Sum of the Heat of any two opposite Days of the Year is less than the Heat of two Equinoctial Days; and therefore the Heat of the whole Year is less in the present Case, than it would be from a constant Equinoctial Sun.

28. Lastly, let it be required to calculate the Heat of a *Polar Day*, or that under the Pole, for the Tropical Sun. In this Case $b=0$, and $x\,g=x\,b\,c=0$; also $a=1$. Whence the Heat of any Day under the Pole will be as d, or *Sine of the Declination*, because z is here always the same, *viz.* a *Semicircle*, or 180 Degrees. And under the Pole the Value of $d\,z$ is thus expressed for the Tropical Sun.

The

ning of *Cancer* ; during which Time, by the inclined Position of the Earth's Axis, the

The Logarithm of Sine	$d = 23^\circ\ 30' =$	.9.600700
The Logarithm of	—— $z = 180 =$	2.255272
The Logarithm of	—— $0{,}01745 =$	8.241795
Total, the Logarithm of	$dz = 1{,}252 =$	0.097767

The Double of which is 2,504 ; which therefore expresses the Heat of a Tropical Day under the Pole, which is greater than the Heat of any Day in any other Latitude. Hence we see the Extreme of Heat, as well as of Cold, is found in the same Place, *viz.* under the Pole.

29. It is a Problem of another Sort, *To find when the Heat is a* Maximum, *or greatest of all, in any given Day.* In order to solve this, let the Semi-diurnal Arch $= a$, $z =$ Arch of the Hour from Noon, $b =$ Rectangle of the Sines of Latitude and Declination, and $c =$ Rectangle of their Co-sines. Then *(per Spherics,* and *Infinite Series)* we have the Co-sine of the Hour from Noon $= 1 - \frac{1}{2}z^2 + \frac{1}{24}z^4 - \frac{1}{720}z^6$, &c. and the Sine of the Sun's Altitude $= c - \frac{1}{2}cz^2 + \frac{1}{24}cz^4 - \frac{1}{720}cz^6$ &c. $+ b$. This multiply by $a + z$ is as $c + cz - \frac{1}{2}acz^2 - \frac{1}{2}cz^3 + \frac{1}{24}acz^4 + \frac{1}{24}cz^5 - \frac{1}{720}acz^6 - \frac{1}{720}cz^7$, &c. $+ ab + zb$; which therefore is proportional to the Sun's Heat. And this is greatest when its Fluxion is equal to Nothing, viz. $c\dot{z} + b\dot{z} - acz\dot{z} - \frac{3}{2}cz^2\dot{z} + \frac{1}{6}acz^3\dot{z} + \frac{5}{24}cz^4\dot{z} - \frac{1}{120}acz^5\dot{z}$, &c. $= 0$. Then dividing by $\dot{z}$, $c + b - acz - \frac{3}{2}cz^2 + \frac{1}{6}acz^3 + \frac{5}{24}cz^4 - \frac{1}{120}acz^5$, &c. $= 0$; whence $\frac{c+b}{ac} = z + \frac{3}{2a}z^2 - \frac{1}{6}z^3 - \frac{5}{24a}z^4 + \frac{1}{120}z^5 = A$. Now putting $r = \frac{3}{2a}$, $s = \frac{-1}{6}$, $t = \frac{-5}{24a}$, $v = \frac{1}{120}$; by reverting the Series we have $z = A - rA^2 + \overline{2rr - s} \times A^3 + \overline{5rs - 5r^3 - t} \times A^4$, &c.

30. From

the *Northern* Parts will be gradually turn'd towards the Sun, and the *Southern* Parts from it; whence the Sun's Rays will fall more and more directly on the former, and pass through a still less Quantity of the *Atmosphere*; but in the *Southern* Parts, the reverse. Also in the *Northern* Parts, the Arches of the Parallels in the *enlighten'd Hemisphere* will continually increase, and those in the *dark* one decrease, shewing the constant Increase of the Days, and Decrease of the Nights; all which will be in their greatest Degree when the Sun is arrived to *Cancer*; and therefore that will be the Middle of that Season we call SUMMER, in *Northern Latitude*; but in *Southern Latitude*

30. From this Theorem it will be easy to compute the Value of z, or the Time from Noon when the Heat is greatest on any given Day. For Example: Let it be required for the Day of the Summer Solstice in the Latitude of 51° 30′, when the Declination is 23° 30′. Then since (by *Art.* 24, 25.) we have $a = 2{,}146$, $b = 0{,}3121$, $c = 0{,}5709$, $b + c = 0{,}883$, and $ac = 1{,}225$; therefore $\frac{b+c}{ac} = A = 0{,}7207$. Whence by the three first Terms of the Series we shall have $z = A - rA^2 + \overline{2rr - s} \times A^3 = 0{,}7862$. Therefore say, As the Circumference $6{,}283 : 360° :: 0{,}7862 : 45°$ nearly. Whence, by allowing 15° to an Hour, it appears that *the hottest Time of the Day is Three o' Clock in the Afternoon.*

tude every Thing will be the reverſe, and their Seaſon *Winter*.

THE *North Frigid Zone* is now wholly enlighten'd, and the Pole turned towards the Sun as far as poſſible; but now as the Earth moves on, the *North* Pole returns, the Diurnal Arches begin gradually to decreaſe, and the Nocturnal to increaſe; and of conſequence the Sun's Rays fall more and more obliquely, and his Heat proportionally diminiſhes till the Earth comes to *Aries*, when the Sun will appear in *Libra*; and thus produce an Equality of Light and Heat, of Day and Night, to all Parts of the World. This will be the Middle of the Seaſon call'd AUTUMN, and that Day the *Autumnal Equinox*.

BUT as the Earth goes on through *Aries*, *Taurus*, and *Gemini*, you will ſee the Sun paſs through the oppoſite Signs of *Libra*, *Scorpio*, *Sagittarius*. The *North* Pole is now in the dark Hemiſphere, and the *Frigid Zone* is now more and more obſcured therein: All *Northern Latitudes* continue gradually turning from the Sun; and his Rays fall more and more obliquely on them, and paſs through a larger Body of the Atmoſphere: The *Nocturnal Arches* continue to increaſe, and the *Diurnal* to decreaſe: All which contribute to make the diſmal dreary

dreary Seaſon we call WINTER; the Midſt whereof is ſhewn by the Sun's entering the firſt Scruple of *Capricorn* on the 10th of *December* (Old-Stile), as by the Index may be ſeen.

LASTLY: As the Earth journeys on from thence through *Cancer*, *Leo*, and *Virgo*, the Sun appears to paſs through *Capricorn*, *Aquarius*, and *Piſces*; and all Things change their Face. The *Northern* Climes begin to return, and receive more directly the enlivening Beams of the Sun, whoſe Meridian Height does now each Day increaſe; the Days now lengthen, and the tedious Nights contract their reſpective Arches; and every Thing conſpires to advance the delightful Seaſon of the SPRING, the Midſt whereof is ſhewn by the Earth's returning again to that Point, where firſt we gave it Motion.

ALL theſe Appearances of the Seaſons, &c. are ſhewn as well for the *Southern Latitudes*, where at the ſame time they happen in Order juſt the reverſe to what we have now obſerved for the *Northern*. Thus, when it is *Summer* with us, it is *Winter* with them, and they have their Days ſhorteſt when ours are longeſt; and *vice verſâ*. All which is moſt diſtinctly ſeen in the Orrery.

AT

At the ſame time the Earth is going round the Sun, the Moon is ſeen conſtantly circulating round the Earth once in 29 Days and a half; which Days are number'd on a ſilver'd Circle, and ſhewn by an Index moving over them. Thus each Day of the Moon's Age, and the *Phaſis* proper thereto, are ſhewn for any required Time; and alſo why we ſee always *one and the ſame Face of the Moon, viz.* on account of her *turning about her own Axis in the ſame Time ſhe takes to revolve about the Earth.*

Again: By placing a Lamp in the Orrery, and making the Room dark, we ſee very naturally how the *Sun is eclipſed* by the New Moon, and the Shadow paſſing over the Diſk of the Earth; and alſo how the Moon, at Full, is eclipſed by paſſing through the Shadow of the Earth. Here alſo we ſee the Manner how *Mercury* and *Venus* tranſit the Sun's Face in Form of a *dark round Spot*; and alſo why they can never appear at a great Diſtance from the Sun; and various other *Phænomena*, of the like Nature. (CXLIII.)

The

(CXLIII.) 1. The *Doctrine of* Eclipses is next to be explained. The Sun being a luminous Body, vaſtly larger than the Earth, will enlighten ſomewhat more than

THE COMETARIUM is a very curious Machine, which exhibits an Idea of the Motion

than One-half of it, and cause the Earth to project a long *conical Shadow*, as is represented in the Figure, where S is the Sun, E the Earth, and H B D its Shadow. Pl. LX. Fig. 2.

2. In order to find the Extent or Magnitude of the Earth's Shadow, the Lines being drawn as in the Figure, in the Triangle S B M, the outward Angle S D A = D S B + D B S, the two inward and opposite Angles; but the first, *viz*. D S B, is that under which the Earth's Semidiameter C D appears at the Sun, which is not sensible; therefore D B S, the Semi-Angle of the Cone, is equal to A D S, which is the Angle under which the Sun's Semidiameter A S appears at the Earth, which in its mean Distance is 16 Minutes.

3. Hence we can find the Height of the shadowy Cone C B; for in the Right-angled Triangle C B D there is given the Side C D = 1, and the Angle C B D = 16 Minutes; therefore to find the Side C B, say,

As the Tangent of C B D	= 00° 16′	= 7.667849
Is to the Radius	90 00	= 10.000000
So is Unity	1	= 0.
To the Length of the Side C B	= 214,8	= 2.332151

4. The Height of the Earth's Shadow being at the mean Distance of the Sun 214,8 Semidiameters, when the Sun is at its greatest Distance it will make C B = 217 Semidiameters of the Earth, which is its greatest Height. Hence we see the Height of the Shadow is near *three times* as great as the mean Distance of the Moon, or 60 Semidiameters: But the Height of the terrestrial Shadow falls far short of the Distance of *Mars*, and therefore can involve no one of the heavenly Bodies but the Moon.

5. After the same Manner it may be shewn that the Angle of the Moon's Shadow (and indeed of all Spheres whose Semidiameters bear no sensible Proportion to their Distance from the Sun) is of the same Dimensions with

Motion or Revolution of a Comet about the Sun; and as this Sort of Motion is not

that of the Earth; whence thoſe Cones are ſimilar Figures, and ſo have their Heights proportional to the Diameters of the Baſes. Therefore ſay, As the Diameter of the Earth 100 is to the Diameter of the Moon 28, ſo is the Altitude of the Earth's Shadow 214,8 to the Altitude of the Moon's Shadow 60 $\frac{144}{1000}$ of the Earth's Semidiameters. The Shadow of the Moon therefore will juſt reach the Earth in her mean Diſtance, which it cannot do in her Apogee; but in her Perigee it will involve a ſmall Part of the Earth's Surface.

Pl. LX. Fig. 3. 6. Beſides the dark Shadow of the Moon, there is another called the *Penumbra*, or Partial Shadow; to repreſent which let S be the Sun, T the Earth, D the Moon; and let K C F and A B E be two Lines touching the oppoſite Limbs of the Sun and Moon; then 'tis evident that C F E B will be the dark or abſolute Shadow of the Moon, in which a Perſon on the Earth's Surface between E and F is wholly deprived of the Sun's Light. Moreover, Let K B G and A C H be two other Lines touching the Sides of the Sun and Moon alternately, and interſecting each other in the Point I above the Moon. Then will H C B G be the *Penumbra* abovementioned, and is the *Fruſtum* of the Cone G I H; for 'tis evident that a Part of the Sun will be ſeen and Part thereof hid to a Spectator on the Earth's Surface between F and H, and E and G; or, in other Words, the Sun in thoſe Parts of the Earth will appear only *partially eclipſed*.

7. To calculate the Angle of the Cone H I G, draw S B, then in the oblique Triangle B I S, the external Angle B I D is equal to both the inward and oppoſite Angles I B S and I S B; but I S B is that under which the Semidiameter of the Moon appears at the Sun, and is therefore inſenſibly ſmall; whence the Angle B I D = I B S or K B S = the apparent Semidiameter of the Sun. Therefore the Part of the Penumbral Cone C I B

not performed in *circular*, but very *elliptic Orbits*, ſo in this Inſtrument, a peculiar Con-

CIB is equal and ſimilar to the dark Shadow of the Moon.

8. Let us now ſee how much of the Earth's Surface can be at any time involved in the Moon's dark Shadow, or the Quantity of the Arch EF. In order to this let us ſuppoſe the Sun to be in Apogee, and the Moon in Perigee; and in that Caſe the Height of the conical Shadow will be about 61 Semidiameters, and the Diſtance of the Moon about 56; that is, (in *Fig*. 4.) Pl. LX.
DK = 61, DT = 56, and TE = 1. In this Caſe alſo the Half-Angle of the Shadow TKE = 15′ 50″, as being leaſt of all. Therefore ſay,

As Unity, or the Side	TE = 1	= 0.
Is to the Side	TK = 5	= 0.698970
So is Sine of the Semi-Angle	TKE = 15′ 50″	= 7.663238
To Sine of the Angle	TEK = 1° 19′ 10″	= 8.362208

Wherefore TEK + TKE = ATE = AE = 1° 35′, and ſo FE = 3° 10′ = 190′ = 220 Miles Statute Meaſure; which is therefore the Diameter of the dark Shadow on the Earth's Surface when greateſt.

9. After a like Manner you find the Diameter of the Penumbral Shadow at the Earth, as GEFH, when greateſt of all, that is, when the Earth is *in Perihelio*, and the Moon in her *Apogee*; for then will the Sun's apparent Diameter be equal to 16′ 23″ = TIG, the greateſt Semi-Angle of the Cone; and thence we ſhall find ID = $58\frac{1}{2}$ Semidiameters of the Earth. In this Caſe alſo the Diſtance of the Moon from the Earth is DT = 64 Semidiameters. Therefore, As TG = 1 : TI = $122\frac{1}{2}$ :: Sine of the Angle TIG = 16′ 23″ : Sine of the Angle IGN = 35° 42′. But IGN = TIG + ITG, and ſo ITG = IGN − TIG = 35° 25′; the double of which is 70° 50′ = GEFH = 4900 *Engliſh* Miles nearly.

10. Since an Eclipſe of the Sun proceeds from an Interpoſition of the Moon, 'tis evident, if the Sun and Moon

Contrivance by *elliptical Wheels* is neceſſary to effect it; of which we have given a Print

Moon were always in the ſame Plane, there would neceſſarily be an Eclipſe of the Sun every time the Moon came between the Sun and Earth, that is, at every *New Moon*. For let X be the Sun, T the Earth, and F B G H the Moon's Orbit in the Plane of the Ecliptic; then when the Moon comes to be at B in the Right Line T X, which joins the Centres of the Sun and Earth, it will be exactly interpoſed between the Sun and a Spectator on the Earth at V; and ſince the apparent Magnitude or Diſk of the Sun is the ſame nearly with that of the Moon, it muſt neceſſarily be hid behind the Sun's Diſk at that time, and ſo eclipſed from the Sight of the Spectator; and this muſt be the Caſe whenever the Moon comes into the ſaid Line or Point B, *viz.* every New Moon.

Pl. LX. Fig. 5.

11. But if (as the Caſe really is) the Orbit of the Moon be not in the Plane of the Ecliptic, but inclined thereto under a certain Angle, there may be a New Moon, and yet no Eclipſe of the Sun at the ſame time. To illuſtrate this, let A B C D E be a Circle in the Plane of the Ecliptic, deſcribed at the Diſtance of the Moon's Orbit A G H, and interſecting the ſame in the Points B and D, making an Angle therewith as A B F, whoſe Meaſure is the Arch G C, as being 90 Degrees diſtant from the angular Points or *Nodes* B and D.

12. Now 'tis evident, if the Arch G C be ſomewhat greater than the Sum of the apparent Semidiameters of the Sun and Moon then at G, and ſome Diſtance from G towards B, there may be a New Moon, and yet no Eclipſe of the Sun, becauſe in this Caſe the Diſk of the Moon G is too much elevated or depreſſed above or below the apparent Diſk or Face of the Sun C to touch it, much leſs to hide or eclipſe any Part thereof; as is evident from the Figure.

13. At a certain Point M in the Moon's Orbit, the Moon will have a Latitude equal to the Sum of the Semidiameters

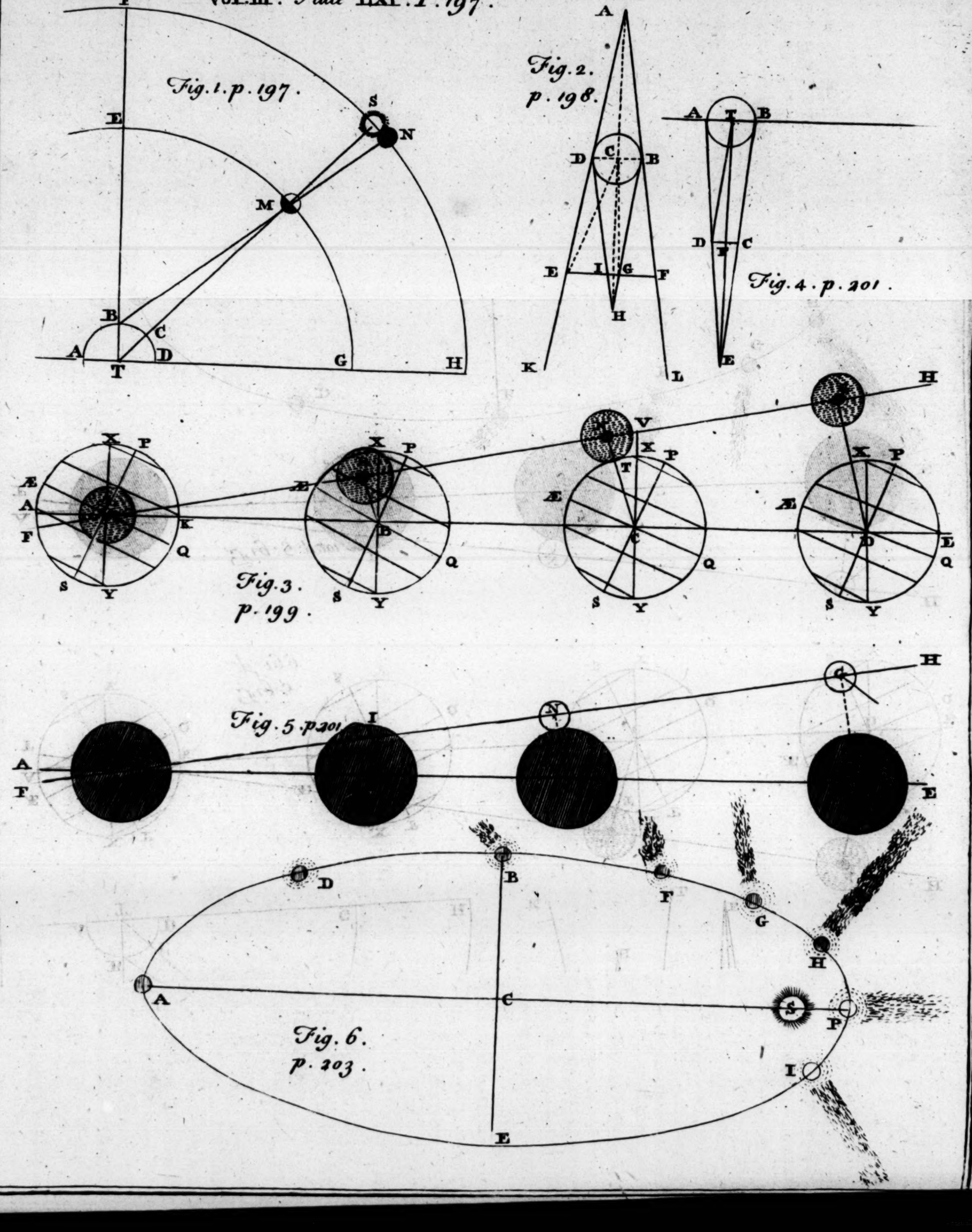


Fig. 1. p. 197.

Fig. 2. p. 198.

Fig. 3. p. 199.

Fig. 4. p. 201.

Fig. 5. p. 201.

Fig. 6. p. 203.

Print in Plate XII. The Comet repre-
sented in this Machine is that which ap-
peared

Semidiameters of the Sun and Moon; and therefore when the Moon is new in that Point, she will appear to a Spectator in the Point Z to touch the Sun only; from whence this Point is called the *Ecliptic Limit*, inasmuch as it is impossible there should happen a New Moon in any Part between this and the Node D (on each Side) without eclipsing the Sun less or more; as you see the *Partial Eclipse* at K, and the *Total Eclipse* in the Node itself B.

14. What we have hitherto said has been with regard to the *Phænomena* of an Eclipse of the Sun as they appear to a Spectator on the Earth's Surface, in whose Zenith the Moon then is, and where there is no Refraction to alter the true Latitude of the Moon: But where the Moon has any Latitude, there the Process of calculating the Appearances of a Solar Eclipse will be somewhat more complex, on account of the Variation of the Moon's Latitude and Longitude for every different Altitude, and consequently every Moment of the Eclipse.

15. But that I may give a clear Idea of this Affair of Pl. LXI.
Refractions, let A B D be the Surface of the Earth, M Fig. 1.
the Moon, S the Sun, seen from the Centre of the Earth T in the same Point of the Heavens with the Moon, and consequently *centrally eclipsed* to a Spectator at C, in whose Zenith the Moon is: But to a Spectator any where else situated, the same *Phænomenon* will not happen in the same Circumstances, if at all. Thus a Spectator at B will view the Moon in the Direction of the Right Line B M N, and so her apparent Place in the Heavens will be at N, where it is evident her upper Limb will but just touch the lower Limb of the Sun, and so will not eclipse it at all: But to a Spectator any where between B and C the Sun will appear to be *partially eclipsed* less or more, as you go from B towards C.

16. This Arch S N in the Heavens is called the *Parallax*, or Difference between the *true and apparent Place*

peared in the Year 1682, whose Period is 75 Years and a half, and therefore will again

of the Body at M, and is equal to the Angle S M N or B M T. Now this Angle or Parallax is constantly diminishing, as the *Phænomenon* at M approaches towards the Zenith at E, where it entirely vanishes; but increases as it approaches the Horizon at G, where it is greatest of all, and is there called the *Horizontal Parallax*, which in the Moon amounts to a *whole Degree*, as was shewn *Annot.* CXXXV.

17. It is here observable, that the Parallax always depresses the Object, and therefore when the Moon has North Latitude it is diminished, but the South Latitude is increased, with respect to us; and so the Ecliptic Limits are variable in every particular Latitude. But a *Solar Eclipse* may in an absolute Manner be best represented by a Projection of the Earth's Disk, and of the Section of the dark and penumbral Shadow of the Moon, as they appear (or would appear) to a Spectator at the Distance of the Moon in a Right Line joining the Centres of the Sun and Earth.

18. In order to this, we are to find the Dimensions of the apparent Semidiameters of the Earth, dark Shadow, and *Penumbra*, at the Distance of the Moon. As to the first, *viz.* the *Earth's Semidiameter*, it is equal to the Moon's *horizontal Parallax*, as we have shewn.
Pl. LXI. That of the *dark Shadow* is thus estimated: Let C be
Fig, 2. the Centre of the Moon, D B its Diameter, D H B its dark Shadow, and K A L the Penumbral Cone. Then let E F be the Diameter of the *Penumbra* at the Earth, and I G that of the dark Shadow, and draw C G and C E; then is the Angle C G B = B H C + H C G, and so G C H = B G C − B H C; that is, the apparent Semidiameter of the dark Shadow is equal to the Difference between the apparent Semidiameters of the Moon and Sun. (See *Art.* 2. and 5.)

19. In like Manner the Angle E C H = D E C + D A C, that is, the apparent Semidiameter of the *Penumbra* at the Earth is equal to the Sum of the apparent

again appear in 1758. By this Piece of Machinery is shewn the *unequal Motion* of a Comet

rent Semidiameter of the Moon and Sun (see *Art.* 7.). Now the Semidiameters of the Sun and Moon, and also the Moon's horizontal Parallax, are already calculated for the various Distances of the Sun and Moon from the Earth, and for least, mean, and greatest Eccentricities of the Lunar Orbit, in the *Astronomical Tables.*

20. Therefore let A E represent a small Portion of the annual Orbit, and F H the visible Path of the Centre of the Lunar Shadows, which will exactly correspond to the Position of the Moon's Orbit with respect to the Ecliptic in the Heavens; and therefore the Point of Intersection ☊ will be the Node, and the Angle H ☊ E the Angle of Inclination of the Lunar Orbit to the Plane of the Ecliptic, which is about 5 Degrees. Pl. LXI. Fig. 3.

21. Hence if Æ P Q S represent the Disk of the Earth (according to the *Orthographic Projection*) in the several Places ☊, B, C, D, whose Semidiameter is made equal to the Number of Minutes in the Moon's Horizontal Parallax at the Time of the Eclipse; and if in the Path of the Shadows in the Points ☊, R, N, G, we describe a small Circle whose Semidiameter is equal to the Difference between the Semidiameters of the Sun and Moon, that shall be the circular Section of the Moon's dark Shadow at the Distance of the Earth (by *Article* 18.). Lastly, if on the same Centre we describe a larger Circle, whose Semidiameter is equal to the Sum of the Semidiameters of the Sun and Moon, that shall represent the Section of the Penumbral Shadow, (by *Art.* 19.) and is here shewn by the dotted Area.

22. Here then it is evident, if the Moon, when New, be at the Distance ☊ G from the Node, the Penumbral Shadow will not fall near the Earth's Disk, and so there cannot possibly happen any Eclipse. If the Moon's Distance from the Node be equal to ☊ N, then the Penumbral Shadow will just touch the Disk, and consequently ☊ C the *Ecliptic Limit*; which may be found

a Comet in every Part of its Orbit, and how from thence it moves with a retarded Velocity

follows. The Line N C, as being the nearest Distance of the Centres of the Shadows and Disk, is perpendicular to the Path F H, and is equal to T C + N T = 62′ 10″ + 16′ 52″ + 16′ 23″, *viz.* the Sum of the Moon's Horizontal Parallax, and of the Semidiameters of the Sun and Moon, all of them when greatest: Also the Angle N ☊ C, when least, is 5° 30′. Therefore in the Right-angled Triangle N ☊ C, to find the Side ☊ C, we have the following Analogy.

As the Sine of the Angle N ☊ C =	5° 30′ = 8.981573
Is to Radius	90° 00′ = 10,000000
So is the Logarithm of the Side	NC=95′,5= 1,980003
To the Logarithm of the Side	☊ C=996′,4= 2.998430

23. The Ecliptic Limit, therefore, is 996′,4 = 16° 36′, beyond which Distance from the Node ☊ there can be no Eclipse; and within that Distance if the Moon be New, the Shadow will fall on some Part of the Disk, as at B; where all those Places over which the Shadows pass will see the Sun eclipsed, *in part only* by the dotted Penumbral Shadow, but *totally* by the dark Shadow; and the Sun will be *centrally* eclipsed to all those Places over which the Centre of the Shadows passeth.

24. If the Moon be new in the Node itself, then will the Centre of the Shadows pass over the Centre of the Disk, as represented at ☊. In this Case if the apparent Diameter of the Moon be greater than that of the Sun, the Face of the Sun will be *wholly obscured* to all Parts over which the Centre passes; but if not, the Sun will only be *centrally eclipsed*, but his Circumference will appear a *bright Annulus*, or luminous Ring, whose Width will be equal to the Difference of the Diameters of the Luminaries.

25. As the Disk of the Earth is here projected, it represents the Case of an Eclipse on an *Equinoctial Day*, so that A K is the Ecliptic, Æ Q the Equator, X Y the Axis

Velocity till it arrives at the *Aphelion* Point, where it moves slowest of all; and from thence

Axis of the Ecliptic, P S the Axis of the Equator or of the Earth, P and S the North and South Poles; besides the Tropics and Polar Circles, here represented by Right Lines, as in the common *Analemma*. And by those who understand this Projection, the Disk of the Earth and the Passage of the Shadows over it may be exhibited for any Place of the Sun, or Declination of the Moon; for which see my *Young Trigonometer's Guide*, Vol. II.

26. LUNAR ECLIPSES are not quite so complicated in Theory, nor near so tedious and difficult in Calculation, as Solar ones. The latter are only *apparent*, the former *really such*; that is, the Moon is really deprived of its Light, and therefore must appear obscured to all the Inhabitants of the Earth equally, by whom she can be seen; whereas the Sun, not being deficient in Light, will ever appear resplendent to those who do not happen to live on that Part of the Earth where the Lunar Shadows pass.

27. As a *Lunar Eclipse* is occasioned by the Immersion of the Moon into the Earth's Shadow, we have only to calculate the apparent Semidiameter of the Earth's Shadow at the Moon, in order to delineate an Eclipse of this Sort. Thus let A B be the Earth, T its Centre, Pl. LXI. AEB its Conical Shadow, D C the Diameter of a Fig. 4. Section thereof at the Moon; and drawing T D, we have the outward Angle $ADT = DTE + DET$; therefore $DTE = ADT - DET$; that is, the Angle DTE, under which the Semidiameter of the Earth's Shadow at the Distance of the Moon appears, is equal to the Difference between the Moon's Horizontal Parallax A D T, and the Semidiameter of the Sun DEF.

28. If therefore A E represent the Path of the Earth's Fig. 5. Shadow at the Distance of the Moon near the Node ☊, and F H a Part of the Lunar Orbit, and the Section of the Earth's Shadow be delineated at ☊, B, C, D, and the Full Moon at ☊, I, N, G; then 'tis evident, where the

thence it is seen continually accelerating its Motion towards the *Perihelium*, in such manner as the Laws of Attraction require. The Comet is represented by a small Brass Ball, carried by a *Radius Vector*, or Wire, in an *elliptic Groove*, about the Sun in one of its *Foci*; and the Years of its Period are shewn by an Index moving with an equable

the least Distance of the Centres of the Moon and Shadow exceeds the Sum of their Semidiameters, there can be no Eclipse of the Moon, as at D. But where that Distance is less, the Moon must be partly or wholly involved in the Shadow, and so suffer an Eclipse, as at B and ☋; in which latter Case the Moon passes over the Diameter of the Shadow.

29. But in a certain Position of the Shadow at C, the least Distance of the Centres N C is equal to the Sum of the Semidiameters; and therefore ☋ C is the *Ecliptic Limit* for Lunar Eclipses: To find which, we have N C $= 63' \; 12''$ nearly when greatest, and the Angle N ☋ C $= 5^\circ \; 00'$. Therefore say

As the Sine of the Angle N ☋ C	$= 5^\circ \, 00' =$	8.940296
Is to Radius	$90^\circ \, 00' =$	10,000000
So is the Logarithm of the Side NC	$= 63', 2 =$	1.800717
To the Logarithm of the Side ☋ C	$= 725', 2 =$	2,860421

Hence if the Moon be at a less Distance from the Node ☋ than $725' = 12^\circ \; 5'$, there will be an Eclipse; otherwise none can happen.

30. If the Earth had no Atmosphere, the Shadow would be absolutely dark, and the Moon involved in it quite invisible; but by means of the Atmosphere many of the Solar Rays are refracted into and mixed with the Shadow, by which the Moon is render'd visible in the midst of it, and of a dusky red Colour.

equable Motion over a graduated ſilver'd Circle: The Whole being a juſt Repreſentation of the preſent Theory of thoſe prodigious and wonderful *Phænomena* of the Planetary Syſtem (CXLIV.).

(CXLIV.) 1. I ſhall here preſent the Reader with as large a Compendium of the NEWTONIAN COMETOGRAPHY as the Limits of this Work will permit, or, perhaps, as he may have an Inclination to read. Sir *Iſaac* has made the Doctrine or *Aſtronomy* of *Comets* the laſt Part of his immortal *Principia*, and declares it to be by far the moſt difficult and intricate Part of Philoſophy.

2. A COMET is a Sort of Planet revolving about the Sun, in a very eccentric Orbit or Ellipſis, and which conſequently approaches very near the Sun in one Part of its Orbit, and recedes to a very remote Diſtance from it in another. Hence 'tis evident, they muſt undergo extreme Degrees of Heat and Cold. Hence it appears that the Comets are ſolid, compact, fixed, and durable Bodies, and not a Vapour or Exhalation of the Earth, Sun, or Planets, as has been uſually ſuppoſed; becauſe if it were ſuch, it muſt inevitably be diſſipated and diſperſed in paſſing ſo near the Sun: For the Diſtance of the Comet of 1680 *in Perihelio* was ſo ſmall, that it conceived a Degree of Heat above 200 times greater than that of red-hot Iron.

3. Yet are they not ſo fixed, but that they emit a fine, thin, lucid Vapour; which at firſt, while the Comet is yet a great way from the Sun, ſurrounds the Body in Form of an Atmoſphere, and begins to render the Comet viſible. As the Comet approaches nearer the Sun, this Vapour begins to aſcend from the Head or *Nucleus*, to Heights greater and greater, as the Comet gets nearer and nearer to the Sun, and makes thoſe amazing Streams of Light we uſually call their *Tails*. All which is eaſy to conceive from a View of the Figure.

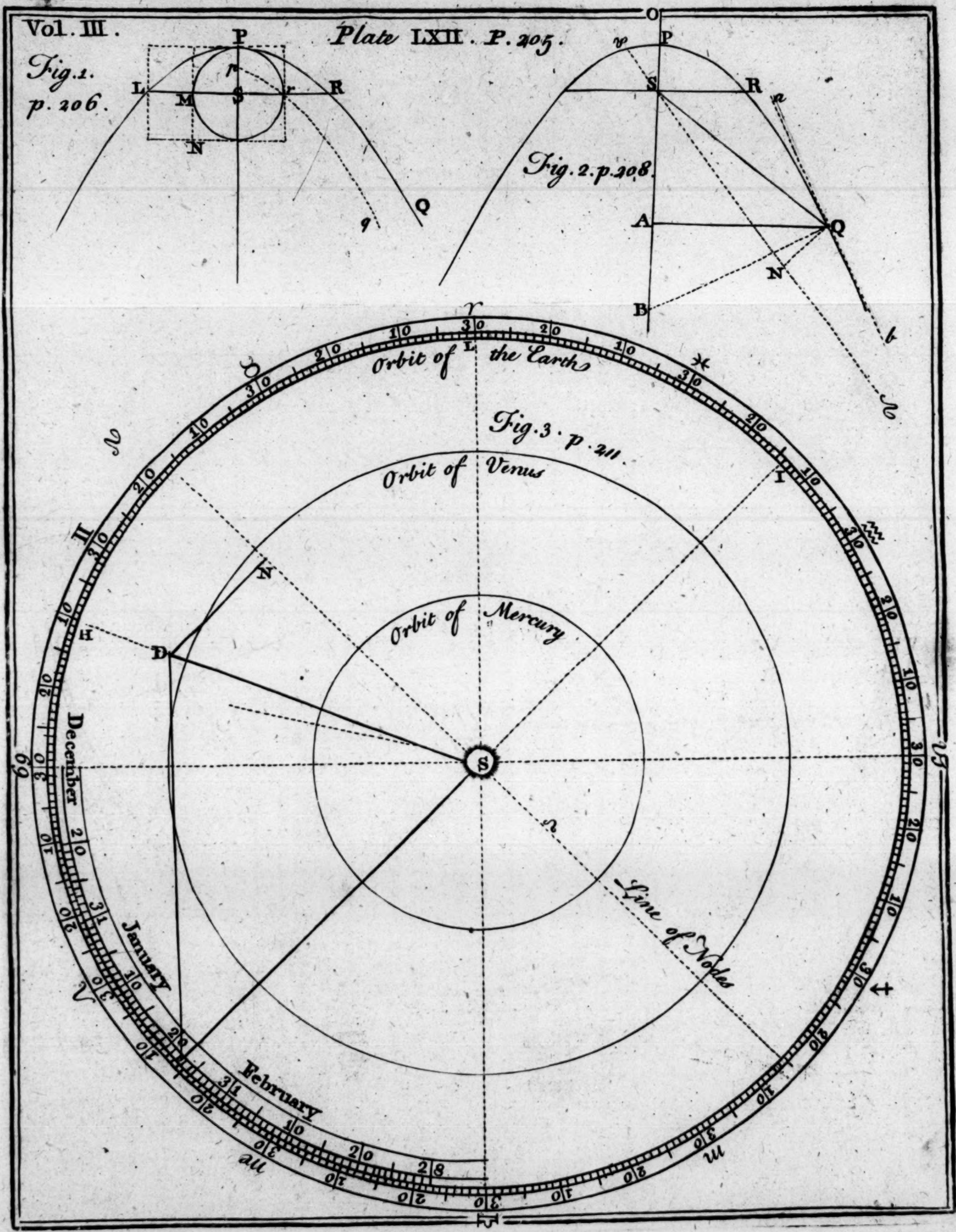
4. Theſe

4. These Tails also are so fine and translucid, that the Stars are distinctly visible through them. As they rise from the Head and ascend, they become rarified, and grow broader towards the upper End. The Form of the Tail is well known to all now living who saw the late Comet; in which we observe the Tail had a small Flexure or Curvature, as they all have, being convex on the anterior Part, and concave behind, which arises from the twofold Motion of the Particles of the Tail, the one of the Ascent from the Head, the other being the progressive Motion in common with the *Nucleus* itself. But as the former is much the greatest, so its Direction is but little alter'd by the latter, and so the Position of the Tail but a little oblique and incurvated.

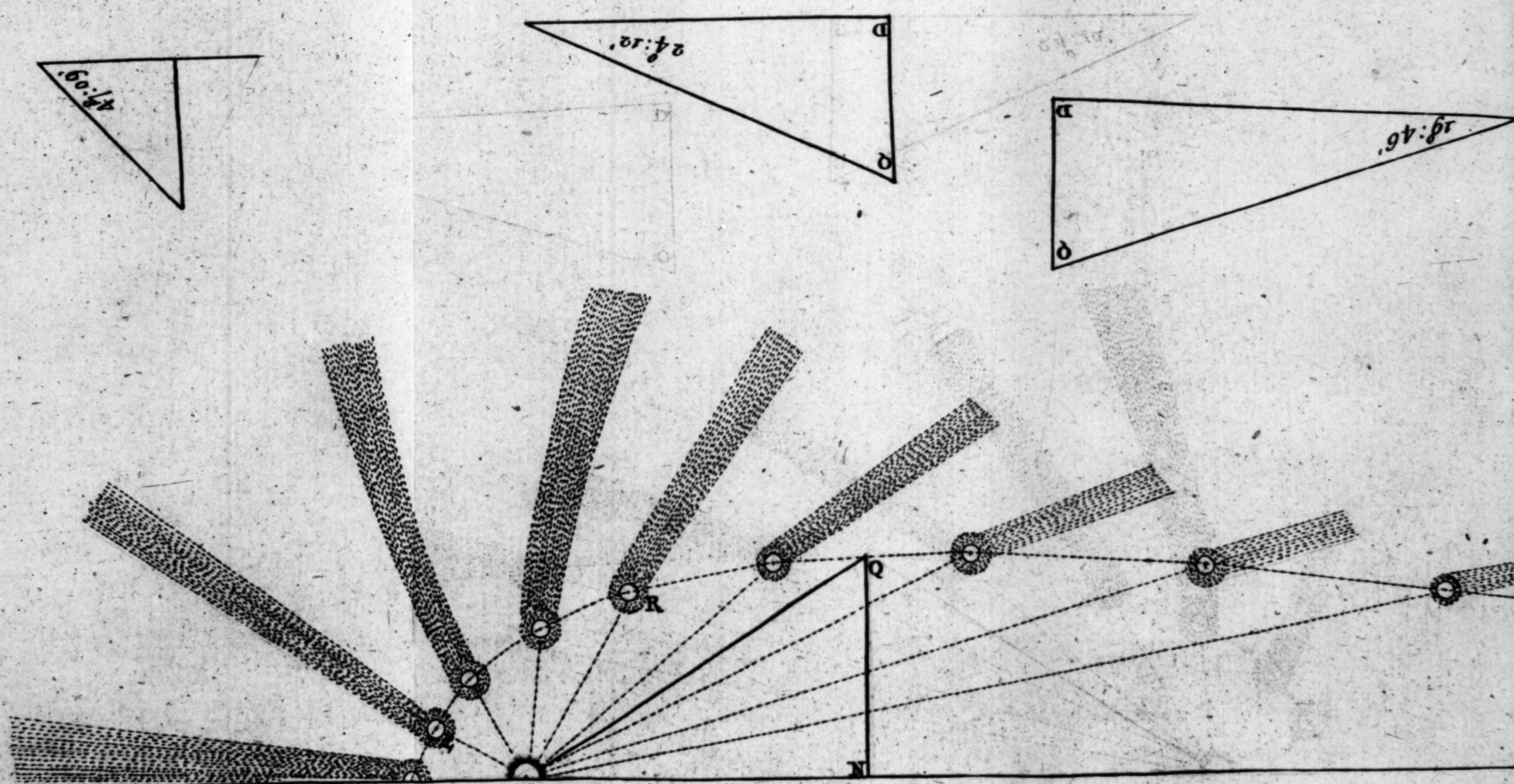
5. As to the Cause of the Ascent of the Cometary Vapour or Tail towards the Parts opposite to the Sun, there have been various Surmises and Conjectures, for so I call them, as not being attended with Certainty and Demonstration. *Kepler* ascribes it to the Action of the Sun's Rays rapidly carrying the Matter of the Tail away with them. And Sir *Isaac* does not think it dissonant to Reason, to suppose the subtil *Æther* in those free Spaces may yield to the Action and Direction of the Sun-Beams. It is certain from Experiments, that the Solar Rays collected by a Burning-Glass to a Focus, impel Light and pendulous Bodies very notably, even so as to make them vibrate backwards and forwards: And though this Impulsion of the Rays of Light with us, in our gross Mediums, and on our sluggish Matter, be inconsiderable; yet in those free Spaces, and on the subtil Effluvia or fine Particles of the Cometary Atmosphere, it may be very great. I know there are other and later Hypotheses to account for the Motion and Form of a Comet's Tail; but on Examination they appear to be insufficient, improbable, and unphilosophical, and therefore shall not trouble the Reader with them.

6. The Bodies of Comets are very small, and above the Orbit of the Moon, as is evident from hence, that they have no perceptible *horizontal* or *diurnal Parallax*, and when view'd with a Telescope at their nearest Distances appear less than to the naked Eye, by having the Splendor

Vol. III. Plate LXII. P. 205.



belonging to Plate 62.



Splendor of their Tails taken off, and that of the Atmosphere abated by being magnified. The *Nucleus* of the last Comet measured but a few Seconds, as I found by measuring the Atmosphere by a Micrometer, and taking a proportional Part.

7. On the other hand, by their *annual Parallax* they are proved to descend within the Regions of the Planets; they also appear sometimes *direct* and slower than they really move, sometimes *retrograde* and swifter than the true Motion, and lastly, they are sometimes *stationary*; all which Phænomena arise from the same Causes as were before explained of the Planets. (See *Annotation* CXXXIX.)

8. Since the Comets by Observation are found to describe curve Lines about the Sun, they must be drawn by some Force from a rectilinear Course by the first Law of Motion. And since this Force in all the Planets tends to the Sun, as being the largest Body in the System, therefore also this Force in the Comets respects the Sun in a more immediate Manner, as being so much less than it than most of the Planets are. And lastly, as this Force in the Planets is inversely in the duplicate Ratio of the Distance from the Sun, the same Law is undoubtedly observed by the Comets, which are in other respects Bodies similar to the Planets. The Comets therefore move in Conic Sections about the Sun, having their Foci in the Sun's Centre. (See *Annot.* CXL.)

9. Hence, if Comets return in an Orbit, those Orbits must be *Ellipses*; and their Periodical Times will be to the Periodical Times of the Planets in the sesquiplicate Ratio of the principal Axes: And therefore the Comets being for the most Part beyond the Planetary Regions, and on that account describing Orbits with much larger Axes than the Planets, revolve more slowly. Thus if the Axis of a Comet's Orbit be 4 times as long as that of *Saturn*'s Orbit, then would the Time of the Period of the Comet be to that of the Planet as $4\sqrt{4}$ to 1, or as 8 to 1, *viz.* $8 \times 30 = 240$ Years.

10. Since it is found by Observations that the Cometary Orbits are extremely eccentric, and that the Portion which a Comet describes during the whole Time of its

its Appearance is but a very ſmall Part of the Whole, the Centre of ſuch an Ellipſis being removed to ſo vaſt a Diſtance muſt occaſion the Curvature at each End to be vaſtly near that of a Parabola having the ſame focal Diſtance; and conſequently the Motion of a Comet may be calculated in a Parabolic Orbit without any ſenſible Error.

Plate LXII. Fig. 1.

11. Therefore the Velocity of a Comet *in Perihelio* (*viz.* in the Vertex of the Parabola P) is to the mean Velocity of a Planet deſcribing a Circle about the Sun, at the ſame focal Diſtance S P, as $\sqrt{2}$ to 1. And ſuppoſing the Earth to be that Planet, let us put the Radius of its Orbit S P = 100000, and then ſay, As the whole Periodical Time of the Earth 365^{1} is to the whole Periphery 628318, ſo is 1 Day to 1720, 2 Parts deſcribed in one Day; and in one Hour it will deſcribe 71,67 Parts. But as $1 : \sqrt{2} :: 1720{,}2 : 2432{,}747$, the Parts deſcribed by the Comet in one Day; and ſo the Parts deſcribed by the Comet in one Hour will be 101,364.

12. Whence if the *Latus Rectum* L R of the Parabola be equal to 4 times the Radius S P of the Earth's Orbit, and we put $\overline{SP}^2 = 100000000$, the Area which the Comet will deſcribe each Day, by a Ray drawn to the Sun, will be $1216373\frac{1}{2}$ of thoſe Parts, and each Hour an Area of $50682\frac{1}{4}$ of thoſe Parts. To demonſtrate this we muſt conſider, that the Square of the Diameter of any Circle is to its Area as $1 : 0{,}7854 :: 4 : 3{,}14159$; therefore the Square of Radius or PM = 1. Whence the Area of the Circle is to the ſaid Square P M as 3,14159 to 1. And the Rectangle P L = 2. But the Parabolic Area $PLS = \frac{2}{3} PL = \frac{2}{3} \times 2 = \frac{4}{3}$. Hence this Area P L S is to the Area of the Circle as $\frac{4}{3}$ to 3,14159. And if the Velocity of the Comet and Planet at P were the ſame, the Time in which the Comet would deſcribe the Arch of the Parabola P L would be to the Time in which the Planet deſcribes its Orbit in the ſame Ratio of $\frac{4}{3}$ to 3,14159. But theſe

Velocities

Velocities are as $\sqrt{2}$ to 1; therefore the ſaid Times will be $\frac{4}{3} \times \frac{1}{\sqrt{2}}$ to $\frac{3,14159}{1}$, that is, as $\sqrt{\frac{16}{18}} = \sqrt{\frac{8}{9}}$ to 3,1459. Wherefore ſay, As $3,14159 : \sqrt{\frac{8}{9}} ::$ 365 D. 6 H. 9′ : 109 D. 14 H. 46′, the Time in which the Comet will deſcribe the Arch P L. If then $PS^2 = PM = 100000000$, we have the Parabolic Area P L S $= 133333333$ Parts deſcribed in 109 D. 14 H. 46′; and therefore the proportional Parts for a Day and Hour as above.

13. What thoſe diurnal and horary Areas are in different Parabolas may be thus ſhewn. Let $p\,r\,q$ be a Parabola ſimilar to the former P R Q; then will the Time T of deſcribing the Arch P R be to the Time t of deſcribing the ſimilar Arch $p\,r$, as the periodical Time S of deſcribing a Circle on P S to the Periodical Time p of deſcribing a Circle on $p\,s$, by the laſt Article. But $P : p :: PS^{\frac{3}{2}} : ps^{\frac{3}{2}} : R^{\frac{3}{2}} : r^{\frac{3}{2}} :: T : t$; alſo the ſimilar Areas $PRS = A$, and $p\,r\,S = a$, are as the Squares of their like Sides P S and p S; that is, $A : a :: R^2 : r^2$. Now ſince in the ſame Figure equal Spaces are deſcribed in equal Times, whatever Number of Days or Hours are contained in T and t, the Areas A and a will conſiſt of as many equal Parts reſpectively; and which therefore we may call the $\frac{1}{T}$ Part of A, and $\frac{1}{t}$ Part of a, or x and y; ſo that $x : y :: \frac{A}{T} : \frac{a}{t} :: \frac{R^2}{R\sqrt{R}} : \frac{r^2}{r\sqrt{r}} :: \sqrt{R} : \sqrt{r}$.

14. Let the Quadrantal Area P S R of the Parabola P R Q be divided into 100 equal Parts, that is, let A $= 100$; then $\frac{A}{100} = 1$ of thoſe Parts, and ſo $\frac{A}{100} : R^2$. Again, let N be the Number of thoſe Parts deſcribed in 1 Day; then will this diurnal Area be $N \times \frac{A}{100}$:

$\frac{A}{T}$

$\frac{A}{T} : N \times R^2 : \sqrt{R}$, (by *Art.* 13.) therefore $N : \frac{1}{R}$.

15. In like Manner it is ſhewn, that if the Quadrantal Area $p\,r\,S$ of the Parabola $p\,r\,q$ be divided into an 100 equal Parts, and $p\,S = r$, and $n =$ Number of thoſe Parts in the diurnal Area; then $n : \frac{1}{r^{\frac{3}{2}}}$. And ſo $N : n :: \frac{1}{R^{\frac{3}{2}}} : \frac{1}{r^{\frac{3}{2}}}$; $n = N \times \frac{1}{r^{\frac{3}{2}}}$, if $R = SP = 1$, or the Radius of the Earth's Orbit.

16. On theſe Principles the *Cometary Calculus* depends; for in any Parabolic Orbit the Quantity $n = N \times \frac{1}{r^{\frac{3}{2}}}$ is the diurnal Area, and may therefore be eſteemed the *mean Motion* or *Anomaly* of the Comet for a Day; which multiplied by the Time (expreſs'd in Days) before or after the Comet is *in Perihelio* at P, will give the whole mean Motion or Area P R Q S for any Place of the Comet Q in its Orbit. In order to this we muſt have the Time aſcertained from Obſervation when the Planet was *in Perihelio* at P, and alſo the Perihelion Diſtance S P from the Sun; as alſo the Place in the Ecliptic at the ſame time, the Poſition of its Nodes, and Inclination of its Orbit: All which Particulars for 24 Comets the Induſtry of the great Aſtronomer of this Age has ſupplied, *viz.* Dr. *Halley*, in his *Cometographia*; which I have tranſcribed, and added thereto the ſame Things for the laſt Comet, as they were determined by the Reverend Mr. *Betts*, from the Obſervations of Mr. Profeſſor *Bliſs* of *Oxford*, at the Obſervatory of the Right Hon. the Earl of *Macclesfield*, at *Sherbourn* in *Oxfordſhire*.

Plate LXII. Fig. 2.

17. From the Place of the Comet Q draw Q A perpendicular to the Axis; and let $a\,b$ be a Tangent to the Curve in the Point Q, and B Q drawn perpendicular thereto; then by the Nature of the Parabola we have $A B = S R$, the *Semi-Latus Rectum*. And putting the given Area $P Q S = a$, and $A Q = x$, we have $\frac{1}{12} x^3 + \frac{1}{4} x = a$, or $x^3 + 3x = 12\,a$; which Cubic Equation reſolved

resolved gives the Ordinate A Q, and thence we have PA; but $PA + PS = SQ =$ Distance of the Comet from the Sun, which therefore is given. Therefore in the Triangle S A Q, right-angled at A, we have S Q and A Q to find the Angle Q S A; and then P S Q the Angle from the *Perihelion* is known. When this is done, all the other Particulars are the same as in the *Planetary Calculus*.

18. These are the Principles or Elements of Calculation; which we shall now proceed to illustrate by Example, that so the *Praxis* may not remain so difficult and obscure as it has hitherto been; and we shall make choice of the last Comet for this Purpose, whose *mean Anomaly*, or diurnal *Area*, is in the first place to be determined.

19. In order to this, we have the constant mean Motion of a Comet moving in a Parabola, whose Perihelion-Distance $PS = R = 1 =$ Semidiameter of the Earth's Orbit, *viz.* $N = \frac{100}{109\,D.\,14\,H.\,46'} = 0.91228$, whose Logarithm 9.960128 is therefore always at hand for constant Use.

20. The Perihelion-Distance $PS = r = 0{,}22206$, and its Logarithm 9.346472, as in the Table, for the Comet of $174\frac{3}{4}$. But we have its mean Anomaly $n = N \times \frac{1}{r^{\frac{3}{2}}}$ (by *Art.* 15.); therefore to find n by Logarithms the Process is as follows:

The Logarithm of Perihelion-Distance	$r =$	9.346472
Which multiply by —	—	3
The Product is the Logarithm of	$r^3 =$	8.039416
Divide by 2, the Quotient is Log. of	$r^{\frac{3}{2}} =$	9.019708
Arithmetical Complement is the Log. of	$\frac{1}{r^{\frac{3}{2}}} =$	0.980292
To which add the Logarithm of	$N =$	9.960128
The Log. of mean Anomaly	$n = 8{,}718 =$	0.940420

21. Having

21. Having thus obtain'd the *diurnal Area*, if we multiply this by any Number of Days and Decimal Parts of a Day, it will give the Area P R Q S, or *mean Anomaly*, for the given Time. Thus let it be required for *January* 23 D. 6 H. 11′.

		D.	H.	M.
Then from the Time of Perihelion,	*Feb.*	19	8	12
Subduct the given Time,	*Jan.*	23	6	11
The Difference will be		27	2	1

Wherefore to Log. of diurnal Area 8.718 = 0.940420
Add the Log. of the given Time 27,0833 = 1.432702

The mean Anomaly requir'd, = 236,1 = 2.373122

22. Having therefore the Area P R Q S = 236,1, we can find A Q = x, from the Equation $x^3 + 3x = 12a$; for if when the Quadrantal Area P S R is 100, we put S R = x = 1, then 'tis plain, $x^3 + 3x = 1 + 3 = 4 = 12a$ in that Case. Therefore when the *mean Anomaly* is but $\frac{1}{100}$ Part of this, we have $x^3 + 3x = \frac{4}{100} =$ 0,04; which will be a constant Multiplier for reducing any given Anomaly to fit it for the Equation. Thus 0,04 × 236,1 = 9,444 = $x^3 + 3x$ in the present Case, which resolved according to the usual Methods gives x = 1,65 nearly.

23. Then by the Nature of the Parabola $\frac{\overline{AQ}^2}{4PS} =$ $AP = \frac{1,65 \times 1,65}{2} = 1,3612$. Also A P + P S = S Q = 1,8612, the Distance of the Comet from the Sun for the given Time. But to express this Distance in the same Parts as the Sun's mean Distance from the Earth contains 1,00000, we must consider that the Perihelion-Distance P S = 0,22206; whence S R = 0,44412. Wherefore say, As 1 : 0,44412 :: 1,8612 : 0,82650, the Expression required.

24. In the Right-angled Triangle Q A S, having all the Sides, we find the Angle Q S A = 62° 36½′; whence the obtuse Angle P S Q = 117° 32½′, which is the Heliocentric Distance of the Comet from the Perihelion.

Now

Now since the Perihelion is in ♎ 17° 12′ 55″, if we subduct 117° 33′ 30″, we have the Heliocentric Longitude in ♉ 19° 39′ 25″.

25. Also the *Descending Node* is in ♏ 15° 45′ 20″, from which subtract the Comet's Place now found, the Difference 147° 05′ 55″ is the Distance of the Comet from the Node. Let the Line of the Nodes be ☋ S ☊; then, since the Perihelion P is 151° 27′ 35″ distant from the Node ☊, it will be but 28° 38′ 25″ distant from the Node ☋. If then from the Angle Q S A = 62° 36½′ we deduct P S ☋ 28° 38′ 25″ = A S ☊, we shall have Q S ☊ = 30° 58′ 5″. Plate LXII. Fig. 3.

26. From Q let fall the Perpendicular Q N on the Line of Nodes; then in the Right-angled Triangle Q S N, having the Angle at S and the Side S Q, we can find Q N as follows:

As Radius	90° =	10.000000
To the Sine of the Angle	Q S N = 33° 58′ =	9.747374
So is the Side	S Q = 0,82650 =	9.917227
To the Length of the Side	Q N = 0,46200 =	9.664601

27. Again: In the Right-angled Triangle Q N D we have the Side now found Q N, and the Angle of the Inclination of the Comet's Orbit, Q N D = 47° 9′, to find the Side or Perpendicular Q D. Thus say,

As Radius	90° =	10.000000
Is to the Sine of Inclination	Q N D = 47° 9′ =	9.805138
So is the Side	Q N = 0,46200 =	9.664601
To the Perpendicular	Q D = 0,33860 =	9.529739

28. We can now find the Heliocentric Latitude of the Comet, or the Angle Q S D; for

As the Side	Q S = 0,82650 =	9.917227
Is to the Side	Q D = 0,33860 =	9.529739
So is Radius	90° =	10.000000
To Sine of Helioc. Lat.	Q S D = 24° 12′ =	9.612512

29. To find the Comet's Curtate Distance from the Sun, *viz.* S D, we have this Analogy from the Right-angled Triangle Q S D.

As Radius $90° = 10.000000$
To the Sine of the Angle $SQD = 65° 48' =$ 9.960052
So is the Side $SQ = 0,82650 =$ 9.917227

To the Curtate Distance $SD = 0,75380 =$ 9.877279

30. To find the Side N D in the Right-angled Triangle QN D, say,
As Radius $90° = 10.000000$
To Co-sine of Inclination $DQN = 42° 51' =$ 9.832616
So is the Side $QN = 0,46200 =$ 9.664601

To the Side $DN = 0,31420 =$ 9.497217

31. Then in the Right-angled Triangle N S D we can find the *Heliocentric Place of the Comet in the Ecliptic*, or Angle D S N, thus:
As the Curtate Distance $SD = 0,75380 =$ 9.877279
To the Side $ND = 0,31420 =$ 9.497217
So is Radius $90° = 10.000000$

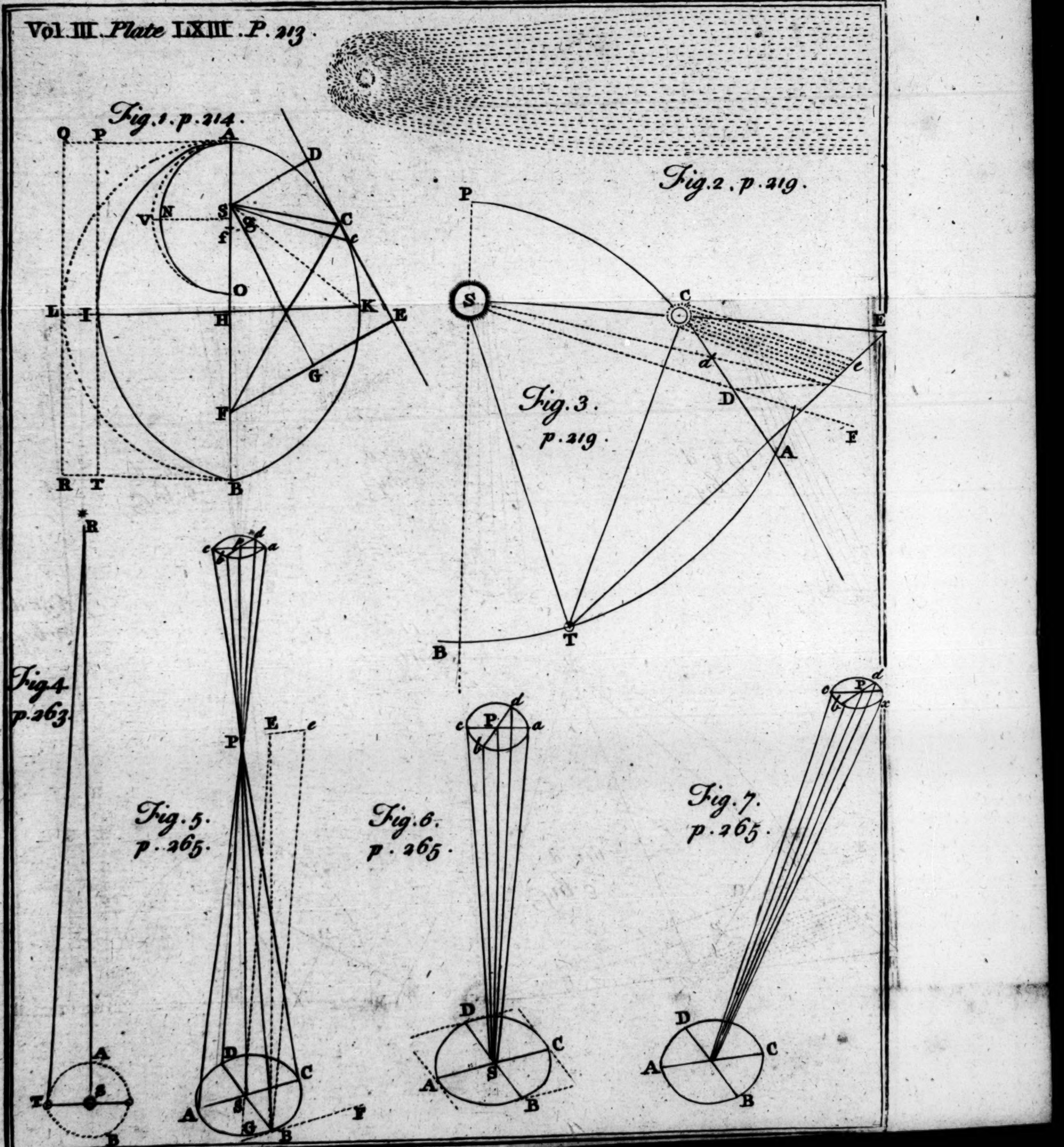
To the Sine of the Angle $DSN = 24° 38' =$ 9.619938
Therefore to the Place of the Node ☊, ♉ $15° 45' 20''$
Add the Angle now found $24\ 38\ 00$

The Sum is the Helioc. Place in the Ecliptic, ♊ $10\ 23\ 20$

32. The next Thing to be done is to find the Place of the Sun, and consequently of the Earth in her Orbit for the given Time; which is calculated from the Tables in the usual Method as follows:

	Mot. of the Sun. S.	°	′	″		*Mot. of Perihelion.* S.	°	′	″	
1741.	9	21	1	58	——	3	8	13	30	
3.	11	29	17	00				2	30	
Jan. 23.	00	22	40	12					3	
Hours 6			14	47						
Min. 11				27						
Mean Mot.	10	13	14	24	——	3	8	16	3	
Equat. add.		1	7	39		10	13	14	24	
True Place	10	14	22	3		7	4	58	21	M. Anom.

Vol. III. Plate LXIII. P. 213.



33. The Sun's Place being found in ♒ $14^\circ\ 22'\ 03''$, the Earth's Place will be in the opposite Part of the Ecliptic, *viz.* in ♌ $14^\circ\ 22'\ 03''$ at T. If therefore from this we subtract the Comet's Heliocentric Place at H, we shall have the Arch H T $= 63^\circ\ 49'\ 43'' =$ D S T, the *Angle of Commutation*. And as the Earth's mean Anomaly is 7 S. $4^\circ\ 58'\ 23''$, the Logarithm of the Earth's Distance S T will be 9,993947. But S D is also known; therefore we can find the Angle D T S, or Elongation of the Comet from the Sun, thus:

As the Sum of the Sides $ST + SD = 1{,}74000 =$ 0.240549
Is to their Difference $ST - SD = 0{,}23240 =$ 9.366236

So the Tang. of $\frac{1}{2}$ the Ang. $\frac{D-T}{2} = 58^\circ 00' =$ 10.204211

To Tang. of $\frac{1}{2}$ their Diff. $\frac{D-T}{2} = 12^\circ\ 03' =$ 9.329898

34. Hence $58^\circ + 12^\circ\ 03' = 70^\circ\ 03' =$ T D S, and $58^\circ - 12^\circ\ 03' = 45^\circ\ 57' =$ S T D, or Longitude of the Comet from the Sun; which added to the Sun's Place at I gives the *Geocentric Longitude* of the Comet at L, in ♈ $00^\circ\ 19'$. And to find the *Geocentric Latitude*, or Angle D T Q, we have this Analogy:

As the Sine of Commutation $TSD = 64^\circ\ 00' = 9.953650$
Is to the Sine of Elongation $STD = 45^\circ\ 57' = 9.856568$
So is Tang. of Helioc. Lat. $DSQ = 24^\circ\ 12' = 9.652650$

To the Tang. of Geo. Lat. $DTQ = 19^\circ 46' = 9.555568$

35. Thus you have the whole Process of Calculation, as it relates to the Phænomena of a Comet moving in a Parabola near the Vertex, and is the same with that used for the Planets (from the 25th *Article* inclusive). And though it is certain (from what will be shewn by and by) that this Comet does not describe a *Parabola*, but an *Ellipsis*, yet the computed Longitude and Latitude are the same which the Comet was observed to have at that very Time; whence the Accuracy of this Method sufficiently appears: But as it is thus limited to a Parabola, and only one small Part of that, and cannot be extended to determine the Axis of the Orbit, or the Time of its Revolution, I shall here supply this

great Deficiency by ſhewing a direct and geometrical Method of Computation of all the Phænomena of a Comet moving in any Conic Section, which was firſt invented by M. *Bonguer*, in *Mon. Pariſ.* An. 1733; which Method I ſhall explain, illuſtrate, and exemplify in the following Articles.

Plate LXIII. Fig. 1.

36. Let A K B I be the Trajectory of a Comet, A B its longeſt Axis, I K the ſhorteſt; S, F, the two *Foci*, in one of which the Sun is at S; C the Place of the Comet, C S its Diſtance from the Sun; D C E a Tangent to the Curve in the Point C; C c the Space paſſed over by the Comet in a ſmall Particle of Time; S D, F E, Perpendiculars from the Foci to the Tangent: And draw S G parallel to D E, and join F C. Alſo let A N O be the elliptic Orbit of any Planet; S, f, its Foci. Laſtly, let A L B be a Circle deſcribed on the longer Axis, A B; A P T B a Rectangle about the Ellipſis A I B; and A Q R B as the Square about the Circle A L B; and put S C $= a$, S D $= b$, C $c = e$, the Time in which it is deſcribed $= f$. The longer Axis of the Cometary Orbit A B $= x$, of the Planetary Orbit A O $= q$, the Circle deſcribed on the ſame Axis A V O $= p$; the periodical Time of the Comet $=$ t, and that of the Planet $= n$.

37. The Space C deſcribed, the Diſtance S C, and the Angle S C D, are all known by Obſervation, and therefore given Quantities. The mean Diſtance of the Comet is A H $= \frac{1}{2} x$, and of the Planet is A g, $= \frac{1}{2} q$. And becauſe the *Squares of the Periodical Times are as the Cubes of the mean Diſtances*, we have $\frac{1}{8} q^3 : \frac{1}{8} x^3 :: n^2 : t^2$; and therefore $t = \frac{nx}{q} \sqrt{\frac{x}{q}}$. (*Ann.* XXXIV. II.)

38. It is neceſſary now to find another Expreſſion of the periodical Time t, thus: Becauſe C c is a very ſmall Portion of the Orbit, it may be eſteemed a Right Line, and the Sector C S c as an evaneſcent Triangle, whoſe Area $\frac{1}{2}$ S D × C $c = \frac{1}{2} b e$ is given; but as the Area $\frac{1}{2} b e$ is to the Time f, ſo is the whole Area of the Ellipſis A K B I $=$ A to the whole periodical Time t; that is, $t = \frac{f}{\frac{1}{2} b e} \times A$.

39. Now

39. Now in order to determine the Area A, we must find the Semi-conjugate HK, thus: Because $AB = SC + FC$, therefore $FC = x - a$; and by similar Triangles SDC and FEC we have $SC : SD :: FC : FE$, that is, $a : b :: x - a : \frac{bx - ab}{a} = FE$; and therefore $FG = (FE - GE) \frac{bx - 2ab}{a}$. Again, $SC : CD :: FC : CE$; or $a : \sqrt{a^2 - b^2} :: x - a : \frac{x - a}{a} \sqrt{a^2 - b^2}$. Hence DE or $SG = CE + CD = \frac{x - a}{a} \sqrt{a^2 - b^2} + \sqrt{a^2 - b^2} = \frac{x}{a} \sqrt{a^2 - b^2}$. But $FG = \frac{bx - 2ab}{a}$; therefore $FS = \sqrt{SG^2 - FG^2}$

$$= \frac{\sqrt{b^2 x^2 - 4ab^2 x + 4a^2 b^2 + a^2 x - b^2 x^2}}{a^2} = \frac{\sqrt{a^2 x^2 - 4ab^2 x + 4a^2 b^2}}{a}.$$

And therefore $SH = \frac{1}{2} SF = \frac{1}{2} \frac{\sqrt{a^2 x - 4ab^2 x + 4ab^2}}{a^2}$.

40. Moreover, by the Nature of an Ellipsis, $SK = AH = \frac{1}{2} x$, and therefore $\sqrt{SK^2 - SH} = HK = \sqrt{\frac{1}{4}x^2 - \frac{a^2 x^2 + 4ab^2 x - 4a^2 b^2}{4a^2}} = \frac{b}{a} \sqrt{ax - a^2}$; therefore $IK = 2HK = \frac{2b}{a} \sqrt{ax - a^2}$. Consequently, $\frac{xb}{a} \sqrt{ax - aa} = APTB$, the Semi-Area of the Ellipse. Let $Q =$ Diameter of the Circle ALB, and P its Periphery; then since $\frac{1}{2} LH \times P = \frac{1}{4} QP$ is the Area of the Circle, we shall have $Q^2 : \frac{1}{4} QP$ ($:: \frac{1}{2} Q^2 : \frac{1}{8} QR$) $:: AQRB : ALB :: APTB : AIB :: q^2 : \frac{1}{4} qp$. That is, $q^2 : \frac{1}{4} qp :: \frac{xb}{a} \sqrt{ax - aa} : \frac{bpx}{4aq} \sqrt{ax - a^2} = AIB$. But $2AIB = AIKB = A = \frac{bpx}{}$

$\frac{b p x}{2 a q} \sqrt{a x - a a}$; therefore the above expression $t = \frac{f}{\frac{1}{2} b e} A = \frac{f p x}{a e q} \sqrt{a x - a a}$. Then $t = \frac{n x}{q} \sqrt{\frac{x}{q}} = \frac{f p x}{a e q} \sqrt{a x - a a}$. And, reducing the Equation, we get $x = \frac{a f^2 p^2 q}{f^2 p^2 q - a e^2 n^2} = AB$, the principal Axis of the Section, or Trajectory of the Comet.

41. If we substitute this Value of x in the Equation above for t, we shall have $t = \frac{p^3 f^3 n^2 a^{\frac{3}{2}}}{(q f^2 p^2 - a e^2 n^2)^{\frac{3}{2}}} =$ the Periodical Time. Also because the Conjugate $IK = \frac{2b}{a} \sqrt{a x - a a} = c$, therefore $x = \frac{c a^2 + 4 b^2 a^2}{4 b^2 a} = \frac{a f^2 p^2 q}{f^2 p^2 q - a e^2 n^2}$; whence $c = IK = 2 b e n \sqrt{\frac{a}{f^2 p^2 q - a e^2 n^2}}$.

42. From these Equations it plainly appears, that when the Velocity of the Comet is such that $f^2 p^2 q = a e^2 n^2$, the Axis x is *infinite*, and consequently the Trajectory will be a *Parabola*; but if $e a^2 n^2$ be greater than $f^2 p^2 q$, it will be an *Hyperbola*; in both which Cases the Comet can never return: But in all Cases where $f p^2 q$ is greater than $e a^2 n^2$, the Comet will describe *Ellipses*; among which we reckon that of the *Circle*, where $x = 2 a = \frac{a f^2 p^2 q}{f^2 p^2 q - e a^2 n^2}$, and hence $e = Cc = \frac{f p}{n} \sqrt{\frac{q}{2 a}}$, the Arch of the Circle described in one Day.

43. Let the Planet we supposed to describe the Ellipsis ANO be the Earth; then will its mean Distance $\frac{1}{2} q = 100000$ equal Parts; and so $q = 200000$, and $p = 628318$. Also the Periodical Time $n = 1$ Year; and then if Cc be the Space described in

in one Day, we have $f = \frac{1}{365,256} = 0.0027378$.
Then also the other Expressions will become for the Principal Axis $x = \frac{591826599535 \times a}{591826599535 - ae^2}$, and for the Perodical Time $t = \frac{4750560000 \times a^{\frac{3}{2}}}{\overline{591826599535 - ae^2}|^{\frac{3}{2}}}$.

44. Hence it appears, that if Observations could be made sufficiently exact to determine the Distance of the Comet, and the Space it moved over in its Orbit in one Day, then the Axes of the Orbit and the Periodical Time of the Comet may as well be computed as those of a Planet; but this is a Matter of the greatest Nicety, and of course the greatest Difficulty, because the *elliptic Orbit* of a Comet, if it be such, can scarcely be distinguished by Observation (however well made) from a Parabolical Orbit, in all that Part of the Orbit which the Comet describes during its Appearance. Hence the Quantity ae^2 will generally come out either equal to, or greater than the Number 591826599535, and so gives the Axis x infinite or negative: And if it chance that ae^2 be less than the said Number, then if a or e be not defined to the last Degree of Exactness, the Axis x, and Periodical Time t, will be very different from the Truth. But more of this in another Place.

45. A *Parabola* therefore is fully sufficient to account for all the Circumstances and Phænomena of a Comet's Motion during the Time of its Appearance; as Sir *Isaac* has shewn with respect to the Comets of 1661, 1680, 1682, 1683, 1723, and Mr. *Betts* for the last Comet of 174$\frac{3}{4}$. And that the Reader may see the wonderful Agreement between the Theory (though grounded on the *Parabolical Hypothesis*) and the Phænomena of Longitude and Latitude of the Comet by Observation, I shall here subjoin a Table exhibiting the same both by Computation and Observation, and the Differences between them severally for each respective Time of Observation.

1743.	Equal Time at *Oxford*. D.	H.	′	Longit. Comet observed.	°	′	″	North Latit. observed. °	′	″	Longit. Comet computed.	°	′	″	North Latit. computed. °	′	″	Diff. in Long. ″	Diff. in Lat. ″
Dec.	23	5	32	♈.	14	10	2	17	33	11	♈.	14	10	3	17	33	37	1 —	26 —
	27	5	7½	♈.	12	2	25	17	51	29	♈.	12	2	26	17	51	47	1 —	18 —
	28	5	1½	♈.	11	32	11	17	55	54	♈.	11	32	14	17	56	8	3 —	14 —
	31	4	44	♈.	10	4	57	18	9	3	♈.	10	5	16	18	8	53	19 —	10 +
		5	53	♈.	10	4	11	18	9	57	♈.	10	3	55	18	9	6	16 +	31 +
Jan.	12	9	10	♈.	4	52	5	18	59	37	♈.	4	52	24	18	59	13	19 —	24 +
	12	6	20	♈.	4	31	40	19	2	31	♈.	4	31	13	19	2	49	27 +	18 —
		8	20	♈.	4	29	27	19	3	32	♈.	4	26	6	19	3	12	21 +	20 +
	16	6	33	♈.	3	18	43	19	15	47	♈.	3	18	27	19	15	13	16 +	34 +
		8	00	♈.	3	17	31	19	16	7	♈.	3	17	00	19	15	30	31 +	37 +
	23	6	11	♈.	0	19	45	19	42	30	♈.	0	19	16	19	42	1	29 +	29 +
	23	7	29	♈.	0	17	58	19	42	47	♈.	0	17	45	19	42	12	13 +	35 +
Feb.	5	7	31½	♓.	21	52	37	19	35	00	♓	21	52	56	19	34	42	19 —	18 +
	11	6	37½	♓.	14	42	45	17	23	30	♓.	14	42	58	17	24	5	13 —	35 —
	12	6	33	♓.	13	10	36	16	38	40	♓.	13	10	52	16	39	17	16 —	37 —
	13	6	25	♓.	11	32	50	15	43	45	♓.	11	33	16	15	44	16	26 —	31 —
	16	23	41½	♓.	5	9	14	10	17	40	♓.	5	9	1	10	18	8	13 +	28 —
	17	23	35	♓.	3	37	37	18	15	39	♓.	3	37	11	18	16	3	26 +	24 —

46. Having

46. Having thus shewn the several Affections of a Comet's Motion, I shall conclude with a Word or two in relation to their *Tails*. The Atmosphere of Comets consisting of a very fine Vapour, will, when the Comet is in its *Aphelion*, be nearly spherical, and its Density greatest. As the Comet approaches the Sun, the Sun's Heat enters the Atmosphere, and rarifies it by Degrees, causing at the same time the finest Part to rise from the Comet, like the Flame from a Candle, towards the Parts averse from the Sun; and as the Comet comes nearer and nearer the Sun, this Fume will rise and extend itself to greater and greater Lengths, and make what is called the *Tail* of the Comet; so that when they are viewed with a Telescope, the *Nucleus*, Atmosphere, and Tail of a Comet appear much like what is represented in the Figure. Plate LXIII. Fig. 2.

47. The Length of the Tail is thus found by Observation. Let S be the Sun, C the Comet, T the Earth, C *e* the Comet's Tail; draw T S, T C, S C, and T *e* touching the End of the Tail, and meeting the Line S C produced in E. The Place of the Sun and Comet being known, the Angle T C E is known (for $TCE = STC + CST$). Also the Angle of Deviation E C *e* is known from Observation; whence T C *e* is known. Moreover the Angle T C *e* is known also by Observation. Therefore in the Triangle T C *e*, having the two Angles T C *e*, and C T *e*, and the Side T C, (from the Theory) we can find the Side C *e*, which is the Length of the Tail. And thus they have been found to be 40, 60, and 80 Millions of Miles. Fig. 3.

48. Draw S *e* cutting the Comet's Orbit in *d*; then because the whole Motion of a Particle from C to *e* may be resolved into two Motions C *d*, and *e d*, 'tis plain, since *d e* is that directly averse to the Sun, the Comet would have possess'd the Point *d* when the Particle at *e* first rose from the *Nucleus*, if the Motion had been every where in the Direction of S *e*, as the Line S *e* kept moving from S *e* to S E.

49. But since this is not the Case, but the Particles move in the oblique Direction C *e*, therefore parallel to C *e* draw S F cutting the Orbit in D, and join D *e*; then will the compound Motion C *e*, arising from the

progressive

progressive Motion of the Comet in the Direction C D, and its Motion of Ascent in the Direction C *e*, give the Point D for the Comet's Place when first the Particle at *e* began to ascend from the *Nucleus*.

50. Now the Time in which the Comet describes any given Part of its Orbit D C may be found from the Theory, and consequently the Time of the Ascent of the Tail of the Comet from the *Nucleus* to the Extremity *e*. Thus I have finished a *compleat Compendium of the* NEWTONIAN *Philosophy of* COMETS.

Jam patet horrificis quæ sit via flexa Cometis;
Jam non miramur barbati Phænomena Astri.

DR. HALLEY.

Mathematical Principles OF CHRONOLOGY.

AN APPENDIX TO LECTURE XI.

Of TIME, *and its* MEASURE *by the* Celeſtial *Motions. Of the* YEAR Tropical *and* Sydereal, *and the Quantity of each. The Time of the* EQUINOXES *and* SOLSTICES *determined by* Calculation. *Of* DAYS, Natural *and* Artificial. *The* EQUATION *of* Time *explained. Of* WEEKS. *Of* MONTHS, Periodical *and* Synodical. *Of* Old *and* New Style. *Of* CYCLES; *the* CYCLE *of the* SUN, *and* Dominical LETTERS; *the* METONIC Cycle, *or* Cycle *of the* MOON, *and* GOLDEN NUMBERS. *The* Cycle *of* INDICTION. *The* Dionyſian PERIOD, *or* Paſchal Cycle. *The* Julian PERIOD. *The* Aſtronomical Principles *of* CHRONOLOGY, *by Sir* ISAAC NEWTON, *explained and exemplified.*

1. I SHALL here give the Reader an Idea of the YEAR, as the grand and original Measure of Time, and derived from the Astronomical Principles of the Earth's Motion; and then afterwards consider its Subdivisions and Distributions into lesser Parts, as *Months*, *Days*, *Hours*, *Minutes*, *Seconds*, *Thirds*, &c. for the Purposes of common Life, and the Uses of Chronology, History, and other Sciences.

2. Time is in itself a flowing Quantity, measuring the Duration of Things; and its Flux is always equable and uniform; and therefore to estimate the Quantity of Time, we should measure it by something that is in its own Nature always of one and the same Tenor. For this Purpose we have no Expedient so convenient as that of Motion; and because the Measure of Time ought to be permanent, we can find no other Motion fit for this Purpose but that of the Heavenly Bodies.

3. AMONG these, none of the Motions are so obvious to Every-body, and plain to common Sense, as that of the *Sun* and *Moon*; which therefore have been agreed upon by the Consent of all Nations for this End; and indeed this seems to have been a principal Part of the Design of their Creation. For we are told they were appointed for

for *Times and Seaſons, for Days and for Years*, Gen. i. That is, the Sun, by his Diurnal Motion, affords the Meaſure for DAYS, and by his annual Motion, the Meaſure for Years; and the Moon, by her Revolutions, gives the Meaſure of another Part of Time we call MONTHS.

4. FOR it is a compleat Revolution of thoſe Luminaries that conſtitutes a Year, a Month, and a Day in the Abſtract, or abſolutely conſidered. Hence it is neceſſary to conſider the Point which is to be eſteemed the *Exordium* or Beginning of theſe Revolutions. And this, with reſpect to the Annual Revolution of the Sun, is fixed in that Point of the Ecliptic, which is the Beginning of *Aries*; and the Time which the Sun takes in going from and returning to this Point again, is called a YEAR.

5. ALSO the Space of Time which the Sun takes to compleat one Revolution about the Earth, is call'd a *Natural Day*, or the *Nychthemeron*, including a Common Day and Night; which Space of Time is ſubdivided into 24 equal Parts, we call HOURS and each of theſe are again ſubdivided into 60 equal Parts or Minutes; each of theſe again into 60 other equal Parts called Second Minutes, or Seconds; each of theſe

into

into Thirds, and ſo on in a Sexageſimal Subdiviſion for any leſſer Parts of Time.

6. Now, if this firſt Point or Beginning of *Aries* were fix'd, each Annual Revolution of the Sun would be conſtantly the ſame, and therefore a juſt and equal Meaſure of the Year, which is call'd the *Periodical Year*, as being the Time of the Earth's Period about the Sun; and which conſiſts of 365 D. 6 H. 9′ 14″. For ſo long is the Earth in departing from any fixed Point in the Heavens, and returning to the ſame again.

7. BUT ſince, as we have ſhewn (*Annot.* CXLI.) the ſeveral Points of the Ecliptic have a retrograde Motion, 'tis eaſy to underſtand, that by this Receſſion of the Equinox it will, as it were, meet the Sun, and cauſe that the Sun ſhall arrive to the Equinox, or firſt Point of *Aries*, before his Revolution is completed. And therefore this Space of Time (which is call'd the *Tropical Year*) is not ſo long as the former; for by Obſervations made at the Diſtance of many Years of the Time of two Equinoxes, and dividing the Time elapſed between by the Number of Revolutions, the Quotient will ſhew the Quantity of this *Tropical Year* to be 365 D. 5 H. 48′ 57″; which is 20′ 17″ leſs than the Periodical Year.

8. THE

Vol III. Plate LXIV. P. 225.

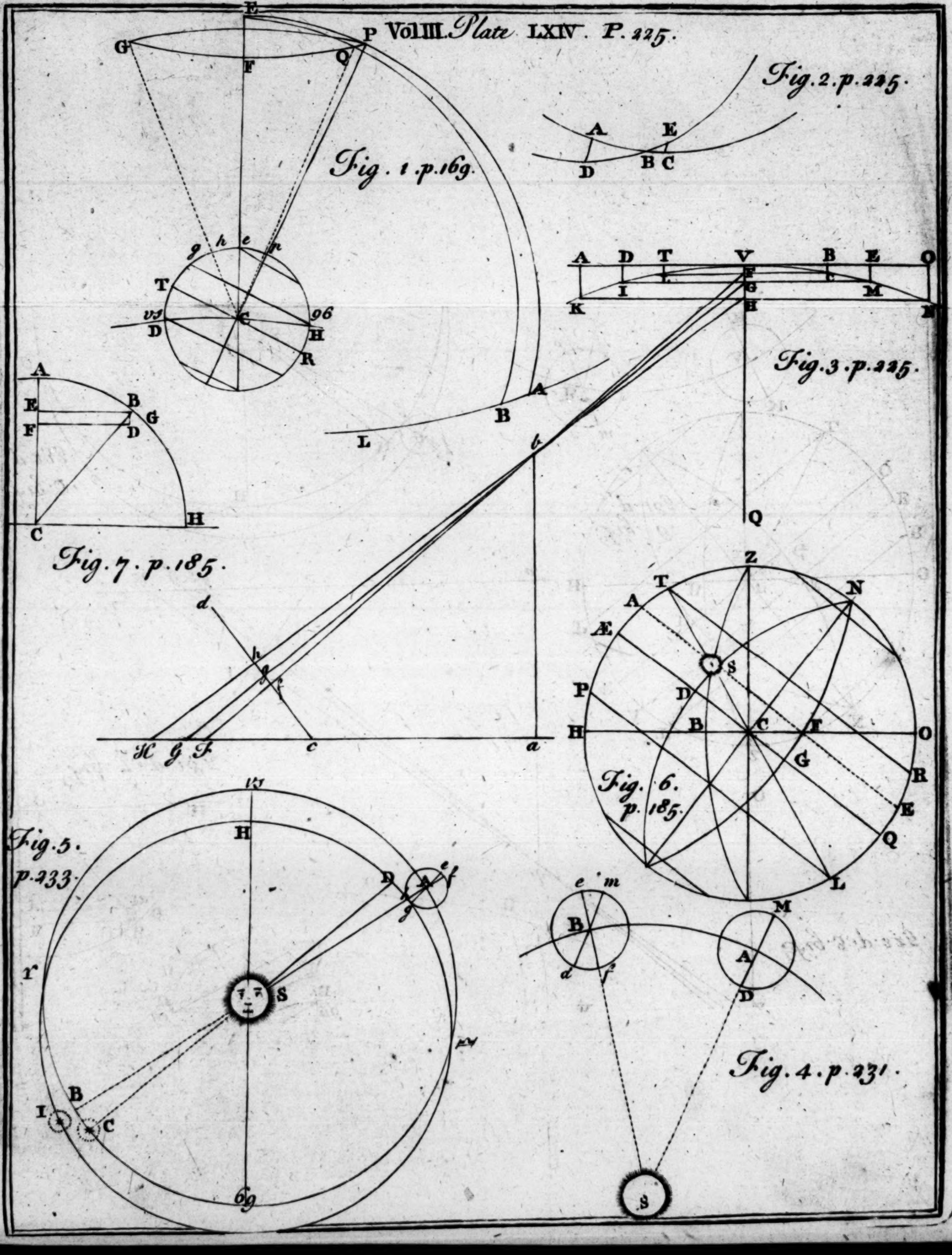


Fig. 1. p. 169.

Fig. 2. p. 225.

Fig. 3. p. 225.

Fig. 4. p. 231.

Fig. 5. p. 233.

Fig. 6. p. 185.

Fig. 7. p. 185.

8. THE Beginning of the Year, or Time when the Sun enters the Equinox, is thus determined by Obſervation. Let ABC be a Portion of the Equinoctial, and DBE an Arch of the Ecliptic; then with a very nice Inſtrument take the Meridian Altitude of the Sun, the Day before and after the Equinox; the Difference between theſe Altitudes and that of the Equator will be the Sun's Declination on thoſe two Days, which ſuppoſe to be AD and EC; which being thus known and the Angle of Obliquity ABD = EBC = 23° 29′, we find the Arch DB and EB; and therefore we ſay, As DB + EB is to DB, ſo is 24 Hours to the Time between the firſt Obſervation and Moment of the Sun's Ingreſs to the Equinoctial Point B. Plate LXIV. Fig. 2.

9. But the Quantity of the Tropical Year is better defined from a Calculation of the Moments of the Solſtices. The Invention of which curious and moſt certain Method was owing to our late celebrated Dr. *Halley*; and is founded on an eaſy Obſervation, and therefore practicable by any Perſon but moderately ſkilled in the Conic Geometry. The Method is as follows: Let AVO repreſent a ſmall Portion of the Tropic, which the Ecliptic KVM touches in the Solſtitial Point V. Suppoſe the Sun Fig. 3.

at ſeveral Times near the Solſtice be in the Points K, I, L, V, C, M, N, then will the Right Lines T L, I D; B C, E M, &c. (perpendicular to the Tropic A O) be the Deficiencies of the Sun's Declination at thoſe Times from his greateſt Declination in V.

10. AND from the Elements of Geometry, the Subtenſes T L, D I, &c. of the Angle of Contact A V K, are as the Squares of the Conterminal Arches V L, V I, &c. that is, of the Lines V T, V D, &c. which are nearly equal to thoſe Arches. Now when the Sun is in L, Part of its Path that Day will be the Line L C; and when in M, the Line I M, drawn parallel to A O. Let V Q be Part of the Solſtitial Colure; then we have V T = L F, and V D = G I, &c. alſo V F = T L, V G = D I, &c. whence $LF^2 : IG^2 :: VF : VG$, &c. ſo that the Figure K V N has really the Property of a Parabola, and may be taken for ſuch, without any ſenſible Error.

11. THEREFORE let three Points F, G, H, in the Axis V Q be determined by Obſervation thus: Let *a b* be an upright Object, *a c* the Ground or Horizon, and *c d* a Plane ſet nearly perpendicular to the Sun's Rays at Noon. Then let the Points *b*, *f*, *g*, on the Plane mark the Shadow of the Apex *b*, on three ſeveral Days at Noon; ſuppoſe

two

two before, and one after the Solstice. By this Means we have the Proportion of Distance between the Points F H and F G, for as $fh : fg :: FH : FG$. By the first Observation from the Point H the Sun's Place at K is given; by the second, from the Point F, we have the Place at L; and by the third having G, we have the Point M in the Curve.

12. Now let the Time between the first and second Observation A T ($= KL$) $= a$; the Time between the second and third Observation T E ($= LVM$) $= b$, $FH = c$, $FG = d$, and $TV = x =$ the Time between the second Observation and the Moment of the Solstice, to be found. Then $AV = a + x$, and $VE = b - x$; and let the *Latus Rectum* of the Parabola be p. Then (*per Conics*) we have $x^2 = VF \times p$, and therefore $VF = \frac{x^2}{p}$. In like Manner $VH = \frac{a^2 + 2ax + x^2}{p}$ and $VG = \frac{b^2 - 2bx + a^2}{p}$, therefore FH ($= VH - VF = \frac{a^2 + 2ax}{p} = c$, and FG ($= VG - VF$) $= \frac{b^2 - 2bx}{p} = d$; wherefore $p = \frac{a^2 + 2ax}{c} = \frac{b^2 - 2bx}{d}$; and redu-

cing the Equation, we have $x = \frac{b^2 c - a^2 d}{2ad + 2bc}$ $=$ T V, the Time required.

13. But if the Order of the Obſervations be ſuch, as that the Obſervation of the Shadow of the *Gnomon* in f is exactly in the Middle between thoſe of the Shadow in h and g; then will A T = T E, and ſo $a = b$, and the Equation will become $x = \frac{ac - ad}{2d + 2c}$ = T V; which gives this Analogy, $2d + 2c : c - d :: a : x$, that is, 2 F G + 2 F H : G :: A T : T V.

14. I shall illuſtrate this Calculation by an Example of each Caſe. In the Year 1500, *Bernard Walker*, in the Month of *June*, at *Nuremberg*, obſerved the Chord of the Sun's Diſtance from the Zenith by a large Inſtrument, as follows:

June 2,	45467	and	*June* 8,	44975
June 9,	44934		*June* 12,	44883
June 16,	44990		*June* 16,	44990

The Differences of theſe Chords are equal very near to the ſmall Diſtances F G and F H; therefore $c = 533$, and $d = 56$, and $c - d = 477$; and ſince the Time was 7 Days between Obſervation, therefore $a = 7$. Whence we have 1178 : 477 :: 7 : 2 D. 20 H.

20 H. 2′, which added to the Time of the middle Obſervation, gives *June* 11 D. 20 H. 2′ for the Time of the Solſtice.

15. AGAIN, by the other three Obſervations, we have $c = 107$, $d = 92$, $c - d = 15$, and $a = 4$; wherefore ſay, As 398 : 15 :: 4 D. = 96 H. : 3 H. 37′, the Difference between the 2d Obſervation, *June* 12, and the Moment of the Solſtice, which therefore muſt be *June* 11 D. 20 H. 23′, which is but 21′ different from the former. The Time of the Tropic therefore, in *Anno* 1500, we may conclude was *June* 11 D. 20 H. 12′.

16. WE will now give an Example of the former Method by the Shadow of a Gnomon 55 Feet high, which *Gaſſendus* at *Marſeilles* made uſe of for determining the Proportion of the Gnomon to its Solſtitial Shade. This he did in the Year 1636, and the Experiments were as follow;

June			
19	New Style, The Shadow	31766	Parts, of which the *Gnomon* was 89428.
20		31753	
21		$31751\frac{1}{2}$	
22		31759	

Here indeed the End of the Shadow, inſtead of being received on the Plane *c d*, perpendicular to the Rays, was taken on

the horizontal Line, where the Points f, g, h, are referred to F, G, H, in three of the Obſervations; yet is the Ratio between FH and FG the ſame nearly as the Ratio between fh and fg, becauſe the Rays at that Diſtance from b, in ſo ſmall an Angle, differ little from parallel Rays.

17. HENCE the Caſe of the Problem is ſtill the ſame. Therefore, let the Shadow, on *June* 19, be $aH = 31766$; on the 21ſt, $aF = 31751$; and on the 22d, $aG = 31759$; then $2c = 30$, $2d = 16$, $c - d = 7$, and $a = 2$, $b = 1$; then the Theorem $\frac{cb^2 - da^2}{2ad + 2bc} = 0{,}274 = 00$ D. 17 H. 25′. which is the Time by which the Solſtice preceded the ſecond Obſervation. The Solſtice therefore was on *June* 20 D. 17 H. 25′, *N. S.* or *June* 10 D. 17 H. 25′ *O. S.*

18. THE Difference between the Time of this and the other Solſtice is 1 D. 2 H. 47′: of which 1 D. 1 H. 12′ ariſes from the Deficiency of the Length of the Tropical Year from that of the *Julian* Year, (as will by and by appear) and the other Part 1 H. 45′ from the Progreſſion of the Sun's Apogæum during that Space of Time, *viz.* 136 Years.

19. THE

19. THE DAYS are the next Part of Time we ſhall conſider. Theſe may be divided into *Solar* and *Sidereal* Days. The *Solar Day* is that Space of Time which intervenes between the Sun's departing from any one Meridian, and its Return to the ſame again. But a *Sidereal Day* is the Space of Time which happens between the Departure of a Star from and its Return to the ſame Meridian again. And each of theſe are divided into 24 equal Parts, or *Hours*.

20. BECAUSE the Diurnal Motion of the Earth about its Axis is equable, every Revolution will be performed in the ſame Time; and therefore all the *Sidereal Days*, and the Hours of thoſe Days, will be equal. And on the other hand, the *Solar Days* are all unequal, and that on two Accounts, *viz.* becauſe of the Elliptic Figure of the Earth's Orbit, and becauſe of the Obliquity of the Ecliptic to the Equator.

21. THIS will appear as follows. Let S be the Sun, A B a Part of the Ecliptic, A the Centre of the Earth, and M D a Meridian whoſe Plane paſſes through the Sun. Now in the Time of one Revolution about its Axis, let the Earth be carried about the Sun from A to B, and then the Meridian Plate LXIV. Fig. 4.

will be in the Poſition *m d*, parallel to the former M D. But 'tis plain, the Meridian *m d* is not yet directed to the Sun, nor will not, till by its angular Motion it has attained the Situation *e f*, deſcribing the Angle *e* B *m* = B S A; whence it appears that all the *Solar Days* are longer than the Time of one Revolution, or *Sidereal Day*.

22. If the Earth revolved in the Plane of the Equator, and in a Circle about the Sun, then would the Angle A S B, and conſequently the Angle *e* B *m* be always of the ſame Quantity, and therefore the Time of deſcribing the ſaid Angle *e* B *m* would always be equal, and ſo all the ſolar Days would be equal among themſelves. But neither of theſe two Caſes have Place in Nature.

23. FOR by the Earth's Theory, founded on the niceſt Obſervations, the Orbit is an *Ellipſis*, and therefore (as we have ſhewn) her annual Motion cannot be equable, or the Angle A S B deſcribed in the ſame Space of Time will not be equal; for in the Aphelion, the Velocity of the Earth will be leſs than in the Perihelion, therefore alſo the Arch A B will be leſs, and conſequently the ſimilar Arch *e m*, and therefore alſo

alſo the Time of deſcribing it ; whence it appears, the Part of Time to be added to the Sidereal Day, to compleat the Solar Day, is always variable.

24. THE other Part of the Equation of Time (and moſt conſiderable) is that which ariſes from the Plane of the Earth's Orbit or Ecliptic being inclined to that of the Equator or Plane of the Diurnal Motion ; to explain which, let ♈ ♑ ♎ be a Semicircle of the Ecliptic, and ♈ H ♎ of the Equinoctial, S the Centre of the Sun, and A that of the Earth in the third Quarter of the Ecliptic ; *h f* the Meridian paſſing through the true Sun S, and its apparent Place at I in the firſt Quarter of the Ecliptic ♈ ♋. Plate LXIV. Fig. 5.

25. SUPPOSE, now, the Motion of the Earth in every Reſpect equable, and firſt that it ſat out from ♎, and proceeded in the Equator in a given Time to D, the Sun would apparently deſcribe in the ſame Time the Arch of the Equator ♈ I. Again, ſuppoſe it ſat out from the ſame Point ♎, and ſpent the ſame Time with the ſame equable Velocity in the Ecliptic, it would arrive to the Point A, ſo that the Arch ♎ A = ♎ D, and ♈ I = ♈ C. Then 'tis evident, as the Earth revolves about its Axis from Weſt to Eaſt,

East, the Meridian of any Place will first arrive at the Sun I in the Ecliptic, and afterwards at the Sun C in the Equinoctial; that is, the Time of Noon by the Sun in the Ecliptic will be sooner than that Noon which would happen by the Sun in the Equinoctial; and that by the Quantity of the Arch *b* D turned into Time.

26. Now the Arch *b* D = B C is the Difference of the Sun's Longitude ♈ I or ♈ C, and his Right Ascension ♈ B. Draw *g e* parallel to DC, and the Angle *e* A *f* will be equal to the Angle D S *b*, and the Arch *e f* similar to the Arch D *b*; therefore the Time in which the Meridian *b f* revolves into the Situation *e f*, is that which is to be added to the Ecliptic Noon to equate it with the Time of the Equinoctial Noon in the first and third Quarters of the Ecliptic. In the second and fourth Quarter, the said Equation is to be subtracted, as would easily appear by making the same Construction there.

27. Now because in different Parts of the Quadrant this Arch D *b* or B C is of a different Length, the *Equation of Time* will be a variable Quantity; and therefore since the Motion and Time measured by the Sun in the Equinoctial is always equal, (there being

being nothing to make it otherwiſe) it follows, that the Times (*i. e.* the Days) meaſured by the Sun in the Ecliptic muſt be always unequal; or, in other Words, the *Solar Days* are ſometimes ſhorter, ſometimes longer, than the *equal Time* meaſured out in the Equinoctial.

28. IT has been ſhewn already, that the True Motion of the Earth precedes the Mean in the firſt Semicircle of Anomaly, and is preceded by the Mean in the ſecond. Therefore while the Earth is going from the Aphelion to the Perihelion, or while the Sun apparently moves from the Apogæum to the Perigæum, the Apparent Time will be before the Mean, and in the other Semicircle of Anomaly it will be after it. The Difference of theſe Motions converted into Time is the *Equation of Time* in this reſpect, and is to be ſubtracted from the Apparent Time to gain the Mean, or added to the Mean to gain the Apparent, in the firſt Semicircle of Anomaly, and *vice verſa* in the latter.

29. Now both theſe Parts of the Equation of Time are calculated by Aſtronomers for every Degree of Anomaly, and for every Degree of the Sun's Longitude in the Ecliptic, and diſpoſed in two ſeveral Tables,

Tables, with Directions for *adding and subtracting*, as the Case requires; so that at all times the true or equal Time may be had. And from thence it appears that the Apparent Time, or that shewn by the Sun, *viz.* by a *Sun-dial*, is but four Days in the whole Year the same with the Mean or equal Time shewn by a good *Clock* or *Watch*, *viz.* about *April* the 15th, *June* the 17th, *August* the 31st, and *December* the 24th. Also about the 23d of *October* the Equation is greatest of all in the Year, being then about 16′ 11″, Clocks being then so much slower than *Sun-dials*.

30. As the Solar Days are unequal, the Hours must be so of course; and hence it appears, that there is no natural Body which can by its Motion measure Time truly or equally; and the only Way to do this is by the artificial Contrivance of Clocks, Watches, Clepsydræ, Hour-Glasses, *&c.*

31. In different Parts of the World, the natural Day has a different Beginning. The ancient *Egyptians* began their Day at Midnight, as do also the modern Nations of *France*, *Spain*, *Great-Britain*, and most Parts of *Europe*. The *Jews*, with the *Germans*, begin their Day at Sun-setting. The

Babylonians

Babylonians began theirs at Sun-riſing. And the Aſtronomers begin the Day at Noon, and reckon on to twenty-four Hours, and not twice twelve, as we do by our Clocks in civil Life.

32. A WEEK is another common Meaſure of Time conſiſting of ſeven Days; and becauſe the Ancients ſuppoſed the ſeven Planets had an Influence upon the Earth and all terreſtrial Things, they allotted the firſt Hour of each Day to the Planet they ſuppoſed then to preſide; from whence the ſeveral Days of the Week received their Names. Thus *Sunday* was *Dies Solis*, i. e. the Day of the *Sun*; *Monday* was *Dies Lunæ*, i. e. the Day of the *Moon*; *Tueſday* was *Dies Martis*, i. e. the Day of *Tueſco* or *Mars*; *Wedneſday* was *Dies Mercurii*, i. e. the Day of *Woden* or *Mercury*; *Thurſday* was *Dies Jovis*, i. e. the Day of *Thor* or *Jupiter*; *Friday* was *Dies Veneris*, i. e. the Day of *Friga* or *Venus*; and *Saturday* was *Dies Saturni*, i. e. the Day of *Saturn*.

33. A MONTH is another Part of Time, ſo call'd from the Moon, becauſe it is the Time of her Revolution about the Earth, and is therefore alſo call'd a *Lunation*. If we reſpect the Revolution of the Moon from any fixed Point in the Heavens (as a Star) to the ſame again, it is call'd a *Pe-*

riodical

riodical Month, and consists of 27 D. 7 H. 43′. But if we regard the Time that passes between one Conjunction or New-Moon and the next following, it is call'd a *Synodical Month*, and is equal to 29 D. 12 H. 44′ 3″.

34. THESE now mentioned are the *Astronomical Years, Months,* and *Days:* But those used in common Life are somewhat different. Thus the Civil Month is a Space of 28, 29, 30, or 31 Days, and 12 Synodic Months make 354 Days, which is call'd a *Civil Lunar Year*; and a Civil Solar Year is the Space of 365 Days. Therefore to equate the *Civil Lunar* to the *Solar Year*, 11 Days are to be added, which were call'd by the *Greeks Epamogenæ*, and by us the *Epacts*.

35. THE *Civil Soli-Lunar* Year of 365 Days, being short of the true by 5 H. 48′ 57″, occasion'd the Beginning of the Year to run forwards through the Seasons one Day nearly in four Years; and in 1460 Years, through all the Months of the Year. On this account *Julius Cæsar* ordain'd that every fourth Year *one Day* should be added to *February*, by causing the 24th Day to be reckon'd twice; and because this 24th of *February* was the Sixth (*Sextilis*) before the Kalends of *March*, there were in this Year two of those *Sextiles*, which gave the Name

of

of *Bissextile* to this Year. The Year, thus corrected, was from thence call'd the *Julian Year*.

36. BUT the six Hours added by *Julius Cæsar* is too much, that is, exceeds 5 H. 48′ 57″ by 11′ 3″, and therefore the Sun each Year begins his Course 11′ 3″ before the *Julian* Year is ended, which in 131 Years amounts to a whole Day. Hence at the Council of *Nice*, A. D. 325, (at which the Time of *Easter* was fixed) the Vernal Equinox being upon the 21st Day of *March*, it was found in the Year 1582 to happen on the 11th of *March*, 10 Days sooner than before.

37. POPE *Gregory* XIII. thought the Kalendar too erroneous, and resolved to reform it, by restoring the Equinox to its former Place in the Year, *viz.* to the 21st of *March*. To do this, he took 10 Days out of the Kalendar, by ordering the 5th of *October* 1582 to be call'd the 15th; and to prevent the Regress of the Equinox for the future, ordered every 100dth Year to consist of only 365 Days, whereas in the *Julian* it has 366, as being *Bissextile*. This Reformation is therefore called the *Gregorian* Account, or New-Stile, and is used by *Papists* in *Italy*, *Spain*, *France*, *Germany*, and by some *Protestants* abroad; but in

in *England* we have lately made a farther Correction, by throwing out 11 Days; which brings us much nearer the Truth than the *Gregorian Account*. This therefore ought to be called the BRITISH or AUGUSTINE Account, in Honour of our own Nation and King.

38. SINCE the Council of *Nice*, to the present Year 1758, there have elapsed upwards of 1433 Years; by which means the Equinox does in the Old-Stile, at this Time, fall on the 10th of *March* nearly, and the *Julian* Account is 11 Days later than our own. But even the Emendation, or New-Stile, (which we now use) is not sufficient; for whereas by that four Days in 400 Years are rejected, a considerable Error is committed; for the odd 11′ 3″, by which the *Julian* Year exceeds the Truth, will not amount to more than three Days in 391 Years. If therefore at the End of every 391 Years we expunge three Days, the Equinox will very nearly always keep to the same Day of the Month.

39. IN Computations of Time, we find it necessary to fix upon some remarkable Transaction, or memorable Event, for the *Exordium* or Beginning of the Reckoning; these are called EPOCHA's or ÆRA's. Thus some compute from the *Creation of the World*: The ancient *Greeks* from the *Institution of the*

the Olympiads, beginning 776 Years before CHRIST: The *Romans* from the *Building of Rome*, about 750 Years before CHRIST. The *Chaldeans* and *Egyptians* uſe the *Æra of Nabonaſſar*, beginning *A. ante C.* 752. The *Turkiſh* Epocha is the *Hegira* or *Flight of Mahomet*, A. C. 622. The *Perſian* Æra is called *Yeſdegird*, A. C. 632. And that of the *Chriſtians* the *Birth of Chriſt*, ſince which Time we reckon 1787 Years.

40. BESIDES the Meaſure of Time by Common Years, we find it became neceſſary to introduce the Uſe of CYCLES (*i. e.* Circles) *of Years*; as the *Metonic Cycle*, the *Cycle of the Sun*, the *Cycle of Indiction*, and the *Julian Period* compounded of all the reſt. Of each of theſe I ſhall give the following ſhort Account.

41. THE CYCLE of the SUN ariſes hence: If the Number 365 be divided by 7, it will have a Remainder of 1, which ſhews the laſt Day of the Year is the ſame Day of the Week with the firſt. Now it was always cuſtomary to place againſt the ſeven Days in the Week, the ſeven firſt Letters of the Alphabet, A, B, C, D, E, F, G; and therefore, as they were continued through the Year, it is evident the ſame Letter muſt

ſtand againſt the firſt and laſt Day of the Year, *viz.* the Letter A.

42. HENCE, if the firſt of *January* will be a *Sunday*, the Letter A will point out all the *Sundays* in that Year; and ſince the 1ſt of *January* in the next Year is *Monday*, the firſt *Sunday* will be on the 7th, againſt which ſtands the Letter G, which therefore will be the *Sunday* Letter for all that Year. Again, the firſt Day of the following Year being *Tueſday*, the firſt *Sunday* will be on the 6th, againſt which ſtands the Letter F, which therefore indicates the *Sundays* through that Year, and ſo on; whence 'tis eaſy to obſerve, that the Letters which point out the *Sundays* in every Year will be in a retrograde Order, *viz.* A, G, F, E, *&c.* And becauſe theſe Letters ſhew the *Dies Domini*, or *Lord's-Days*, they have been called DOMINICAL LETTERS.

43. NOW, if all the Years were common ones, the ſame Letter would not be the *Dominical*, or the *Sundays* would not be upon the ſame Days of the Week, till after a Cycle or Revolution of ſeven Years; and ſince every 4th Year has a Day extraordinary, this Day will interrupt the Succeſſion of the *Dominical Letters*, and cauſe that the

the ſame Days will not be ſhewn again by the ſame Letters after a *Cycle* of ſeven Years, but of $4 \times 7 = 28$ Years, which is called the *Cycle* of the *Sun*.

44. BECAUSE in every *Biſſextile* Year the 24th or 25th of *February* is reckoned twice, and both thoſe Days have the ſame Letter, it follows, that that Letter which ſhewed the *Sundays* before the 24th of *February* will not ſhew it afterwards, and therefore in every ſuch Year there will be two *Dominical Letters.* For Example, the Year 1744 was *Biſſextile*, *January* 1 *Sunday*, the *Dominical Letter* A; but the 24th of *February* being *Friday* had the Letter F, and alſo *Saturday* the 25th; therefore *Sunday* the 26th muſt have G, which for that reaſon was the *Sunday* Letter the remaining Part of the Year.

45. To find what Year of the *Cycle* the preſent or any Year of CHRIST is, add 9 to the given Year, (becauſe the firſt Year of CHRIST was the 9th of the *Cycle*) and divide by 28, the Remainder is the Year of the *Cycle* required. Example: The Year $1746 + 9 = 1755$, then 1755 divided by 28 leaves 19, the Year of the *Cycle* required, whoſe *Dominical Letter* is E, according to the following Table:

R 2 *Cycle*

Cycle	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	1.
Dom. Let.	G F.	E.	D.	C.	B A.	G.	F.	E.	D C.	B.	A.	G.
Cycle	13.	14.	15.	16.	17.	18.	19.	20.	21.			
Dom. Let.	F E.	D	C	B	A G.	F.	E.	D.	C B.			
Cycle	22.	23.	24.	25.	26.	27.	28.					
Dom. Let.	A.	G.	F.	E D.	C.	B.	A.					

46. THE METONIC CYCLE, (so called from the Inventor *Meton*) otherwise called the *Cycle of the Moon*, is a Period of nineteen Years, after which the *New* and *Full Moons* were supposed to return on the same Days of the Month, and Hours, as before; because if the Solar and Lunar Year began together at any Time, these Years being to each other as 365 to 354, could not coincide again at their Beginning till after a certain Time, *viz.* 235 Lunations, which make 6939 D. 16 H. 31′ 45″, and in nineteen Solar Years are 6939 D. 18 H.; the Difference being only 1 H. 28′ 15″ shews the two Years will then begin again very nearly at the same Time, and the *New* and *Full Moons* come round again upon the same Days of the Month.

47. Yet

47. YET this Deficiency of an *Hour and half* will cauſe the *New* and *Full Moons* to happen ſo much ſooner each *Cycle* in the Heavens than by this Reckoning, and this in 304 Years amounts to a whole Day; and therefore, at this Time, they happen almoſt five Days ſooner than they ſhould do, by the Rule ſettled by the *Nicene* Council for finding the ſame by the *Golden Numbers*; the Nature and Uſe of which are to be underſtood as follows.

48. TAKING any Year for the Firſt of the *Cycle*, the Ancients obſerved all the Days on which the *New Moons* happened through the Year, and againſt each ſuch Day they placed the Number 1; in the 2d Year of the *Cycle* they did the like, and to each Day of the *New Moon* annexed the Number 2. In like Manner to every *New Moon* Day in the 3d Year of the *Cycle* they ſubjoined the Number 3; and ſo on, thro' all the Years of the *Cycle*. This being done for one *Cycle*, the ſame Numbers were fitted to the Kalendar to ſhew the *New Moons* in each Year of any future *Cycle*; and, upon Account of this their excellent Uſe, they were in *Gold*, and were therefore called the *Golden Numbers* for thoſe Years reſpectively.

49. BUT becauſe theſe Numbers for the obſerved *New Moons* are not of laſting Uſe (as above ſhewn), the beſt Way of diſpoſing theſe Numbers is by the *Mean Lunations*, as they may be found from Aſtronomical Tables for each Year of the *Cycle*, which are the ſame in every *Cycle*, and do not vary greatly from the true. But however advantageous this may be in civil Life, we are not to expect this Innovation ſhould take Place in the Liturgy of the Church of *England*, which ſtill continues to compute the Moons, as it does the Equinoxes, by the old erroneous Rule eſtabliſhed by the Council of *Nice*, which are called *Eccleſiaſtical New Moons*, in Contradiſtinction to the true ones in the Heavens.

50. BESIDES theſe, there was another Period called the CYCLE OF INDICTION, conſiſting of 15 Years; it was ſo called becauſe the Numbers of this Cycle *indicated* the Time of *Eaſter*. But as this *Cycle* has no Connection with the Motions of the Heavenly Bodies, I ſhall ſay no more of it here, but refer the Reader for a farther Account of this and other Matters purely *Chronological*, to the Authors who have wrote on *Chronology*, or, if they pleaſe, to an Epitome

of

of that Science in my *Philological Library of Literary Arts and Sciences.*

51. THE DIONYSIAN PERIOD is one that is made by multiplying together the *Cycles* of the *Sun* and *Moon*, and therefore consists of 532 Years, for $28 \times 19 = 532$. After the Completion of this Period, not only the New and Full Moons return to the same Days of the Month, but also the Days of the Month return to the same Days of the Week; and therefore the *Dominical Letters* and the *Moveable Feasts* all return again in the same Order. Hence this *Cycle* was called the GREAT PASCHAL CYCLE.

52. THE JULIAN PERIOD is the last I shall mention, and the largest of all, consisting of 7980 Years, being composed of the *Cycles of the Sun, Moon, and Indiction*; thus $28 \times 19 \times 15 = 7980$. The Beginning of this Period was 764 Years before the Creation, and is not yet compleated; and therefore comprehends all other *Periods, Cycles* and *Epochas*, and the Times of all memorable Actions and Histories. It had its Name from its Inventor *Julius Scaliger*, who has eternized himself thereby.

53. I CAN'T conclude this Essay without laying before the Reader the *Astronomical*

Principles of CHRONOLOGY, which Sir *Iſaac Newton* makes uſe of for ſettling the Grand EPOCHA of the *Argonautic Expedition*, and which he makes the Baſis of his Chronology. He obſerves, that *Eudoxus*, in his Deſcription of the Sphere of the Ancients, placed the Solſtices and Equinoxes in the Middle of the Conſtellations *Aries*, *Cancer*, *Chelæ*, and *Capricorn:* And alſo that this Sphere or Globe was firſt made by *Muſæus*, and the *Aſteriſms* delineated upon it by *Chiron*, two of the *Argonauts*.

54. Now it has been ſhewn, that by the Preceſſion of the Equinoxes the Stars go back 50″ *per Annum*. And ſince at the End of the Year 1689, the *Equinoctial Colure* paſſing through the middle Point, between the firſt and laſt Star of *Aries*, did then cut the Ecliptic in ♉ 6° 44″, it is evident, that the Equinox had then gone back 36° 44′; therefore, as 50″ is to one Year, ſo is 36° 44′ to 2645 Years, which is the Time ſince the *Argonautic* Expedition to the Beginning of the Year 1690; that is, 955 Years before CHRIST is the Æra of the *Argonautic Expedition*.

55. BUT our great Author is more particular and ſubtile in this Affair. He finds the Mean Place of the *Colure of the Equinoxes*

noxes and Solſtices, by conſidering the ſeveral Stars they paſs'd through among the other Conſtellations, as follows, according to *Eudoxus*.

56. IN the Back of *Aries* is a Star of the 6th Magnitude, marked γ by *Bayer*; in the End of the Year 1689, its Longitude was ♉ 9° 38′ 45″; and the Equinoctial Colure paſſing through according to *Eudoxus*, cuts the Ecliptic in ♉ 6° 58′ 57″.

57. IN the Head of *Cetus* are two Stars of the 4th Magnitude, called ν and ξ by *Bayer*. *Eudoxus*'s Colure paſſing in the Middle between them, cuts the Ecliptic in ♉ 6° 58′ 51″, at the End of the Year 1689.

58. IN the extreme Flexure of *Eridanus* there was formerly a Star of the 4th Magnitude (of late it is referred to the Breaſt of *Cetus*). It is the only Star in *Eridanus* through which this Colure can paſs; its Longitude was at the End of the Year 1689 ♈ 25° 22′ 10″, and the Colure of the Equinox paſſing through it cuts the Ecliptic in ♉ 7° 12′ 40″.

59. IN the Head of *Perſeus*, rightly delineated, is a Star of the 4th Magnitude, called τ by *Bayer*; its Longitude was ♉ 23° 25′ 30″ at the End of the Year 1689; and the

the Colure of the Equinox paſſing through it cuts the Ecliptic in ♉ $6^\circ\ 18'\ 57''$.

60. In the Right Hand of *Perſeus*, rightly delineated, is a Star of the 4th Magnitude, whoſe Longitude at the End of the Year 1689 was ♉ $24^\circ\ 25'\ 27''$, and the Equinoctial Colure paſſing through it cuts the Ecliptic in ♉ $4^\circ\ 56'\ 40''$.

61. Now the Sum of all theſe Places of the Colure, *viz.*

		°	′	″
	♉	6	58	57
	♉	6	58	51
	♉	7	12	40
	♉	6	18	57
	♉	4	56	40
Is =	1	2	26	05
The 5th Part of which is =	♉	6	29	13

which is therefore the Mean Place, in which the Colure in the End of the Year 1689 did cut the Ecliptic.

62. After a like Manner he determines the Mean Place of the *Solſtitial Summer Colure* to be ♌ $6^\circ\ 28'\ 47''$, which as it is juſt 90 Degrees from the other, ſhews it to be rightly deduced. The *Equinoxes* having then departed $1^s\ 6^\circ\ 29'$ from the Cardinal Points of *Chiron*, ſhews that 2628 Years have elapſed ſince that Time, which is more

more correct than the former Number (*Article* 55.) though leſs by only ſeventeen Years.

63. By ſome other Methods, of a like Nature, he alſo ſhews the *Æra of the Argonauts* ought to be placed in that Age of the World; and having fixed this moſt ancient *Epocha*, he makes his Computation with Reference thereto in the future Part of his Book.

64. And thus our great Author has, with his uſual Sagacity, ſo conducted his Deſign, as to make his Chronology ſuit with the *Courſe of Nature*, with the *Principles of Aſtronomy*, with *Sacred Hiſtory*, with *Herodotus*, the Father of Profane Hiſtory, and with itſelf. And though many have thought fit to cavil, and find great fault with his Chronology, yet how little Regard ought to be paid to them may from hence appear, that Sir *Iſaac Newton* was undoubtedly equal to any Man *in all the common Qualifications of a Chronologiſt*, and *vaſtly ſuperior to all* in thoſe which were eſſential. Gentlemen ſhould have the Modeſty not to criticiſe on the greateſt Man that ever lived, till they have convinced the World, at leaſt, *that they underſtand him.*

LECTURE XII.

The Use of the GLOBES.

Of the GLOBES *in general. The* CIRCLES *of the* SPHERE *described. The* POSITIONS *of the* SPHERE. *The* SOLUTION *of* PROBLEMS *on the* CELESTIAL GLOBE. *The* TERRESTRIAL GLOBE *described.* PROBLEMS *on the same. Of the* CONSTELLATIONS *of the* Northern *and* Southern HEMISPHERE. Flamsteed's CATALOGUE *of the* STARS. *Of the* DISTANCE *and other* Phænomena *of the* STARS. *A* Calculation *of the surprising* VELOCITY *of* LIGHT. *Of the* ABERRATION *of* Light, *and the* Telescopic MOTION *of the Stars by Dr.* BRADLEY. *The* PRINCIPLES *of* GNOMONICKS, *or* Art *of* DIALING *demonstrated, by a* DIALING-SPHERE. Astronomical Doctrine *of the* SPHERE, *and Method of calculating* SPHERICAL TRIANGLES. *The* HARVEST-MOON *explain'd. How to find a* MERIDIAN LINE. *The* FIGURE *and* DIMENSIONS *of the Earth determined by* actual

actual Mensuration *of a* DEGREE *under the* ARCTICK CIRCLE *and at* PARIS. *A new* CALCULATION *on that Head. Of the* ORTHOGRAPHICAL PROJECTION. *Of the* STEREOGRAPHICAL PROJECTION. *The* Globular Projection. *Of* MERCATOR'*s* CHART, *and a new Method of Constructing the Table of* MERIDIONAL *Parts by Fluxions. The Nature of the* RHUMB-LINE *investigated, and applied in* Sailing. *A new* MAP *of the* WORLD *on the* Globular Projection. *A* MAP *of the* Country *in* Lapland *where the Arch of the* Meridian *was measured by the* French *King's* Mathematicians.

IN this Lecture I shall explain the *Use of both the Globes*, by giving a succinct Account of the Nature and Design of each, and a Solution of the *Principal Problems* that are usually performed thereby.

EACH Globe is suspended in a General Meridian, and moveable (within an Horizon) about its Axis, in the same Manner as the *Armillary Sphere* of the Orrery; and the Circles of that Sphere, already described, are laid on the corresponding Parts of the Surface of each Globe; and are therefore supposed to be known. Plate LXVI. Fig. 4.

THE

THE Surface of the CELESTIAL GLOBE is a Repreſentation of the Concave Surface of the *Starry Firmament*, there being depicted all the Stars of the Firſt and Second Magnitude, and the moſt noted of all the reſt that are viſible. So that by this Globe we may ſhew the *Face of the Heavens* for any required Time, by Day or Night, throughout the Year, in general; or in regard to any particular Body, as the *Sun*, *Moon*, *Planet*, or *Fixed Star*.

THE Stars are all diſpoſed into Conſtellations, under the Forms of various Animals, whoſe Names and Figures are printed on the Paper which covers the Globe; which were invented by the ancient Aſtronomers and Poets, and are ſtill retained for the ſake of Diſtinction and better Arrangement of thoſe Luminaries, which would be otherwiſe too confuſed and promiſcuous for eaſy Conception, and a regular Method of treating on them. (CXLV.)

(CXLV.) 1. The Surface of this CELESTIAL GLOBE may be eſteemed a juſt and adequate Repreſentation of the concave Expanſe of the Heavens, notwithſtanding its Convexity; for 'tis eaſy to conceive the Eye placed in the Centre of the Globe, and viewing the Stars on its Surface, ſuppoſing it made of Glaſs, as ſome of them are; and alſo, that if Holes were made in the Centre of each Star, the Eye in the Centre of the Globe, properly

IN order to understand the following Problems, it will be necessary to premise the

properly posited, would view through each of those Holes the very Stars in the Heavens represented by them.

2. Because it would be impossible to have any distinct or regular Ideas or Notions of the Stars in respect of their Number, Magnitude, Order, Distances, *&c.* without first reducing them to proper Classes, and arranging them in certain Forms, which therefore are call'd ASTERISMS or CONSTELLATIONS. This was done in the early Ages of the World by the first Observers of the Heavens, and those who made Spheres or Delineations; of whom Sir *Isaac Newton* reckons *Chiron* the Centaur the first who formed the Stars into Constellations, about the Time of the *Argonautic Expedition*, or soon after; and that the several Forms or *Asterisms* were, as it were, so many symbolical Histories, or Memorials of Persons and Things remarkable in that Affair. Thus *Aries*, the Ram, is commemorated for his *Golden Fleece*, and was made the first of the Signs, being the Ensign of the Ship in which *Phryxus* fled to *Colchis*; *Taurus*, the Bull, with Brazen Hoofs, tamed by *Jason*; *Gemini*, the Twins, *viz.* *Castor* and *Pollux*, two of the *Argonauts*; the Ship *Argo* and *Hydrus* the Dragon, *&c.* which all manifestly relate to the Affairs of that Expedition, which happened about forty or fifty Years after *Solomon*'s Death.

3. By this Means they could make Catalogues of the Stars, record their Places in the Heavens, and call them all by their Names. *Hipparchus* is said to be the first who framed a Catalogue of the Stars, which was afterwards copied by *Ptolomy*, and adjusted to his own Time, *A. D.* 140. The Number in this was 1026. After this *Ulug Beigh* made a Catalogue of 1022, reduced to the Year 1437. *Tycho Brahe* rectified the Places of 1000 Stars; but his Catalogue, published by *Longomontanus*, contains but 777, for the Year 1600. *Bayer* published a Catalogue of 1160. *Hevelius* composed a Catalogue

the following Definitions in relation thereto, *viz.*

I. THE DECLINATION of the Sun and Stars is their Diſtance from the *Equinoctial* in Degrees of the general Meridian, towards either Pole, *North* or *South.*

II. RIGHT ASCENSION is that Degree of the Equinoctial reckon'd from the Beginning of *Aries*, which comes to the Meridian with the Sun or Star.

III. OBLIQUE ASCENSION is that Degree of the Equinoctial which comes to the Horizon when the Sun or Star is riſing: And *Oblique Deſcenſion* is that Point which comes to the Horizon on the Weſt Part, when the Sun or Star is deſcending or ſetting in an oblique Sphere.

IV. ASCENSIONAL DIFFERENCE is the Difference between the *Right* and *Oblique Aſcenſion.*

V. THE

Catalogue of 1888 Stars adjuſted to the Year 1660. But the largeſt and moſt compleat of all is the *Britiſh* Catalogue by Mr. *Flamſteed*, containing about 3000, of which ſcarce 1000 can be ſeen by the naked Eye in the cleareſt and darkeſt Night. They are rectified for the Year 1689. They are diſtinguiſhed into ſeven Degrees of Magnitude, in their proper Conſtellations; whoſe Names, Latitudes, and Longitudes here follow, together with the Number of Stars in each, and of each particular Magnitude, as I have taken them from the third Volume of the *Hiſtoria Cœleſtis.* Note, the firſt Latitude is South, the other North, in the Twelve Signs, unleſs marked to the contrary.

4. The

V. THE LONGITUDE of the Sun or Star is an Arch of the Ecliptic, between the first Point of *Aries*, and that Point of the Ecliptic to which the Luminary is referred by the Meridian passing through it; and is therefore reckoned in Signs and Degrees of the Ecliptic.

4. The *Constellations* of the TWELVE SIGNS.

Names	Long. °	'	Lat. °	'	*Num.*	1	2	3	4	5	6	7
Aries.	♈ 26 ♉ 21	48 06	00 21	01 31	65	0	1	2	5	6	28	23
Taurus.	♉ 16 ♊ 26	49 36	18 09	27 46	135	1	1	4	13	21	44	51
Gemini.	♊ 14 ♋ 12	11 33	10 13	07 18	79	1	2	4	6	12	32	22
Cancer.	♋ 22 ♌ 12	49 19	10 14	19 59	71	0	0	0	6	7	39	19
Leo.	♌ 10 20	57 42	07 17	39 38	95	2	2	6	15	10	50	10
Virgo.	♍ 00 ♎ 29	10 23	00 21	24 24	89	1	0	5	10	19	45	9
Libra.	♎ 04 ♏ 28	11 35	18 11	34 *N.* 27 *S.*	49	1	2	7	5	11	21	2
Scorpio.	♏ 26 ♐ 20	48 46	12 13	46 *N.* 57 *S.*	51	0	2	2	12	5	25	5
Sagittarius.	♐ 22 ♑ 26	55 29	10 07	59 31	50	0	1	5	6	11	23	4
Capricornus.	♑ 27 ♒ 21	26 29	08 07	53 27	51	0	0	3	3	9	34	2
Aquarius.	♒ 07 ♓ 21	24 57	21 23	04 02	99	1	0	4	7	31	50	6
Pisces.	♓ 11 ♈ 26	06 47	23 09	06 05	109	0	0	1	6	27	54	21

VI. THE LATITUDE of a Star is its Distance from the Ecliptic towards the North or South Pole.

VII. AMPLITUDE is the Distance at which the Sun or Star rises or sets, from the East or West Point of the Horizon, towards the North or South.

5. The *Constellations* of the NORTHERN HEMISPHERE.

Andromeda.	♈ ♉	3 18	29 4	15 49	55 53	66	0	3	2	12	13	34	[illegible]
Aquila cum Antinoo.	♑ ♒	29 8	46 6	10 43	5 27	70	1	0	10	7	15	32	5
Anser cum Vulpe	♑ ♓	20 1	20 14	37 47	39 46	34	0	0	0	4	12	18	0
Auriga.	♊ ♋	11 12	22 30	2 32	29 13	68	1	2	1	10	18	31	5
Bootes.	♍ ♏	22 00	34 54	25 60	15 33	55	1	0	8	10	11	17	8
Cassiopeia.	♈ ♊	21 8	6 4	38 59	18 53	56	0	0	5	7	9	30	5
Camelopardus.	♊ ♋	10 10	40 39	29 45	24 43	58	0	0	0	4	18	27	9
Cepheus.	♈ ♉	00 26	39 37	59 75	32 27	35	0	0	3	7	8	14	3
Coma Berenices	♍ ♎	16 4	53 48	15 33	14 56	40	0	0	0	8	14	14	4
Corona Septen.	♏	00 20	58 54	44 56	21 25	21	0	1	0	6	8	6	0
Cygnus.	♑ ♓	20 23	55 17	37 74	39 10	107	0	1	6	21	31	48	0
Delphinus.	♒	8 16	49 31	23 33	00 44	18	0	0	6	0	2	9	1
Draco.	*Per totam Circumf.*			57 87	13 25	49	0	1	7	8	13	19	1
Equuleus.	♒	14 21	12 7	20 25	9 13	10	0	0	0	4	1	5	0

VII. AZI-

VIII. AZIMUTH is the Diſtance between the North Point of the Horizon, and the Point where the Vertical Circle, paſſing through the Body of the Sun or Star, cuts the Horizon.

IX. THE ALTITUDE of the Sun or Star is its Height above the Horizon, meaſured

Hercules.	♎ ♐	28 28	7 29	25 69	15 33	95	0	0	11	15	31	38	0
Leo Minor.	♋ ♍	29 4	8 4	9 30	21 50	53	0	0	1	5	11	33	3
Lacerta.	♓	19 6	49 27	43 55	12 34	16	0	0	0	3	6	7	0
Lynx.	♊ ♌	28 9	24 45	17 10	3 39	44	0	0	0	3	12	21	8
Lyra.	♑	3 26	35 14	54 66	28 13	21	1	0	2	2	5	11	0
Perſeus. C. M.	♉ ♊	8 10	8 48	11 41	17 13	67	0	2	5	11	15	28	6
Pegaſus.	♒ ♈	23 7	37 17	9 44	13 24	93	0	4	3	10	13	58	5
Sagitta.	♑ ♒	20 8	00 37	35 43	35 15	23	0	0	0	4	1	18	0
Serpens Ophiuchi.	♏ ♑	7 11	38 31	7 42	59 28	59	0	1	7	6	3	32	10
Scutum.	♑	0 10	23 8	4 18	59 17	7	0	0	0	2	4	1	0
Serpentarius, or *Ophiuchus.*	♏ ♑	27 1	58 29	6 37	54 18	69	0	1	7	15	13	26	7
Triangulum.	♉	0 13	5 15	13 20	55 54	15	0	0	0	3	1	7	4
Urſa Major.	♊ ♎	10 6	41 58	17 61	6 3	215	0	6	5	5	58	91	20
Urſa Minor.	♊ ♌	21 17	43 19	65 77	42 50	24	0	0	4	3	5	6	6
Canes Venatici	♍	0 25	5 43	52 33	52 56	25	0	1	0	2	5	14	3

in the Degrees of the *Quadrant of Latitude*, or moveable *Azimuth Circle*.

X. A Star is ſaid to riſe or ſet *Coſmically*, when it riſes or ſets when the Sun riſes.

XI. A Star riſes *Acronically*, if it riſes when the Sun ſets.

6. *Conſtellations in the* SOUTHERN HEMISPHERE.

Ara cum Thuribulo.	♐	15 27	6 18	23 37	5 15	9	0	0	1	6	2	0	0
Argo, or *Navis.*	♋ ♌	24 14	57 2	22 10	24 14	25	0	0	4	6	6	9	0
Apus.	♐	9 21	45 24	44 62	32 4	11	0	0	0	4	3	4	0
Canis Major.	♊ ♋	3 25	7 12	34 59	44 14	32	1	7	1	4	11	5	3
Canis Minor.	♋ ♌	16 0	48 49	9 23	45 47	15	1	0	1	0	3	9	1
Cetus.	♓ ♉	18 14	36 30	2 34	42 14	78	0	2	6	13	9	44	4
Centaurus cum Lupo.	♎ ♏	25 28	42 30	11 21	28 59	13	0	1	0	5	6	1	0
Cameliontis.	♏ ♐	26 3	30 39	63 75	35 24	10	0	0	0	0	9	1	0
Columba Noahi.	♊ ♋	14 6	54 46	55 60	42 41	10	0	2	0	1	6	1	0
Corona Auſtr.	♑	1 10	18 14	12 22	28 36	12	0	0	0	1	3	8	0
Corvus.	♎ .	6 13	26 3	10 21	21 44	10	0	0	3	2	2	3	0
Crater.	♍ ♎	19 3	26 58	11 22	18 42	11	0	0	0	8	2	2	0
Eridanus.	♈ ♊	16 11	38 15	18 54	26 33	68	0	0	12	15	20	20	1
Grus.	♒	14 18	54 2	39 41	43 55	23	0	0	0	2	1	0	0
Hydrus.	♑ ♈	26 3	59 37	64 78	10 5	10	0	0	4	2	3	1	0

XII. A

Vol. III. Plate LXV. P. 261.

A Direct Sphere

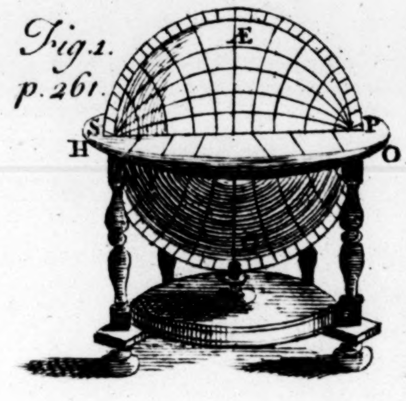


Fig. 1. p. 261.

A Parallel Sphere

Fig. 2. p. 263.

An Oblique Sphere

Fig. 3. p. 265.

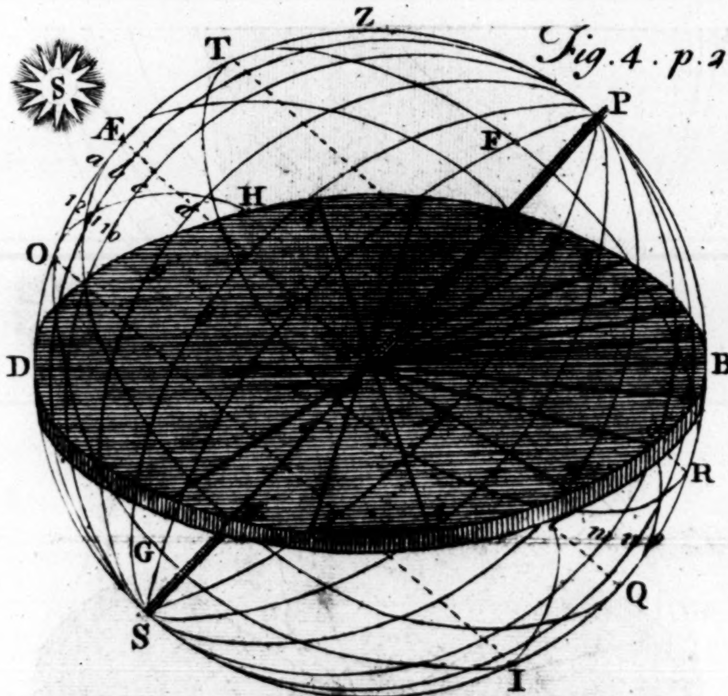


Fig. 4. p. 269.

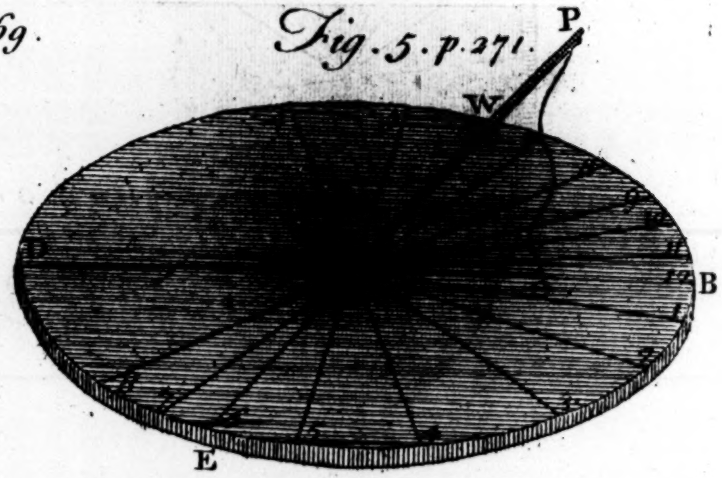


Fig. 5. p. 271.

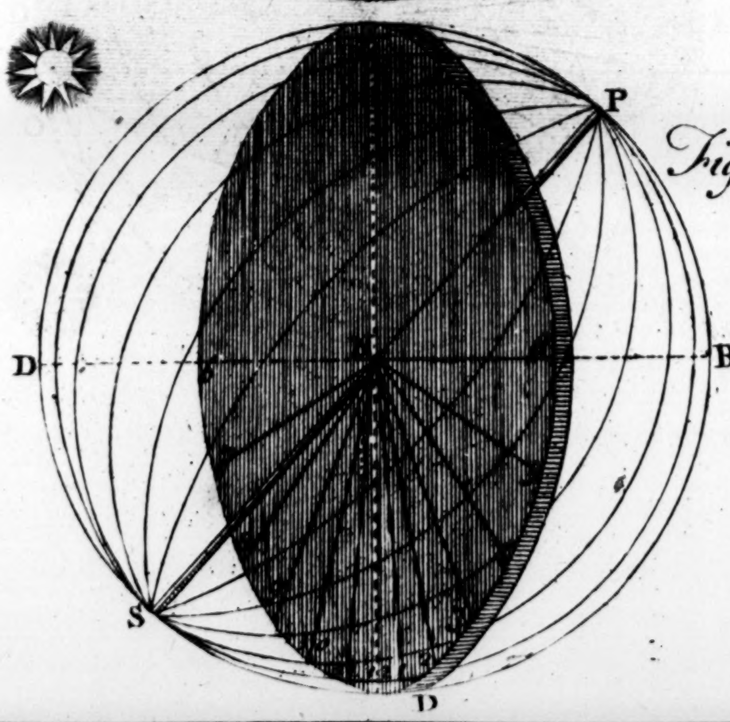


Fig. 6. p. 271.

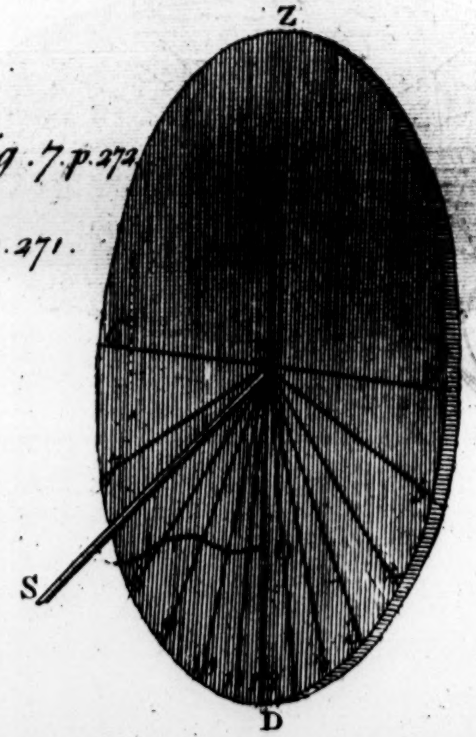


Fig. 7. p. 272.

XII. A Star riſes *Heliacally*, when it emerges out of the Sun-beams, and is ſeen in the Morning before Sun-riſing: And it ſets *Heliacally*, when it is ſo near the Sun that it cannot be ſeen.

XIII. A *Right Sphere* is that whoſe Poles are in the Horizon and the Equinoctial, Pl. LXV. Fig. 1, 2, 3.

Lepus.	♊	6 28	44 9	34 45	45 46	19	0	0	3	7	3	6	0
Muſca.	♏	16 22	20 22	55 58	11 47	4	0	0	0	2	2	0	0
Monoſceros.	♊ ♌	29 10	34 50	13 31	13 11	19	0	0	0	10	7	2	9
Orion.	♉ ♊	7 15	32 11	3 34	11 4	80	2	4	4	25	20	25	0
Pavo.	♐ ♑	24 24	7 41	36 50	11 49	14	0	1	3	5	4	1	0
Phœnix.	♒ ♓	29 24	47 14	31 55	39 5	13	0	1	5	6	1	0	0
Piſcis Volans.	♎ ♏	11 14	19 11	67 82	52 35	8	0	0	0	0	7	1	0
Robur Carolinæ	♎ ♏	3 7	34 6	51 72	2 12	12	0	1	2	7	2	0	0
Sextans.	♌ ♍	19 13	59 5	1 19	21 43	41	0	0	0	1	7	32	1
Toucan.	♒	3 22	16 43	45 59	27 46	9	0	0	4	2	3	0	0
Triangulum.	♐	5 17	35 2	41 48	32 1	5	0	1	2	0	2	0	0
Xiphias.	♐ ♉	7 18	36 32	70 88	12 14	6	0	0	1	2	1	2	0

	Num	1	2	3	4	5	6	7
7. In the *Zodiac*,	943	7	11	43	94	169	445	174
In *Northern Hemiſphere*,	1511	4	23	93	227	356	695	113
In *Southern Hemiſphere*,	547	4	20	56	136	145	176	10
Sum of all the STARS,	3001	15	54	192	457	670	1316	297

 and

and all its Parallels cut the Horizon at Right Angles.

8. The Uſe of ſuch a Catalogue of Stars is very great, for from hence we learn, (1.) If any *new Stars* at any Time appear, which never have been obſerved before. (2.) If any Star, which now appears, ſhall in Time to come diſappear. (3.) If the *new Star* which ſhall appear be the ſame with a Star that has appeared formerly; and therefore, (4.) If the Stars have any periodical Times of Apparition. Hence (5.) The Means or Method of predicting the Appearing or Diſappearing of Stars. (6.) By a Catalogue of the Stars we compare their reſpective Places, Situations, and Diſtances with Eaſe. (7.) By this Means we alſo compare and determine the true Places and Motions of the heavenly Bodies in general, and of the Sun, Moon, Planets, and Comets in particular, with many other uſeful Purpoſes it ſerves beſides.

9. Now it is actually Fact, that ſome new Stars appear, and that others diſappear; yea, that they change their apparent Magnitude, and diſappear by Degrees. *Hipparchus* the firſt of Men obſerved a new Star, (120 Years before *Chriſt*) which occaſioned his making a Catalogue of the Stars. Another is ſaid to have appear'd *A. D.* 130; another *A. D.* 389; one exceeding bright in the 9th Century, and another in the Year 1264.

10. But the firſt *new Star*, of which we have any good Account, is that in the Chair of *Caſſiopeia*, firſt obſerved by *Cornelius Gemma* on the 9th of *November* 1572, and by *Tycho Brahe* on the 11th. Sir *Iſaac Newton* ſays it equalled *Venus* in Brightneſs at its firſt Appearance, and gradually declined in its Luſtre, till it totally diſappeared in the *March* following. This Star is ſuppoſed to be the ſame that appeared in the Years 945 and 1264, having its Period about 310 or 320 Years.

11. In *Aug.* 13, 1596, *D. Fabricius* obſerved another *new Star* in the *Neck of the Whale*; and through the 17th Century this Star was obſerved to appear and diſ-

appear

XIV. A *Parallel Sphere* is that whose Poles co-incide with the Poles of the Horizon,

appear periodically, its Period being equal to 333 Days. The Phænomena of this and the like Stars are supposed to be owing to the Spots on their Surface, which sometimes increase, and sometimes decrease, in the Manner as we have observed they do on the Surface of our Sun.

12. For that the Stars are really *Suns*, and have each a System of Planets, *&c.* about them, like ours, can be no Doubt to those who understand the Rules of Reasoning rightly, as I have before observed, *Annot.* CXXXI. And therefore, as they revolve about their Axes, those Spots may cause a great Alteration of Lustre, and sometimes wholly obscure them for a Time. But it is no Wonder if Bodies at such a Distance should have Appearances produced by Causes quite unknown to us. See more on this Head in Dr. *Long's Astronomy*.

13. As to the Distance of the Fixed Stars, we had but small Hopes of any Estimation of it, till Dr. *Bradley* began his Observations on them with an Instrument so very exact, as that he is of opinion, if the Parallax of a Star amounted to but one single Second, he must have observed it; and therefore that such a Star must be above 400000 Times farther from us than the Sun.

14. For if S represent the Sun, T the Earth, A T B its Orbit, and R a Star at such a Distance S R or T R, that the Semidiameter of the Orbit S T shall subtend an Angle T R S = $30'''$, or half a Second, then we find the Distance S R by this Analogy: Plate LXIII. Fig. 4.

As the Tangent of the Angle	TRS = $30''$ =	4.371914
Is to Radius	$90°$ =	10.000000
So is the Sun's Distance	S T = 1 =	0.000000
To the Distance of the Star	S R = 424700 =	5.628086

15. But the Distance of the Sun S T = 20000 Semidiameters of the Earth (see *Annot.* CXXXIV.); and

rizon, or *Zenith* and *Nadir*; and the Equinoctial with the Horizon; and all the Parallels parallel thereto.

XV. AN

ſuppoſing S R = (T R =) 400000 S T, then is the Diſtance of the Star from the Earth T R = 400000 × 20000 = 8000000000 Semidiameters of the Earth, or 8000000000 × 4000 = 32000000000000 Miles of *Engliſh* Meaſure. Hence it appears, that though the Velocity of Sound be ſo very great as at the Rate of 1142 Feet *per* Second, or 7000000 Miles *per Annum*, yet it would take up 4571430 Years to paſs from the neareſt Star to us. A Cannon-Ball would take up twice that Time to paſs from us to the Star; (ſee *Annot.* XXV. 4.) yea Light itſelf, with the inconceivable Velocity of 10000000 Miles *per Minute*, takes up more than 6 Years in coming from the Star to us. Therefore how immenſely great muſt thoſe Luminaries be, which appear ſo bright, and of ſuch different Magnitudes, at ſuch immenſe Diſtances!

16. The different apparent Magnitudes of the Stars are owing to their different Diſtances from us. Had we Teleſcopic Eyes, we ſhould ſee many more. *Seventy Stars*, and more, have been diſcovered in the *Pleiades* (commonly called the *Seven Stars*); and all that Tract of the Heavens called the *Milky Way* (or *Galaxy*) is well known to be owing to the Refulgence of a prodigious Multitude of Stars diſſeminated through thoſe Parts of the Univerſe, though at ſo great a Diſtance as to be inviſible to the naked Eye; yet are they diſcernable in great Numbers through a Teleſcope, and more in Proportion as the Inſtrument is better.

17. Hence likewiſe we account for that particular Phænomenon we call a *nebulous Star*, or cloudy faintiſh bright Spots that appear like Stars in an indirect View; for in order to this you have no more to do than only to direct a good Teleſcope to any one of them, and you will be agreeably ſurpriſed with a View of a great Multitude of very ſmall Stars, which were the Cauſe of the *luminous Spot* to the naked Eye.

18. To

XV. AN *Oblique Sphere* is that, one of whoſe Poles is above the Horizon, and the

18. To the very ſmall apparent Magnitude of the Stars we owe their conſtant *Twinkling*; for being but lucid Points, every opake Corpuſcle or Atom floating in the Air will be big enough to cover and eclipſe them, when they get in the right Line between the Star and the Eye; which Alternations of momentary Occultations and Apparitions make the Twinkling of the Stars we now ſpeak of.

19. I ſhall here give a fuller Account of the ſmall elliptic apparent Motion of each Star about its true Place, which I have already begun in a former *Annotation*. And in order to underſtand the Force of the Argument, the following Repreſentations are neceſſary, *viz.* Let S be the Sun, A B C D the Earth's Orbit; and from S ſuppoſe a Perpendicular erected, as S P, paſſing through a Star at P. Now if the Spectator were at S, he would view the Star in the ſame Perpendicular, and in its true Place P, projected in the Point *p* in the viſible Surface of the Heavens. But if the Spectator be carried about the Sun in the Circle A B C D, whoſe Diameter is ſenſible at the Diſtance P, or ſubtends a ſenſible Angle A P C, then in the Poſition A he will ſee the *Phænomenon* P in the Right Line A P *a*, projected in the Point *a*. For the ſame Reaſon in the Points B, C, D, the Star will appear in *b*, *c*, *d*; ſo that it will ſeem to have deſcribed the little Circle *a b c d*.

Plate LXIII. Fig. 5, 6, 7.

20. If the Diſtance of the Star S P be ſo great, that the Diameter of the Earth ſubtends no ſenſible Angle, but appears as a Point, then will alſo the ſmall Circle *a b c d* become inſenſible; and all the Lines A P, B P, *&c.* may be eſteemed perpendicular to the Plane of the Ecliptic, and be directed to the ſame Point in the Heavens with the Perpendicular S P, as to Senſe. So that in this Caſe the Star P would ever appear in the ſame Point *p*, if Light were propagated in an Inſtant.

21. But if in this very Caſe, in which the Star is ſo remote, Light be propagated in Time or with a certain Velocity, then as the Earth deſcribes its Orbit a Spectator

the other below it; and the Equinoctial
and

tator will ſee the Star in an oblique Direction, and not in the Perpendicular, as we have formerly ſhewn: That is, if G F be a Tangent to the Earth's Orbit in B, and B E perpendicular to the Plane of the Ecliptic in the Point B, then while the Earth moves through the indefinitely ſmall Arch G B, a Star at E will appear to move from E to *e*, or to be in *e* when the Earth arrives at B.

22. Now ſince the Diſtance S B is but a Point with reſpect to the great Diſtance S P of the Star, it follows, that we may refer the Spectator from the ſeveral Points A, B, C, D, to the central Point S, for obſerving the *Phænomena* of the Star at P, which will not be altered thereby. Therefore if *c a* be parallel to A C, and you make the Angle P S *a* equal to the Angle E B *e*, 'tis plain the Star P muſt appear in *a*, in the Direction S *a*. Alſo when the Earth is at D, the Star will be ſeen in the oblique Direction S *c* at *c*, the Spectator being referred to S.

23. For the like Reaſon, *viz.* becauſe *b d* is parallel or alike ſituated in reſpect of D B, and to the Tangents in D and B, therefore the Star at P will appear in *d* and *b* when the Earth is at C and A; and ſo during the Space of one Year the Star P will appear to deſcribe a ſmall Circle *a d c b*, ſuppoſing the Star in the Zenith E of the Spectator; but if the Star be at any Diſtance from the Zenith, the ſaid ſmall Circle will become an Ellipſe, as in *Fig.* 7.

24. Theſe ſmall elliptic Motions of the Stars occaſioned their Declinations to vary, and alſo their Diſtances from the Poles of the World, and that by the Space of $20'' \frac{1}{4}$ on one Side and on the other. Now this could not happen on any account of Refraction, becauſe the ſame Thing was as well obſerved of Stars near the Zenith, where there is no Refraction, as elſewhere ſituated. Nor could it reſult from any *Nutation* of the Earth's Axis; for that would have made the equal Diſtances of the Stars on oppoſite Sides of the Pole unequal, which never happened.

25. Neither

and its Parallels obliquely cutting the ſame (CXLVI).

THE

25. Neither can this be a *Parallactic Motion* of the Stars; for then while the Earth deſcribed the Half of its Orbit A B C, the Star would deſcribe the Semicircle *a b c*; whereas it is found by Obſervation, that the Star deſcribes the ſaid Semicircle *a b c* while the Earth deſcribes its Semi-Orbit B C D. (See *Art.* 22, 23.) Therefore it muſt ariſe ſolely from *the Velocity of Light bearing a ſenſible Proportion to the annual Motion of the Earth.*

(CXLVI.) 1. The three Poſitions of the Sphere here deſcribed are repreſented in ſo many Figures; the firſt of which is the *Direct* or *Right Sphere*, which is proper to thoſe People only who live under the Equinoctial Circle Æ Q, becauſe to them the Poles of the World P and S will both be in the Horizon H O. Plate LXV. Fig. 1.

2. The ſecond Figure repreſents the *Parallel Sphere*, where the Axis of the Earth P S is perpendicular to the Horizon, or the Poles P, S, are in the *Zenith* and *Nadir.* This Poſition of the Sphere is peculiar to the Parts of the Earth under each Pole; whoſe Inhabitants, if any there were, would perceive no circular Motion of the Sun, Moon, or Planets, nor any Motion of the Stars at all. But this muſt be underſtood of a Perſon ſtanding preciſely on the Ends of the Earth's Axis, which are the only Points on the Earth's Surface which have no real Motion, and conſequently which can produce no apparent Motion. Fig. 2.

3. The *Oblique Sphere* is repreſented in the third Figure. In this the Axis of the World P S makes an Angle P E O with the Horizon H O, of a greater or leſſer Number of Degrees according to the Latitude of the Place. Hence it appears, that all the Inhabitants of the Earth have ſuch a Poſition of the Sphere, except thoſe under the *Equinoctial* and the *Poles.* Fig. 3.

4. The Arch P O meaſures the *Altitude* or Height of the Pole, or what is commonly called the *Pole's Elevation*; and this Arch P O is ever equal to the Latitude of the Place Æ Z, as will eaſily appear thus: It is Æ Z

+ Z P

THE *Problems* on the *Celestial Globe* are the following.

PROB.

+ ZP = (ÆP = Quadrant =) ZP + PO = ZO; if therefore from the two equal Quadrants ÆP = ZO you subduct the common Part or Arch ZP, the remaining Arches ÆZ = PO; which was to be shewn.

5. Hence appears also the Reason of the Method of rectifying the Sphere or Globe for any given Place Z, or Latitude ÆZ, *viz.* because if the Pole P be elevated so high above the Horizon as the Place is distant from the Equator, the said Place will then be the highest Point of the Globe, and consequently that to which alone all the *Phænomena* of the Heavens and the Earth, in such a Position of the Globe, can agree.

6. Hence also we observe, that the Complement of the Latitude ZP is equal to the Elevation of the Equator ÆH above the Plane of the Horizon. For ÆZ + ZP = (ÆP = ZH =) ZÆ + ÆH; therefore subduct the common Part ÆZ, and there remains on each Side ZP = ÆH; which was to be shewn. Whence the Angle ZEP = ÆEH.

7. Any great Circle of the Sphere passing through the Zenith and Nadir Z and N, as ZEN, ZAN, are called *Azimuths* or *Vertical Circles*; of which that which passes through the East and West Points of the Horizon, as ZEN, is called the *Prime Vertical*. The Arch of the Horizon AE is the *Amplitude* of a Phænomenon emerging above the Horizon at the Point A; this is called the *Ortive Amplitude*, because it is *rising*; as on the Western Side it is called the *Occasive Amplitude*, because it is there *setting*. The Arch AB measured by a Quadrant of Altitude ZA is the *Altitude* of any Celestial Body at B, above the Horizon.

8. As I judge this a proper Place, I shall here explain the *Philosophical Principles* of GNOMONICS, or the *Art of* DIALLING. In order to this we are to consider, that as the Time which passes between any Meridian's leaving the Sun and returning to it again is divided into 24 Hours, so if we conceive a Sphere to be constructed with

PROB. I. *To rectify the Globe.*

ELEVATE the Pole to the Latitude of the Place, and every Thing as directed under

with 24 of these Meridians, the Sun will orderly come upon or be in one of them at the Beginning of every Hour. Such a Sphere may be represented by the Figure P D S B, where the several Meridians are represented by P 1 S, P 2 S, P 3 S, and so on to twice 12, or 24 in all. Plate LXV. Fig. 4.

9. Since these Meridians divide the Equinoctial into 24 equal Parts, each Part will contain just 15°, because $15 \times 24 = 360° =$ the whole Circle; and since all the Meridians pass through the Poles of the World, the Planes of those Meridians all intersect each other in one common Line P S, which is the Axis of the Sphere, therefore the said Axis P S is in the Plane of each of the 12 Meridians.

10. Suppose Z to be the Zenith of any Place, as *London*, and D W B E the Plane of the Horizon fixed within the Sphere, constructed with the said 12 *Meridians* or *Hour-Circles*, 1, 1, 2, 2, 3, 3, 4, 4, *&c.* then will the Axis of the Sphere P S pass through the Centre of the Plane at N, so that One-half N P will be above the Plane, and the other Half N S below it.

11. Suppose now this *Dialling-Sphere* to be suspended by the Point Z, and moved about so as to have the Points D and B exactly in the *South* and *North* Points of the Horizon, and E and W in the *East* and *West* Points; then will the Sphere have a Situation every way similar to that of the Earth and Heavens with respect to the given Place *London*, and the Axis of the Sphere to that of the Earth.

12. Therefore the Sun shining on such a Sphere will be attended with all the same Incidents, and produce all the same Effects, as would happen if the said Sphere were at the Centre of the Earth, or the Centre N of the Sphere coincided with the Centre of the Earth; because the Distance betwixt the Surface and Centre of the Earth is insensible at the Distance of the Sun.

13. Now

der PROB. II. of the *Terrestrial Globe*, which see.

PROB.

13. Now 'tis evident, as the Sun revolves about such a Sphere, it will every Hour be upon One-half or other of the 12 Hour-Circles, *viz.* from Midnight to Noon it will be on those Parts of the Circles which are in the *Eastern Hemisphere*, and from Noon to Midnight it will pass over all those in the *Western*. It is also farther evident, that while the Sun is in the Eastern Hemisphere it will be first below and then above the Plane of the Horizon, and *vice versa* on the other Side.

14. Again: When the Sun is upon any one of these Hour-Circles, by shining upon the Axis it causes it to cast a Shadow on the contrary Side, on the Plane of the Horizon, on the nether or upper Surface, as it is below or above the said Plane. This Shadow of the Axis will be precisely in the Line in which the Plane of the Hour-Circle would intersect the Plane of the Horizon: If therefore Lines were drawn through the Centre N, joining the Points on each Side the Plane where the Hour-Circles touch it, as 4 N 4, 5 N 5, 6 N 6, *&c.* the Shadow of the Axis will fall on those Lines at the Beginning of each respective Hour, and thereby indicate the Hour-Circle the Sun is in for every Hour of the Day.

15. These Lines are therefore properly called *Hour-Lines*; and amongst the rest, that which represents the Hour of 12 at Noon is N B, half the Meridian-Line D B; whence it appears, that the Hour-Lines N 1, N 2, N 3, *&c.* which serve for the Afternoon, lie on the East Side of the Plane, and are numbered from the North to the East; and on the contrary.

16. It also appears, that as the Sun's Altitude above the Plane is greater or less, the Number of Hour-Circles the Sun will possess above the Horizontal Plane will be also greater or less. Thus when the Sun is at S in the Equinoctial, its *diurnal Path* for that Day being the Equinoctial Circle itself Æ E Q W, 'tis plain, since the Arch Æ E = E Q, the Sun will apply to six Hour-Circles

PROB. II. *To find the* SUN's PLACE *in the Ecliptic.*

FIND

Circles below the Horizon, and to six above it, in each Half of the Day; and consequently, that on that Day the Shadow will occupy but 12 of the Hour-Lines on each Surface of the Plane, beginning and ending at 6.

17. But when the Sun is in the Tropic of *Cancer*, its diurnal Path for that Day being the Tropic itself T C R F, 'tis manifest the Sun in the Forenoon ascends above the Plane in passing between the Hour-Circles of 3 and 4 in the Morning, and descends below it in the Afternoon between the Hours of 8 and 9: Therefore in the Summer-Tropic the Shadow will pass over 16 of those Hour-Lines. And *vice versa*, when the Sun is in the Winter-Tropic at O, its Path being then O G I H, it rises above the Plane between 8 and 9, and leaves it between 3 and 4.

18. From what has been said 'tis evident, that if the Circles be supposed removed, and only the horizontal Plane remain, with the Half of the Axis N P above it, in the same Position as before, then should we have constituted an HORIZONTAL DIAL, every way the same with those in common Use, as represented in the next Figure, with only the Addition of a Substyle P O, to render the Style N P very firm. Plate LXV. Fig. 5.

19. Hence appears the Reason why the *Gnomon* or Style N P in those Dials is always directed to the North Pole, and always contains such an Angle P N O with the Hour of 12 N B as is equal to the Latitude of the Place: Lastly, the Reason why the Number of Hour-Lines on these Dials exceeds not 16, and are all drawn from 6 to 12 and 6 again on the Northern Part, the rest on the Southern; and why the Hour-Line of 6 lies directly *East* and *West*, as that of 12 does *North* and *South*.

20. If a Plane be fixed with the same Sphere in a Vertical Position, or perpendicular to the Horizon, and coinciding with the Plane of the *Prime Vertical*, i. e. facing full South and North; then will the Axis P S Fig. 6.

still

FIND the Day of the Month in the Calendar on the Horizon, and right againſt it is the Degree of the Ecliptic which the Sun is in for that Day.

PROB.

ſtill paſs through the Center of the Plane N, and the lower Semi-axis N S will by its Shadow mark out the Hour-Lines on the Southern Surface, and the upper Semi-axis N P will do the ſame on the Northern. Theſe Hour-Lines are determined in the ſame Manner as thoſe on the Horizontal Dial; and it is plain, the Sun cannot come on the Southern Face of this Plane before Six in the Morning, nor ſhine on it after Six in the Evening.

21. Alſo it is evident, that all the Hours before Six in the Morning, and after Six at Night, will be ſhewn on the Northern Face or Side of this Plane, for the Time of the Sun's being above the Horizon in any Place. Hence the Reaſon of a *Direct South and North Vertical Dial* eaſily appears; the latter of which is here repreſented apart from the Sphere, with its Style N S, Subſtyle, and Hour-Lines: And the ſame may be conceived for a *North Erect Dial.* Plate LXV. Fig. 7.

22. The *Gnomon* N S contains an Angle S N D = Z N P with the Meridian or Hour-Line of 12, *viz.* Z D, which is exactly the Complement of the former P N B to 90 Degrees; or the Elevation of the *Gnomon* is in theſe equal to the Complement of the Latitude of the Place: And what has been ſaid about the Reaſon of the Hour-Lines is the ſame for the Half-Hours, Quarters, &c. Likewiſe if the *Rationale* of a *Direct South Dial* be underſtood, Nothing can be difficult to underſtand of a Dial which does not face the South or North directly, but declines therefrom any Number of Degrees towards the Eaſt or Weſt. But they who would know more of the Mathematical Structure and Calculations for all Sorts of Dials, may have recourſe to the Second Volume of my *Mathematical* INSTITUTES.

PROB. III. *To find the Sun's* DECLINATION:

RECTIFY the Globe, bring the Sun's Place in the Ecliptic to the Meridian, and that Degree which it cuts in the Meridian is the Declination required.

PROB. IV. *To find the Sun's* RIGHT ASCENSION:

BRING the Sun's Place to the Meridian, and the Degree in which the Meridian cuts the Equinoctial is the Right Ascension required.

PROB. V. *To find the Sun's* AMPLITUDE:

BRING the Sun's Place to the Horizon, and the Arch of the Horizon between it and the East or West Point is the Amplitude, North or South.

PROB. VI. *To find the Sun's* ALTITUDE *for any given Day and Hour:*

BRING the Sun's Place to the Meridian; set the Hour-Index to the upper XII. then turn the Globe till the Index points to the given Hour, where let it stand; then screwing the Quadrant of Altitude in the *Zenith*, lay it over the Sun's Place, and the Arch contained between it and the Horizon will give the Degrees of Altitude required.

PROB. VII. *To find the Sun's* AZIMUTH *for any Hour of the Day:*

EVERY Thing being done as in the laſt Problem, the Arch of the Horizon contained between the North Point and that where the Quadrant of Altitude cuts it, is the *Azimuth* Eaſt or Weſt as required.

PROB. VIII. *To find the Time when the Sun riſes or ſets:*

FIND the Sun's Place for the given Day; bring it to the Meridian, and ſet the Hour-Hand to XII. then turn the Globe till the Sun's Place touches the Eaſt Part of the Horizon, the Index will ſhew the Hour of its Riſing: After that turn the Globe to the Weſt Part of the Horizon, and the Index will ſhew the Time of its Setting for the given Day.

PROB. IX. *To find the Length of any given Day or Night:*

THIS is eaſily known by taking the Number of Hours between the Riſing and Setting of the Sun for the Length of the Day; and the Reſidue to 24, for the Length of the Night.

PROB. X. *To find the Hour of the Day, having the Sun's Altitude given:*

BRING the Sun's Place to the Meridian, and ſet the Hour-Hand to XII.; then turn the

the Globe in ſuch Manner, that the Sun's Place may move along by the Quadrant of Altitude, (fixed in the *Zenith*) till it touches the Degree of the given Altitude; where ſtop it, and the Index will ſhew on the *Horary* Circle the Hour required.

PROB. XI. *To find the Place of the* MOON *or any* PLANET, *for any given Day:*

TAKE *Parker's* or *Weaver's* *Ephemeris*, and againſt the given Day of the Month you will find the Degree and Minute of the Sign which the Moon or Planet poſſeſſes at *Noon*, under the Title of *Geocentric Motions*. The Degree thus found being marked in the Ecliptic on the Globe by a ſmall Patch or otherwiſe, you may then proceed to find the *Declination*, *Right Aſcenſion*, *Latitude*, *Longitude*, *Altitude*, *Azimuth*, *Riſing*, *Southing*, *Setting*, &c. in the ſame Manner as has been ſhewn for the Sun.

PROB. XII. *To explain the Phænomena of the* HARVEST-MOON.

IN order to this we need only conſider, that when the Sun is in the Beginning of *Aries*, the Full Moon on that Day muſt be in the Beginning of *Libra*: And ſince when the Sun ſets, or Moon riſes, on that Day, thoſe Equinoctial Points will be in the Horizon, and the Ecliptic will then be

Vol. III. *Plate* LXVI. P. 277.

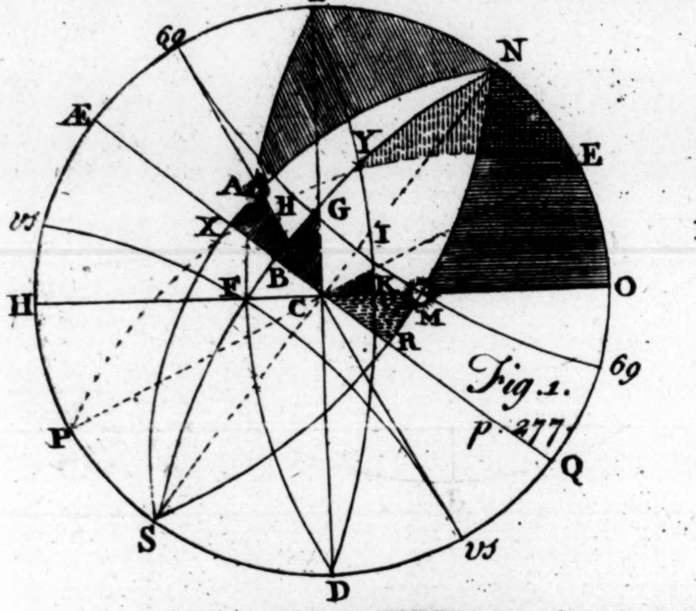


Fig. 1. p. 277.

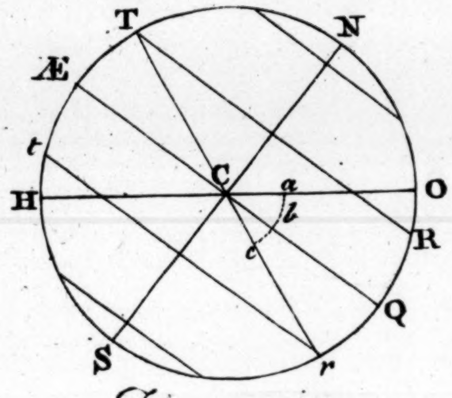


Fig. 2. p. 279.

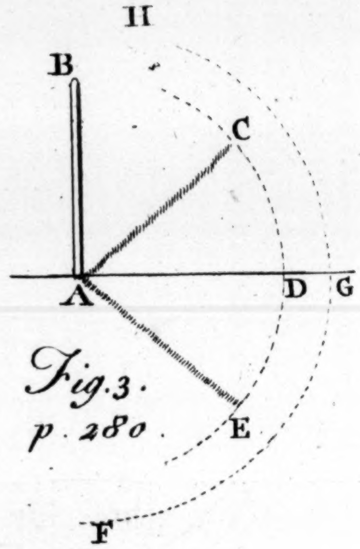


Fig. 3. p. 280.

Fig. 4. p. 281.

Fig. 5. p. 281.

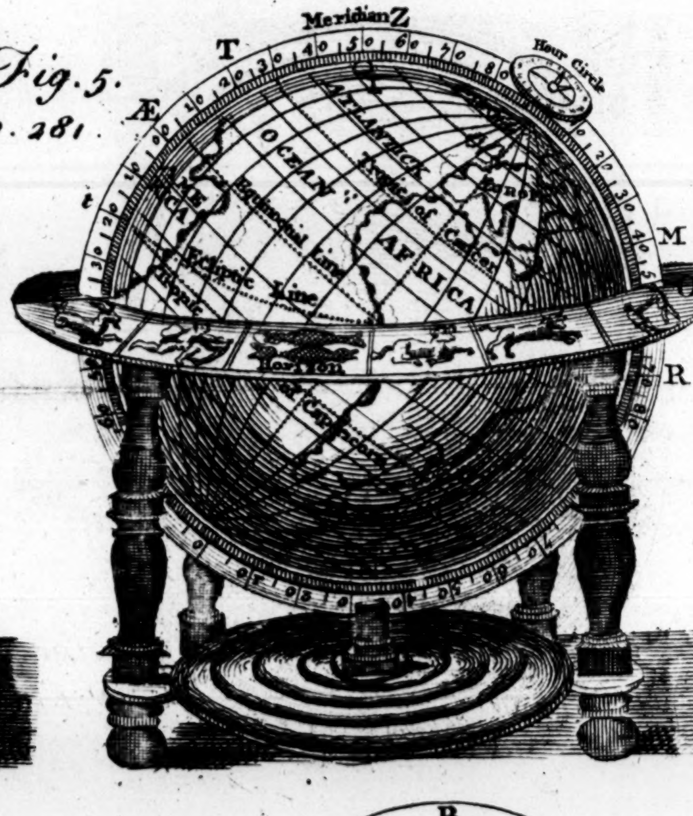


Fig. 6. p. 298.

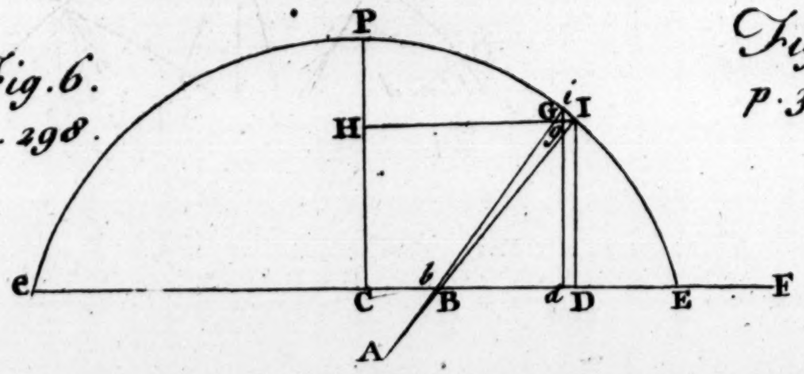
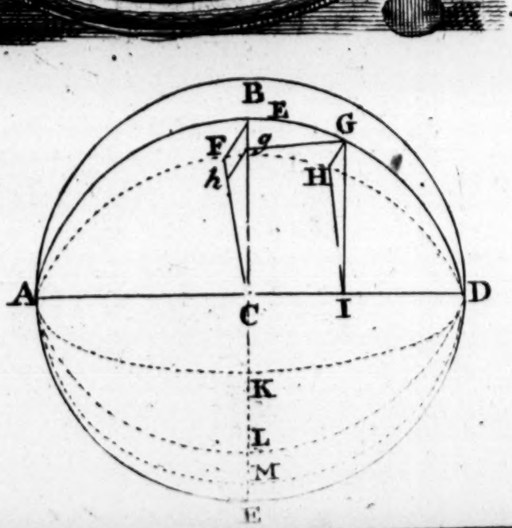


Fig. 7. p. 306.



vens, and all the Stars on the Globe will be in ſuch Situations as exactly correſpond to thoſe in the Heavens; which may therefore be eaſily found, as will be ſhewn.

PROB. XIV. *To find the Hour when any known Star will riſe, or come upon the Meridian:*

RECTIFY the Globe, and ſet the Index to XII.; then turn the Globe till the Star comes to the Horizon or Meridian, and the Index will ſhew the Hour required.

PROB. XV. *To find at what Time of the Year any given Star will be on the Meridian at* XII. *at Night.*

BRING the Star to the Meridian, and obſerve what Degree of the Ecliptic is on the North Meridian under the Horizon; then find in the Calendar on the Horizon the Day of the Year againſt that Degree, and it will be the Day required. (CXLVII.)

THESE

(CXLVII.) 1. I ſhall here repreſent the Caſes of theſe Aſtronomical Problems, as they are performable by the Circles of the Celeſtial Globe, or by the *Stereographical Projection* of the Sphere *in Plano*. Thus;

Let Æ N Q S be the General Meridian.

N S the Axis of the Sphere. Plate LXVI. Fig. 1.

Æ Q the Equinoctial Line.

H O the Horizon of *London*.

♋ C ♑ the Ecliptic, or Sun's Path.

Z D the *Prime Vertical* or *Azimuth*.

THESE are the chief *Problems* on the *Celestial Globe*: We now proceed to those on

E P the Axis of the Ecliptic.
N A S an Hour-Circle or Meridian.
Z A D an Azimuth Circle.
E Y P a Circle of Longitude.
♋ I ♋ the Tropic of *Cancer*.
♑ ♑ the Tropic of *Capricorn*.

2. By Means of these Circles various Spherical Triangles are formed for Calculation. Thus let A be the Place of the Sun in the Ecliptic; then in the Right-angled Triangle A X C we have

C A the Sun's *Place*, or Longitude from the Equinox C.
A X the Sun's *Declination* North.
C X the Sun's *Right-Ascension*.
A C X the Angle of Obliquity of the *Ecliptic*.

3. And supposing the Sun rising in the Horizon at M on the Day of the Summer Tropic, and N M S an Hour-Circle; then there is formed the Right-angled Triangle N O M, in which we have

N O = Æ Z = the *Latitude* of the Place Z.
M O the *Amplitude* from the North.
N M the Complement of the Sun's *Declination* R M.
O N M the Angle of the Hour from Midnight.
O M N the Angle of the Sun's *Position*.

4. On the same Tropical Day the Sun is at I at Six o'Clock, because the Hour-Circle of Six is projected upon the Axis N C S; therefore in the Right-angled Triangle I C K we have

I K the Sun's *Altitude* at Six.
C K the Azimuth from the East at Six.
C I the *Declination* North.
I C K the *Latitude* of the Place.

5. Again; when the Sun on the same Day comes to the *Prime Vertical* Z C D, his Place when due *East* and *West* is at G; therefore in the Right-angled Triangle G B C we have

G B the Sun's *Declination* North.
G C the Sun's *Altitude* when East or West.

B C

on the *Terrestrial*; but shall first premise the following Definitions relating thereto.

I. THE

BC the *Hour* of his being due East or West.
BCG the *Latitude* of the Place.

6. Suppose the Sun in the Horizon at M once more; then in the Right-angled Triangle MCR we have
CM the *Amplitude* from East to West.
MR the *Declination* North.
CR the *Ascensional Difference*.
RCM the *Co-Latitude* of the Place.
RMC the Angle of *Position*.

7. In the Oblique Triangle AZN we have
ZN the *Co-Latitude* of the Place Z.
AN the *Co-Declination*.
AZ the Complement of the *Altitude* AF.
ANZ the *Hour* from Noon equal to ÆX.
AZN the Azimuth from the North.

And the same may be done for any Star at A, or any other Place.

8. Lastly, let Y be any Star, then in the oblique Triangle YNE we have
YE the *Co-Latitude* of the Star.
YN the *Co-Declination* of the Star.
NE = Æ ♋ = the *Obliquity* of the Ecliptic.
NEY the Star's *Longitude* in the Ecliptic.
ENY the *Hour* from Midnight.

9. For the Canons and Method of Calculation I shall refer the Reader to the second Volume of my *Young Trigonometer's Guide*, what I have done being as much as the Nature of the Subject at present requires: And those who have no Globes may solve most of these (and many other) Problems by my *Synopsis Scientiæ Cœlestis*, at a very small Expence, and with the greatest Exactness.

10. The Reason of the *Phænomenon* we call the HARVEST-MOON is extremely easy by the Globe, and may also be represented in a Diagram thus. Let HO be the Horizon, ÆQ the Equinoctial; then will T r be the Ecliptic, when the Beginning of *Aries* is in the Western Plate LXVI. Fig. 2.

I. THE LATITUDE of any Place is its Diſtance from the Equator towards either Pole;

Weſtern Horizon; but when the other Equinox is there, *t* R will be the Poſition of the Ecliptic. On the *Vernal Equinox* if a Full-Moon happens, it will be at C in the Eaſtern Horizon at Riſing; in one Day the Moon will deſcribe the Arch C *c*; wherefore the following Night ſo much Time will intervene between Six o'Clock and the Hour of the Moon's riſing, as is ſpent in the Motion of the Globe while the Arch C *c* is aſcending above the Horizon.

11. Whereas at the oppoſite Time of the Year, *viz.* at the *Autumnal Equinox*, if a Full-Moon happen, then the next Night the Moon's diurnal Arch to be elevated above the Horizon is C *b* = C *c*; but ſince the Poſition of C *b* is ſo much nearer to the Horizon than C *c*, it will aſcend much ſooner above it, *viz.* in about one-fifth Part of that Time, and ſometimes in leſs, becauſe the Moon's Orbit ſometimes makes a greater Angle with the Horizon than T C H = *a* C *c*, and ſometimes a leſs Angle than *t* C H = *a* C *b*. But for more on this Subject ſee my *Philoſophical Grammar*.

12. Becauſe in many Caſes it is abſolutely neceſſary to have a MERIDIAN-LINE at hand, I ſhall here ſhew the beſt Way of making or drawing ſuch a one on any Plane where the Sun can ſhine, thus. Let a ſtrait Plate LXVI. Fig. 3. Braſs Pin or Steel Wire A B be fixed upright in the Point A, on which Point as a Centre you had before deſcribed ſeveral concentric Circles, as C D E, F G H, &c. Now to make the Pin A B exactly perpendicular, let three Points be choſen in the outmoſt Circle, as F, G, H, in which place one Foot of the Compaſſes, and extend the other to the Top of the Pin B. The Pin is to be bent one way and the other, till the ſaid Point of the Compaſſes will fall nicely on the Middle of the Top B from each Point of the Circle F, G, H, and then is the Pin well adjuſted.

13. Then obſerving in the Forenoon where the Top of the Shadow A C touches any one Circle, there make a Mark;

Pole; and is reckoned in Degrees of the General Meridian, beginning at the Equator.

II. LONGITUDE is the Diſtance between the Meridian of any Place, and the firſt or ſtanding Meridian, reckoned in the Degrees of the Equator towards the Eaſt or Weſt.

III. A CLIMATE is a Space of the Earth's Surface, parallel to the Equator, where the Length of the Day is *half an Hour* longer in the Parallel which bounds it on the North, than in that which terminates it on the South.

IV. A ZONE is alſo a Diviſion of the Earth's Surface parallel to the Equator, in regard of the different Degrees of *Heat and Cold*, which we have deſcribed in the preceding Lecture.

a Mark, as at C; and then in the Afternoon make a Mark at E, where the Shadow's Point is in the ſame Circle again. Then biſect the Arch C E in the Point D, through which and the central Point A draw the Line A D, and it will be the *Meridian Line* required. If this be done in ſeveral Circles, the Operation will be the more exact and certain.

14. I have here added the Figures of the Celeſtial and Terreſtrial Globe, with all the principal Circles and their Names, as they are rectified for the Latitude of *London*. *Note*, Theſe Globes I now make from the late Mr. *Senex*'s Plates, of 3, 9, 12, 17, and 27 Inches Diameter, and with many new Additions, Corrections, and Improvements. Plate LXVI. Fig. 4, 5.

V. THE

V. THE ANTOECI are thoſe Inhabitants of the Earth who live under the ſame Meridian, but on oppoſite Parallels, and are therefore equally diſtant from the Equator. Their Noon and Midnight are at the ſame Time; the Days of one are equal to the Nights of the other; and their Seaſons of the Year are contrary.

VI. THE PERIOECI are thoſe People who live under the ſame Parallel, but oppoſite Meridians. The ſame Pole is elevated and depreſs'd to both; are equally diſtant from the Equator, and on the ſame Side; when Noon to one, it is Midnight to the other; the Length of Days to one is the Complement of Night to the other, and the contrary; and the Seaſons of the Year are the ſame to both, at the ſame Time.

VII. THE ANTIPODES are thoſe who live *Feet to Feet*, or under *oppoſite Parallels and Meridians*. They are equally diſtant from the Equator on different Sides; have the contrary Poles equally elevated; the Noon of one is Midnight to the other; the longeſt Day or Night to one is ſhorteſt to the other; and the Seaſons of the Year are contrary, *&c.*

VIII. ALSO the Inhabitants of the Torrid Zone are called AMPHISCII, becauſe their *Shadows fall on both Sides* of them.

IX.

IX. THOSE of the Frigid Zone are called PERISCII, becauſe their *Shadows fall all around* them.

X. AND the Inhabitants of the Temperate Zones are called HETEROSCII, becauſe they caſt *their Shadows only one way*.

XI. A CONTINENT is the largeſt Diviſion or Space of Land, comprehending divers Countries and Kingdoms, not ſeparated by Water.

XII. AN ISLAND is any ſmall Tract of Land ſurrounded by Water.

XIII. A PENINSULA is a Part of Land encompaſſed with Water all around, except on one Part, which is called

XIV. AN ISTHMUS, being that narrow Neck of Land which joins it to the Continent.

XV. A PROMONTORY is a mountainous Part of Land ſtanding far out in the Sea; whoſe Fore-part is call'd a *Cape* or *Head-Land*.

XVI. THE OCEAN is the largeſt Collection of Waters, which lies between, and environs the Continents.

XVII. THE SEA is a ſmaller Part of the aqueous Surface of the Earth, interceding the Iſlands, Promontories, *&c.*

XVIII. A GULF is a Part of the Sea every where environed with Land, except on one ſmall Part call'd

XIX. A STRAIT, which is that narrow Paſſage joining it to the adjacent Sea.

XX. A LAKE is any large Quantity of ſtagnant Water entirely ſurrounded by Land.

THE other Parts of Land or Water need no explanation.

I SHALL now proceed to the Solution of the moſt uſeful *Problems* on the *Terreſtrial Globe*, firſt premiſing, that *the Latitude of a Place is equal to the Elevation of the Pole at that Place*; for if the Arch of the Meridian between the Place and the Pole be added to the Latitude of the Place, it makes 90 Degrees; alſo if it be added to the Pole's Elevation, or Arch between the Pole and Horizon, the Sum is 90 Degrees: Whence the Propoſition is evident.

PROB. I. *To find the Latitude of any Place:*

BRING the given Place to the Brazen Meridian, and obſerve what Degree it is under, for that is the Latitude required.

PROB. II. *To rectify the Globe for any Place:*

RAISE

RAISE the Pole ſo high above the Horizon, as is equal to the Latitude of the Place; ſcrew the Quadrant of Altitude in the *Zenith*; find the Sun's Place, and bring it to the Meridian; ſet the Hour-Hand to the upper XII.; and place the Globe North and South by a Needle; then is it a juſt Repreſentation of the Globe of the Earth, in regard of that Place, for the given Day at Noon.

PROB. III. *To find the Longitude of a given Place:*

BRING the Place to the Brazen Meridian, and obſerve the Degree of the Equator under the ſame, for that expreſſes the Longitude required.

PROB. IV. *To find any Place by the Latitude and Longitude given:*

BRING the given Degree of Longitude to the Meridian, and under the given Degree of Latitude you will ſee the Place required.

PROB. V. *To find all thoſe Places which have the ſame Latitude and Longitude with thoſe of any given Place:*

BRING the given Place to the Meridian, then all thoſe Places which lie under the Meridian have the ſame Longitude: Again, turn the Globe round on its Axis; then all thoſe

those Places, which pass under the same Degree of the Meridian with any given Place, have the same Latitude with it.

PROB. VI. *To find all those Places where it is Noon at any given Hour of the Day in any Place:*

BRING the given Place to the Meridian; set the Index to the given Hour; then turn the Globe, till the said Index points to the upper XII.; and observe what Places lie under the Brass Meridian, for to them it is Noon at that Time.

PROB. VII. *When it is Noon at any one Place, to find what Hour it is at any other given Place:*

BRING the first given Place to the Meridian, and set the Index to the upper XII.; then turn the Globe till the other given Place comes to the Meridian, and the Index will point to the Hour required.

PROB. VIII. *For any given Hour of the Day in the Place where you are, to find the Hour of the Day in any other Place:*

BRING the Place where you are to the Meridian; set the Index to the given Hour; then turn the Globe about, and when the other Place comes to the Meridian, the Index

dex will ſhew the Hour of the Day there, as required.

PROB. IX. *To find the Diſtance between any two Places on the Globe in* Engliſh *Miles:*

BRING one Place to the Meridian, over which fix the Quadrant of Altitude; and then laying it over the other Place, count the Number of Degrees thereon contained between them; which Number multiply by 69 and a half, (the Number of Miles in one Degree) and the Product is the Number of *Engliſh* Miles required.

PROB. X. *To find how any one Place bears from another:*

BRING one Place to the Braſs Meridian, and lay the Quadrant of Altitude over the other; and it will ſhew on the Horizon the Point of the Compaſs on which the latter bears from the former.

PROB. XI. *To find thoſe Places to which the Sun is vertical in the Torrid Zone, for any given Day:*

FIND the Sun's Place in the Ecliptic for the given Time, and bring it to the Meridian, and obſerve what Degree thereof it cuts; then turn the Globe about, and all thoſe Places which paſs under that Degree of the Meridian are thoſe required.

PROB.

PROB. XII. *To find what Day of the Year the Sun will be vertical to any given Place in the Torrid Zone.*

BRING the given Place to the Meridian, and mark the Degree exactly over it; then turn the Globe round, and obſerve the two Points of the Ecliptic which paſs under that Degree of the Meridian: Laſtly, ſee on the Wooden Horizon, on what Days of the Year the Sun is in thoſe Points of the Ecliptic; for thoſe are the Days required.

PROB. XIII. *To find thoſe Places in the North Frigid Zone, where the Sun begins to ſhine conſtantly without ſetting, on any given Day between the 10th of* March *and the 10th of June.*

FIND the Sun's Place in the Ecliptic for the given Day; bring it to the general Meridian, and obſerve the Degrees of Declination; then all thoſe Places which are the ſame Number of Degrees diſtant from the Pole, are the Places required to be found.

PROB. XIV. *To find on what Day the Sun begins to ſhine conſtantly without ſetting, on any given Place in the North Frigid Zone, and how long:*

RECTIFY the Globe to the Latitude of the Place; and, turning it about, obſerve what Point of the Ecliptic between *Aries* and

and *Cancer*, and alſo between *Cancer* and *Libra*, coincides with the North Point of the Horizon; then find, by the Calendar on the Horizon, what Days the Sun will enter thoſe Degrees of the Ecliptic, and they will ſatisfy the Problem.

PROB. XV. *To find the Place over which the Sun is vertical, on any given Day and Hour:*

FIND the Sun's Place, and bring it to the Meridian, and mark the Degree of Declination for the given Hour; then find thoſe Places which have the Sun in the Meridian at that Moment; and among them, that which paſſes under the Degree of Declination is the Place deſired.

PROB. XVI. *To find, for any given Day and Hour, thoſe Places wherein the Sun is then riſing, or ſetting, or on the Meridian; alſo thoſe Places which are enlighten'd, and thoſe which are not.*

FIND the Place to which the Sun is vertical at the given Time, and bring the ſame to the Meridian, and elevate the Pole to the Latitude of the Place; then all thoſe Places which are in the Weſtern Semicircle of the Horizon have the Sun *riſing*, and thoſe in the Eaſtern Semicircle ſee it *ſetting*; and to thoſe under the Meridian it is *Noon*.

 Laſtly,

Lastly, all Places above the Horizon are enlightened, and all below it are in Darkness or Night.

PROB. XVII. *The Day and Hour of a Solar or Lunar Eclipse being given, to find all those Places in which the same will be visible:*

FIND the Place to which the Sun is vertical at the given Instant, and elevate the Globe to the Latitude of the Place; then in most of those Places above the Horizon will the Sun be visible during his Eclipse; and all those Places below the Horizon will see the Moon pass through the Shadow of the Earth in her Eclipse.

PROB. XVIII. *The Length of a Degree being given, to find the Number of Miles in a great Circle of the Earth, and thence the Diameter of the Earth:*

ADMIT that one Degree contains $69\frac{1}{2}$ *English* Statute Miles; then multiply 360 (the Number of Degrees in a great Circle) by $69\frac{1}{2}$, and the Product will be 25020, the Miles which Measure the Circumference of the Earth. If this Number be divided by 3.1416, the Quotient will be $7963\frac{86}{100}$ Miles for the Diameter of the Earth.

PROB. XIX. *The Diameter of the Earth being known, to find the Surface in Square Miles, and its Solidity in Cubic Miles:*

ADMIT

ADMIT the Diameter be 7964 Miles; then multiply the Square of the Diameter by 3.1416, and the Product will be 199250205 very near, which are the Square Miles in the Surface of the Earth. Again, multiply the Cube of the Diameter by 0.5236, and the Product 264466789170 will be the Number of Cubic Miles in the whole Globe of the Earth.

PROB. XX. *To express the Velocity of the diurnal Motion of the Earth:*

SINCE a Place in the Equator describes a Circle of 25020 Miles in 24 Hours, 'tis evident the Velocity with which it moves is at the Rate of $1042\frac{1}{2}$ in one Hour, or $17\frac{3}{10}$ Miles *per* Minute. The Velocity in any Parallel of Latitude decreases in the Proportion of the *Co-Sine of Latitude to the Radius*. Thus, for the Latitude of *London*, 51 Deg. 33 Min. say,

As Radius — —	10.000000
To the Co-sine of Lat. 51 Deg. 30 M.	9.794149
So is the Velocity in the Equator, $17\frac{3}{10}$ M.	2.232046
To the Velocity of the City of *London* $10\frac{8}{10}$ M.	2.032195

U 2 That

That is, the City of *London* moves about the Axis of the Earth at the Rate of $10\frac{6}{10}$ Miles every Minute of Time. But this is far ſhort of the Velocity of the annual Motion about the Sun; for that is at the Rate of 60000 Miles *per* Hour, or about 1000 Miles each Minute, ſuppoſing the Diameter of the annual Orbit to be 82 Millions of Miles (CXLVIII).

THERE

(CXLVIII.) 1. I might here ſhew how the ſeveral Spherical Triangles are formed for the Solution of moſt of theſe *Geographical Problems*, as I did before for the *Aſtronomical* ones; but as the Method is the ſame, I need not again repeat it. However, to facilitate the Ideas of the above Definitions, *&c.* I have added (as before mentioned) a Print of each Globe. The *Rationale* of the ſeveral Methods of ſolving Problems of this Sort cannot be well ſhewn without an Eye upon the Globe, and a Praxis *cum vivâ Voce* with a Demonſtrator.

2. I ſhall here ſubjoin a few Things relating to the Magnitude of the Earth, and the Dimenſions of the ſeveral Parts, together with the Manner of acquiring the Knowledge thereof. Firſt then, the moſt natural, eaſy, and certain Method of doing this is, by firſt *meaſuring the Length of a Degree of Latitude* in the Meridian of any Place; becauſe if the Meaſure of *one Degree* be once found, the Earth being ſuppoſed round, 'tis plain all the other Meaſures may eaſily be deduced from this.

3. Thus if I take the Height of the North Pole-Star in this Place with a very good Quadrant or Sextant, and then proceed directly Northward or Southward, till by

THERE is a geometrical Method of describing the Superficies of the Celestial and Terrestrial

by the same Instrument I find the said Star raised or depressed just *one* Degree; then 'tis evident I must have pass'd over just one Degree on the Earth's Surface, which therefore might be known by actual Mensuration, were it possible to find such a Part of the Earth's Surface as is exactly even and spherical, and truly in the same Meridian.

4. Now this is scarcely to be expected any where, except in such a Country as *Holland*, which is level, and when overflowed with Water, and that frozen into Ice, the icy Surface may be near the Truth; and a Degree measured in the Meridian upon this Ice must of course be pretty exact, if due Regard be had to Refractions in taking the Height of the Pole. Thus *Snellius* actually measured the Distance between a Tower at *Leyden* and another at *Souterwode* three Times over, and then a straight Line in the Meridian on the Ice, whence by a Trigonometrical Process he measured a Degree; but as some Mistakes had been made in the Calculations, the indefatigable Mr. *Muschenbroek* attempted the Thing anew, and formed Triangles upon the fundamental Base of *Snellius* in the Year 1700, and found 57033 *Toises* to a Degree.

5. Now this was but 27 Toises less than had been found by the Royal Academy of *Paris*; and this was but little different from the Measure of a Degree some Time before by our Countryman *Norwood*, which resulted from his measuring the Distance between *London* and *York*, which he did in the Year 1653; and according to him the Length of a Degree is 69½ of *English* Miles.

6. Mr. *Greaves* compared the *English* Foot taken from the Iron-Standard in *Guild-Hall*, *London*, with the Standards of divers Nations. The Proportion between some of them is as follows:

Terreſtrial Globe on a Plane; and this is call'd the *Projection of the Sphere in Plano.*

Thus,

The *Engliſh* Foot,	1.000
The preſent *Roman* Foot,	0.967
The *Grecian* Foot,	1.007
The *Paris* Foot,	1.068
The *Leyden* or *Rhinland* Foot,	1.033
The *Bologna* Foot,	1.250

7. If the *French* Meaſure of a Degree, *viz.* 57069 Toiſes, be corrected by making proper Allowances for the *Preceſſion of the Equinoxes*, the *Aberration of Light* in the Stars be made uſe of, and the *Refraction of Light* through the Air (all which were neglected by *Picard*) the true Meaſure of a Degree at *Paris* will be 56925,7 Toiſes.

8. Now ſince the famous *Caſſini* and Sir *Iſaac Newton* had both of them ſhewn the Earth could not naturally have a *ſpherical Form*, but muſt be a *Spheroid*; and ſince theſe great Men differed in their Accounts of what Sort the Spheroid was; Sir *Iſaac* ſhewing it to be an *Oblate Spheroid*, and *Caſſini* ſtrongly contending for the *Oblong Spheroid*; the King of *France* was nobly inclined to have this important Affair decided, and accordingly ordered the Length of a Degree to be meaſured at the Equator, and at the Polar Circle; that by comparing them with the Length of a Degree near *Paris*, it might be known whether the Earth were oblong or flat towards the Poles.

10. And ſince a Determination of the Figure of the Earth, and its Dimenſions by actual Menſuration, is a Problem of the higheſt Concern in *Navigation*, *Aſtronomy*, *Geography*, *Levelling*, *Hydraulics*, &c. I think it quite neceſſary the Reader ſhould have an Idea of the Manner in which this was effected by the *French* Mathematicians, and which therefore I ſhall give from the Book entitled, *The Figure of the Earth determined*, &c. by *Maupertuis*.

11. The

Vol. III. *Plate* LXVII. P. 295.

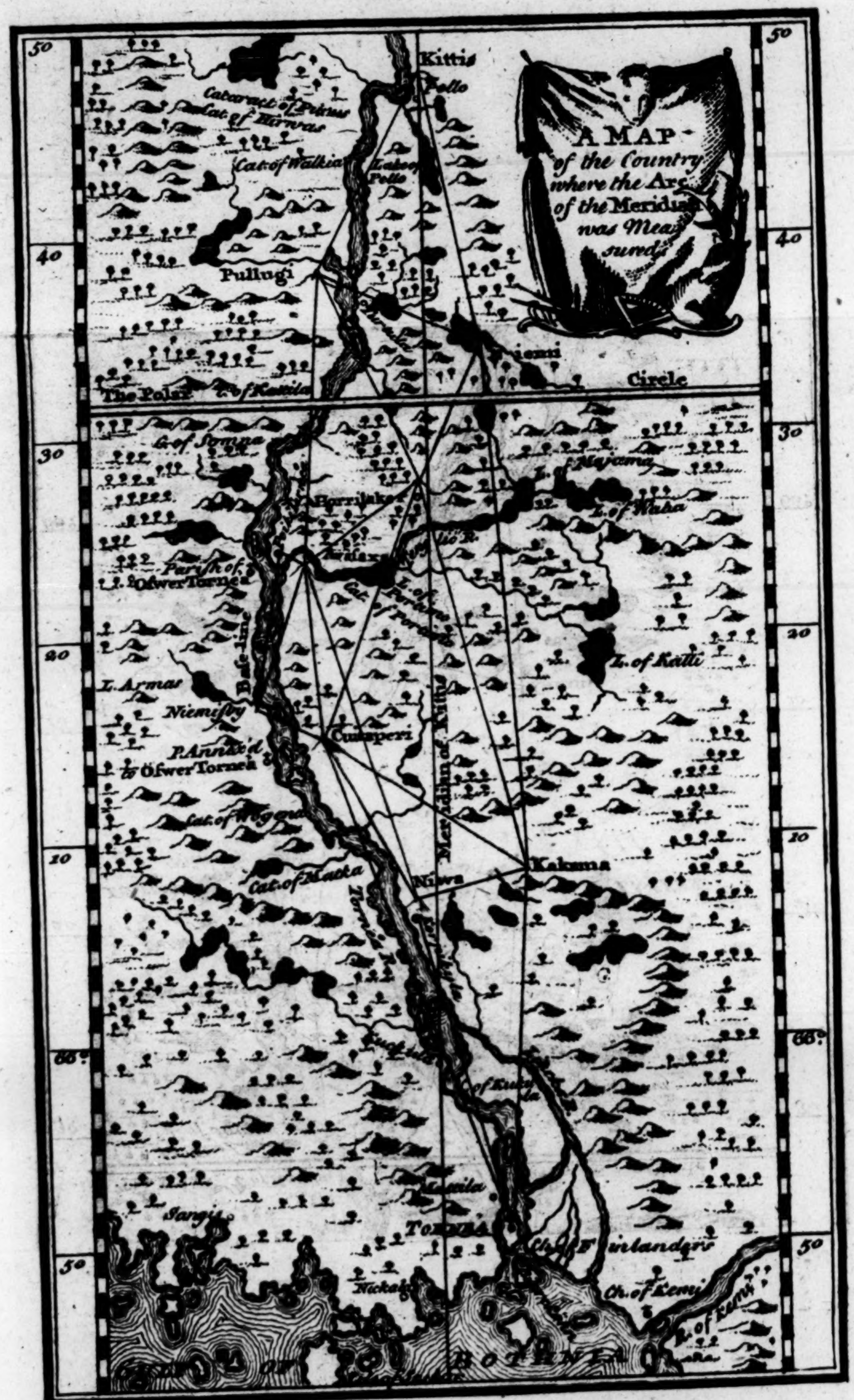
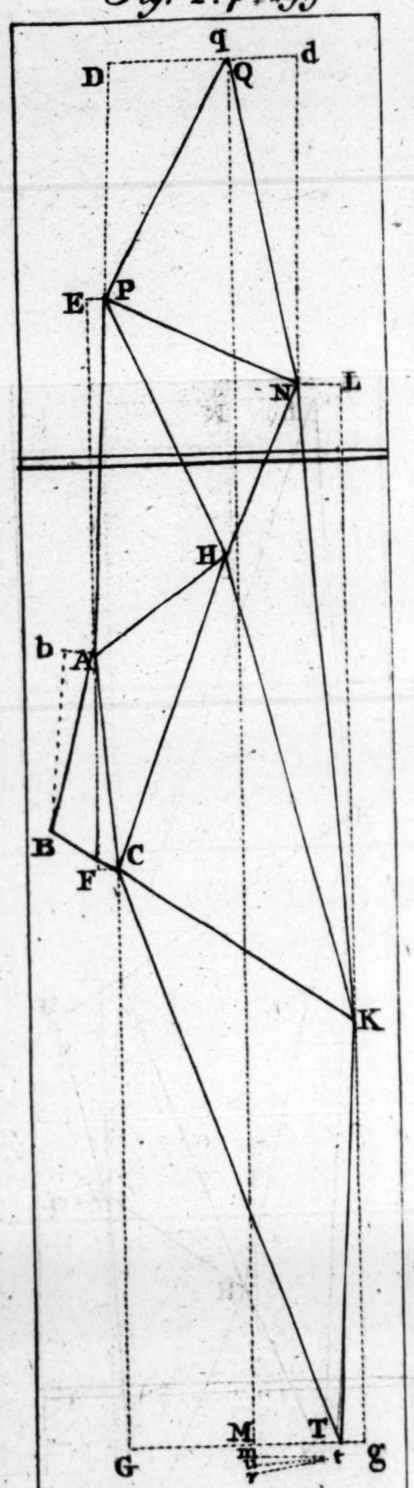


Fig. 2. *p.* 295.



DP + EF + CG = 54940.39.t

dN + Lg = 54944.76.

at a mean. QM = 54942.57.

Thus, One-Half of the Globe is projected on one Side of the Plane, and the other Half

11. The arduous Task was performed in *Lapland* by Messieurs *Clairaut*, *Camus*, *Le Monnier*, *Maupertuis*, the *Abbé Outhier*, and M. *Celsius* of *Upsal*. They sat out for *Stockholm*, and from thence for the Bottom of the Gulph of *Bothnia*. Being arrived at *Tornea*, they began their Work; for from thence they sat out, *July* 6, 1736, to reconnoitre the Country, of which I have here added their Map, by which the Affair is made easy to understand. Plate LXVII.

12. After twelve Hours Voyage up the River, they came to the Hamlet *Korpykila*, and from thence through the Forest they went on Foot to the steep Mountain *Niwa*, whose Summit (a bare Rock) they made their first Station. Farther up the River they met with another high Mountain called *Avasaxa*, on the Top of which they built a *Signal*. They then went up the River *Tenglio*, and crossed a Morass to the great Mountain *Horrilakero*, where they built another *Signal*. From hence they returned back again, and in their Way crossed the Forest to another very steep Mountain call'd *Cuitaperi*, which afforded a very fair Prospect to all the rest.

13. After this they went some to one Part and some to another, and built Signals on the Summits of other Mountains, *viz.* *Kakama*, *Pullingi*, *Niemi*, and *Kittis*, near the Village *Pello*. Then taking the Angles which the visual Rays made, connecting the several Signals by a Quadrant of two Feet Radius, furnished with a Micrometer, they constituted a *Heptagonal* Figure T C A P Q N K, extending from the Tower of the Church of *Tornea* at T, to *Kittis* at Q. Fig. 2.

14. And because the Truth of their Work may the better appear, I shall here set before the Reader the Sum of all the Angles, of which the several Angles of the Heptagon did consist, *viz.*

Half on the other; and if the Plane be that of the Ecliptic or Equinoctial, as in the

		°	′	″
1. The Angle	CTK =	24	22	54,5
2. The Angle TCA =	KCT =	37	9	12
	KCH =	100	9	56,8
	HCA =	30	56	53,4
3. The Angle CAP =	CAH =	112	21	48,6
	HAP =	53	45	56,7
4. The Angle APQ =	APH =	31	19	55,5
	HPN =	37	22	2,1
	NPQ =	87	52	24,3
5. The Angle	PQN =	40	14	52,7
6. The Angle QNK =	QNP =	51	53	4,3
	PNH =	93	25	7,5
	HNK =	27	11	53,3
7. The Angle NKT =	NKH =	9	41	47,7
	HKC =	43	45	35,6
	CKT =	118	28	12
	The Sum of all,	900	1	37

15. But since the Angles of any Polygon are equal to twice the Number of Right Angles that the Figure has Sides, abating 4, therefore the Sum of the Angles of a Heptagon is $14-4 = 10 \times 90° = 900°$. Hence if their Heptagon had been taken on a Plane, it would have exceeded the Truth but by 1′ 37″; but since the Figure lay on a convex Surface, the Sum ought to be a little more than 900°. And thence it appears to what a surprising Degree of Exactness they attained in this Undertaking.

16. Now in order to measure the *Meridian-Line* QM, which lay through the Middle of the Heptagon, or rather the Line q m, which was the correct Distance between the two Parallels where they made their *Astronomical Observations* with a Sector, (whose Accuracy is incredible, and of a Structure not here to be described) I say,

the Case of the Celestial Globe, these Projections are then call'd the *Celestial Hemispheres.*

I say, in order to measure this Line q m, it was necessary to begin with some Base-Line to be first of all measured, and then to compute a fundamental Triangle or two, for the Grounds of their future Work.

17. Thus they pitched on the Distance between *Niemisby* and the Village *Poiki*, for the *Base-Line* B *b*, because it lay along the River, and could be most accurately measured on the Ice. It was measured twice over.

	Toises.	*Feet.*	*In.*
The first Mensuration gave —	7406	5	0
The second ———	7406	5	4
The mean Length therefore is	7406	5	2

18. Having this Base-Line known, they calculated the two Triangles A B *b* and A B C, from which they found the Distance between *Avasaxar* and *Cuitaperi* to be 8659,94 = A C; from whence they proceeded to find the Sides and Angles of all the other Triangles round the Figure, as A H C, A H P, P Q N, C T K, *&c.* and from thence having found the Sides A P, P Q, N K, K T, T C, they formed the Right-angled Triangles A E P, A F C, P D Q, C G M, by drawing E F, G C, P D, at Right Angles to the Parallels passing through Q and M, and parallel to the Meridian Line Q M; and the same they did on the other Side the Figure, as is there represented.

19. Having thus measured the several Lines, they were found as follows:

		On the other Side.
P D =	9350,45	N d = 13297,88
A E =	14213,24	K L = 24995,83
A F =	8566,08	K g = 16651,05
Total,	54940,39 — —	54944,76
		54940,39

Therefore at a Mean the Meridian Line is Q M = 54942.57

spheres. But with regard to the Terrestrial Globe, they are generally made on the Plane

20. By very accurate Methods they deduced the Length of the Line q m = 55020,09 Toises, and the still more correct Distance q u = 55023,47. But this Distance or Arch q u, by the nicest Astronomical Observations and Corrections, was found to be equal to 57′ 28″,67 of a Degree. Therefore as 57′ 28″,67 is to 55023,47 Toises, so is 60′, or 1 Degree, to 57437,9 Toises in one Degree at the *Arctic* Circle.

21. If therefore from the Length of a Degree here, *viz.*

at the *Arctic* Circle, — — —	57437,9
you subduct the Length of a Degree at *Paris*, by *Picard*,	56925,7
the Difference will be — —	512,2 Toises,

or 3282,878 Feet of *English* Measure.

22. Hence, having the Length of a Degree, the Radius of Curvature is found for any Part of the Elliptic Meridian. For let R denote that Radius, then it is 3,1416 : 1 :: 360 : 2 R :: 180 : R; therefore $R = \frac{180}{3{,}1416}$, or $R = \frac{189 \times 57437{,}9}{3{,}1416}$ Toises, for the Curvature of the Earth's Surface in the Latitude of 66° 20′ at *Lapland*; and in *France* the Radius of the Earth's Curvature is $R = \frac{180 \times 56925{,}7}{3{,}1416}$ Toises; or in *English* Miles those Radii are R = 3994, and *R* = 3958,4.

23. We are now prepared to assign the Proportion of the Axis of the Earth to the Diameter of the Equator from this actual Mensuration; in order to which we must first of all premise some *Theorems*, which result from the Properties of the Ellipse. Therefore let E P *e* denote the Elliptic Surface of the Earth, E *e* the Diameter of the Equator, and C P the Semi-axis of the Earth. Let A I be the Radius of Curvature to any Point I of the Ellipsis, I F a Tangent; and draw H I

Plate LXVI. Fig. 6.

and

Plane of the *General Meridian or Horizon*, and then they are commonly call'd *Maps* *of*

and I D perpendicular to C P and C E. Take the Arch I i = 1 Degree, and Draw A i and the Perpendicular $i\,d$; there A is the Centre of a Circle touching the Ellipsis in the Points I, i. The Angle I B E = D I F is the Latitude of the Place I. Now put z = C E, u = C P, x = C D = H I, y = D I; and let 1, t, s, denote the Radius, Tangent, and Secant of the Angle I B D; and lastly, let A I = r.

24. Therefore	1	$tt + 1 = s^2$, and so $t^2 - s^2 = 1$. *Theor.* I.
Let	2	$z^2 : u^2 :: 1 : a$, $\because \frac{u^2}{z^2} = a$. *Theor.* II.
Hence	3	$\frac{u^2}{z} = a\,z = p$, the Parameter. *Theor.* III.
Again	4	$1 : s :: y :$ I F $= s\,y$. *Theor.* IV.
Also	5	$1 : t :: y :$ D F $= t\,y$. *Theor.* V.
And	6	$t : 1 ::$ I F $= s\,y :$ I B $= \frac{s\,y}{t}$. *Theor.* VI.
And	7	$t : 1 :: y :$ D B $= \frac{y}{t}$. *Theor.* VII.
Then *per Conics*	8	E D × D e : D I^2 :: C E^2 : C P^2.
That is	9	$zz - xx : yy :: zz : uu :: 1 : a$. *Theor.* VIII.
Also because	10	C D : C E :: C E : C F, *per Conics*,
That is,	11	$x : z :: z : x + t\,y$,
We have	12	$z\,z = x^2 + t\,y\,x$, or $z^2 - x^2 = t\,y\,x$.
Therefore (9)	13	$1 : a :: t\,y\,x : y^2 :: t\,x : y$.
Hence	14	$y = a\,t\,x$, or $x = \frac{y}{a\,t}$. *Theor.* IX.
And also	15	$a\,x = \frac{y}{t} =$ D B. *Theor.* X.
Since (14)	16	$y = a\,t\,x \because z^2 = x^2 + x\,t\,y = x^2 + a\,t^2\,x^2$.
Wherefore	17	$z^2 = x^2 \times \overline{1 + a\,t\,t}$, $\because z = x\sqrt{1 + a\,t\,t}$. *Th.* XI.
Whence also	18	$x^2 = \frac{z^2}{1 + a\,t\,t}$, $\because x = \frac{z}{\sqrt{1 + a\,t\,t}}$. *Theor.* XII
Therefore also	19	$y = \frac{a\,t\,z}{\sqrt{1 + a\,t\,t}}$. *Theor.* XIII.

Because

of the World: And the ſeveral Circles, and Parts of the Surface of one Hemiſphere are ſo

Becauſe	20	$DC : DB :: x : ax :: 1 : a :: z^2 : u^2$
Theref. Conv.	21	$CD : CB :: 1 : 1-a :: Cd : Cb$.
Whence	22	$Dd(=gI) : Bb :: 1 : 1-a$.
Alſo we have	23	$GI : gI :: BF : DF$, becauſe $IGi : FBI$.
Wh. conjointly	24	$GI : Bb :: BF : DF - a \times DF$.
Again	25	$GI : Bb :: AI : AB :: BF : DF - a \times DF$.
Conv. and Inv.	26	$IB : AI :: BD + a \times DF : BF = BD + DF$.
In Species	27	$\frac{sy}{t} : r :: \frac{y}{t} + aty : \frac{y}{t} + ty$.
That is,	28	$\frac{sy}{t} : r :: 1 + att : 1 + tt = ss$.
Whence (15, 18)	29	$r = \frac{s^3 y}{t + at^3} = \frac{s^3 ax}{1 + att} = \frac{s^3 az}{\overline{1+att}^{\frac{3}{2}}}$. *Theor.* XIV.
Whence alſo	30	$z = \frac{r \times \overline{1+att}^{\frac{2}{3}}}{s^3 a}$, and $zz = \frac{r^2 \times \overline{1+att}^3}{s^6 a^2}$. *T.* XV.
Therefore	31	$z^{\frac{2}{3}} = \frac{r^{\frac{2}{3}} + r^{\frac{2}{3}} att}{s^2 a^{\frac{2}{3}}} = \frac{r^{\frac{2}{3}} + r^{\frac{2}{3}} att}{a^{\frac{2}{3}} + a^{\frac{2}{3}} tt}$. *Theor.* XVI.
Whence alſo	32	$tt = \frac{r^{\frac{2}{3}} - a^{\frac{2}{3}} z}{a^{\frac{2}{3}} z^{\frac{2}{3}} - ar^{\frac{2}{3}}}$. *Theor.* XVII.
For any other Lat. we have	33	$zz = \frac{r^2 \times \overline{1+att}^3}{s^6 a^2} = \frac{\mathrm{r}^2 \times \overline{1 + a\mathrm{tt}}^3}{\mathrm{s}^6 a^2}$
Whence it is	34	$r^{\frac{2}{3}}\mathrm{ss} + r^{\frac{2}{3}} att\mathrm{ss} = \mathrm{r}^{\frac{2}{3}} ss + \mathrm{r}^{\frac{2}{3}} a\,\mathrm{tt}ss$.
Therefore laſtly	35	$a = \frac{r^{\frac{2}{3}} ss - r^{\frac{2}{3}}\mathrm{ss}}{r^{\frac{2}{3}} ttss - \mathrm{r}^{\frac{2}{3}}\mathrm{ttss}}$. *Theor.* XVIII.

25. From theſe Theorems we can calculate whatever relates to the Figure and Magnitude of the Earth; and firſt to determine the Value of a, or the Ratio of z^2 to n^2, that is, of CE to CP. In order to this, we have r, s, t, for the Latitude of 60° 20′ at *Lapland*; and r, s, t, for the Latitude of 49° 22′, being the Middle of

so delineated on the said Plane, as they would appear thereon to an Eye placed in the

of the Degree measured in *France*. (See *Art.* 22.) For having $r = 3994$, and $r = 3958{,}4$; whence by Logarithms we have $r^{\frac{2}{3}} ss = 593$, and $r^{\frac{2}{3}} ss = 1552{,}9$; also $r^{\frac{2}{3}} ttss = 3090{,}1$, and $r^{\frac{2}{3}} ttss = 2109$: Therefore $a = \frac{959{,}3}{981{,}1} = \frac{u^2}{z^2}$. Whence we get $z : u :: 313{,}22 : 309{,}72 :: CE : CP$. Therefore by Mensuration it appears, that CE exceeds CP in a greater Proportion than that of 230 to 229, as was observed in the *Scholium* of *Annot.* XXXIV.

26. Put $\begin{cases} r = az \text{ at E}, \\ r = \frac{z}{\sqrt{a}} \text{ at P}; \end{cases}$ then $az : \frac{z}{\sqrt{a}} :: d : D$; hence $\frac{d}{\sqrt{a}} = aD$; and $d = a\sqrt{a} \times D = \frac{u^3}{z^3} D$; and $z^3 d = u^3 D$; therefore $d : D :: z^3 : u^3$.

27. The Theorem $z = CE = \frac{r \times \overline{1 + att}^{\frac{3}{2}}}{s^3 a}$ is (by Logarithms) equal to 3971,1 Miles; and so the Diameter of the Equator is equal to 7942,2 Miles. Whence because $\frac{u}{z^2} = a$, $u = z\sqrt{a} = CP = 3926{,}2$ Miles; and so the Axis of the Earth is equal to 7852,4 Miles; so that the Equatorial Diameter exceeds the Axis by 89,8 or 90 Miles, which is near three times as much as the Theory gave it. See *Annot.* XXXIV. 36.

28. In any given Latitude the Radius of Curvature is found by *Theorem* XIV. *viz.* $r = \frac{as^3 z}{\overline{1 + att}^{\frac{3}{2}}}$; and because under the Pole P the Angle IBE is a Right one, s and t will in that Case become infinite and equal; and therefore $r = \frac{z}{\sqrt{a}} = 4016{,}6$ Miles, which is the greatest

the Pole or Middle Point of the other Hemisphere. Hence it will come to pass, that the

greatest of all. And under the Equator that Angle vanishes, and there $s = 1$, and $t = 0$; and so $r = az =$ 3881,8 Miles the least of all.

29. The Radius of Convexity being known, we find the Length of a Degree in any Latitude by this Analogy; As 180 : 3,1416 :: $r : \frac{3,1416\, r}{180}$ = the Length of the Degree required. Thus under the Equator we have $\frac{3,1416}{180} \times 3881,8 = 67\frac{1}{4}$ Miles, for the least Degree; and under the Pole we have $\frac{3,1416}{180} \times 4016,6 = 70\ \frac{1}{10}$ Miles, for the greatest Degree of Latitude: A *mean Degree* therefore is 68,92 Miles. Thus also a Degree in the Latitude 49° 22′ is $\frac{3,1416}{180} \times 3858,4 = 69,087$ Miles; and in the Latitude 66° 20′ it is $\frac{3,1416}{180} \times$ $3994,1 = 69,709$ Miles.

29. If the Length of a Degree be known, the Radius of Convexity may be determined, and thence the Latitude of the Place, by *Theor.* XVII. $tt = \frac{r^{\frac{2}{3}} - a^{\frac{2}{3}} z^{\frac{2}{3}}}{a^{\frac{2}{3}} z^{\frac{2}{3}} - a r^{\frac{2}{3}}}$; for if the Tangent of an Angle be known, the Angle itself, that is, the Latitude, is known also.

31. Hence also the Radius of any Parallel of Latitude may be discovered; for, by *Theorem* XII. HI = CD $= x = \frac{z}{\sqrt{1 + att}}$; and 180 : 3,1416 :: x : a Degree of Longitude in the given Parallel. In the Equator $x = z$; hence $\frac{3,1416}{180} \times 3971,1 = 69,309$ Miles, the Length of a Degree in the Equator.

32. Hence

the Stars and Constellations of the Hemispheres, and the Parts of Land and Water in

32. Hence the Circumference of the Earth under the Equator is 360 × 69,309=24951 Miles. I might now proceed to calculate the Surface and Solidity of the Earth as a Spheroid; but the Process would be tedious, and answer no great Purpose, enough having been said for any Person to form a proper Idea of the Magnitude and Figure of the Earth. I conclude with observing, that there is about $2\frac{1}{4}$ Miles between the greatest and least Degree of Latitude within the Compass of our common Charts: *Quære* then, *If our Theory of Navigation, founded upon an Hypothesis of their being all equal, be not very erroneous; and if it be not necessary to have one corrected according to the foregoing Measures?* See my *New Treatise* of GEOGRAPHY and NAVIGATION lately published, containing new Tables of *Meridional Parts* and *Sea Charts*, adapted to the *Spheroid* of the Earth, and fitted for Practice; being calculated to every Minute of a Degree.

33. *A* TABLE *of Arcs of the Meridian to the Spheroid, in Minutes of the Equator.* By the Rev. Mr. *Murdoch.*

D.	Spheroid.	Sphere.	Diff.	D.	Spheroid.	Sphere.	Diff.
1	58.7	60.0	1.3	10	587.0	600.0	13.0
2	117.3	120.0	2.7	11	645.8	660.0	14.2
3	176.0	180.0	4.0	12	704.5	720.0	15.5
4	234.7	240 0	5.3	13	763.3	780.0	16.7
5	293.4	300.0	6.6	14	822.1	840.0	17 9
6	352.1	360.0	7.9	15	880.9	900.0	19.1
7	410.8	420.0	9.2	16	939.7	960.0	20.3
8	469.6	480.0	10.4	17	998.5	1020.0	21.5
9	528.3	540.0	11.7	18	1057.4	1080.0	22.6

in the Maps are not repreſented in their natural and juſt Diſtances, and in their due Magnitudes

D.	*Spheroid.*	*Sphere.*	*Diff.*	*D.*	*Spheroid.*	*Sphere.*	*Diff.*
19	1116.3	1140.0	23.7	39	2299.2	2340.0	40.8
20	1175.2	1200.0	24.8	40	2358.7	2400.0	41.3
21	1234.1	1260.0	25.9	41	2418.2	2460.0	41.8
22	1293.0	1320.0	27.0	42	2477.7	2520.0	42.3
23	1350.0	1380.0	28.0	43	2537.3	2580.0	42.7
24	1411.0	1440.0	29.0	44	2596.8	2640.0	43.2
25	1470.0	1500.0	30.0	45	2656.6	2700.0	43.4
26	1529.0	1560.0	31.0	46	2716.4	2760.0	43.6
27	1588.1	1620.0	31.9	47	2776.2	2820.0	43.8
28	1647.2	1680.0	32.8	48	2835.9	2880.0	44.1
29	1706.3	1740.0	33.7	49	2895.5	2940.0	44.5
30	1765.5	1800.0	34.5	50	2955.3	3000.0	44.7
31	1824.7	1860.0	35.3	51	3015.2	3060.0	44.8
32	1883.9	1920.0	36.1	52	3075.0	3120.0	44.0
33	1943.1	1980.0	36.9	53	3135.0	3180.0	45.0
34	2002.4	2040.0	37.6	54	3194.9	3240.0	45.1
35	2061.7	2100.0	38.3	55	3254.9	3300.0	45.1
36	2121.0	2160.0	39.0	56	3314.9	3360.0	45.1
37	2180.4	2220.0	39.6	57	3370.0	3420.0	45.0
38	2239.8	2280.0	40.2	58	3435.1	3480.0	44.9

Magnitudes and Forms, as on the Globes themſelves: Yet moſt of the Problems of either Globe are performable on theſe artificial Projections, by thoſe who underſtand their Nature and Uſe. But theſe Things will be beſt underſtood from a View of

D.	Spheroid.	Sphere.	Diff.	D.	Spheroid.	Sphere.	Diff.
59	3495.2	3540.0	44.8	75	4460.8	4500.0	39.2
60	3555.3	3600.0	44.7	76	4521.3	4560.0	38.7
61	3615.7	3660.0	44.5	77	4581.9	4620.0	38.1
62	3675.7	3720.0	44.3	78	4642.5	4680.0	37.5
63	3736.0	3780.0	44.0	79	4703.1	4740.0	36.9
64	3796.2	3840.0	43.8	80	4763.7	4800.0	36.3
65	3856.5	3900.0	43.5	81	4824.3	4860.0	35.7
66	3916.8	3960.0	43.2	82	4884.9	4920.0	35.1
67	3977.2	4020.0	42.8	83	4945.5	4980.0	34.5
68	4037.5	4080.0	42.5	84	5006.2	5040.0	33.8
69	4097.9	4140.0	42.1	85	5066.8	5100.0	33.2
70	4158.4	4200.0	41.6	86	5127.5	5160.0	32.5
71	4218.8	4260.0	41.2	87	5188.2	5220.0	31.8
72	4279.3	4320.0	40.7	88	5248.8	5280.0	31.2
73	4339.8	4380.0	40.2	89	5309.5	5340.0	30.5
74	4400.3	4440.0	39.7	90	5370.2	5400.0	29.8

thoſe Prints, and a Specimen of the Praxis of their Uſe (CXLIX).

(CXLIX.) 1. The Solution of moſt of theſe GEOGRAPHICAL PROBLEMS may be performed by a *Trigonometrical Calculation*, as is evident from the Original Diagram we before made uſe of for the Solution of *Aſtronomical Problems*. Thus if A and Z be any two Places on the Surface of the Globe, then in the Triangle AZN we have

Plate LXVI. Fig. 1.

Z N the *Co-Latitude* of the Place Z.

A N the *Co-Latitude* of the Place A.

Z A the *Diſtance* of the Places A and Z from one another.

Z N A the *Difference of Longitude*.

A Z N the Angle of *Poſition* or *Bearing* of A from Z.

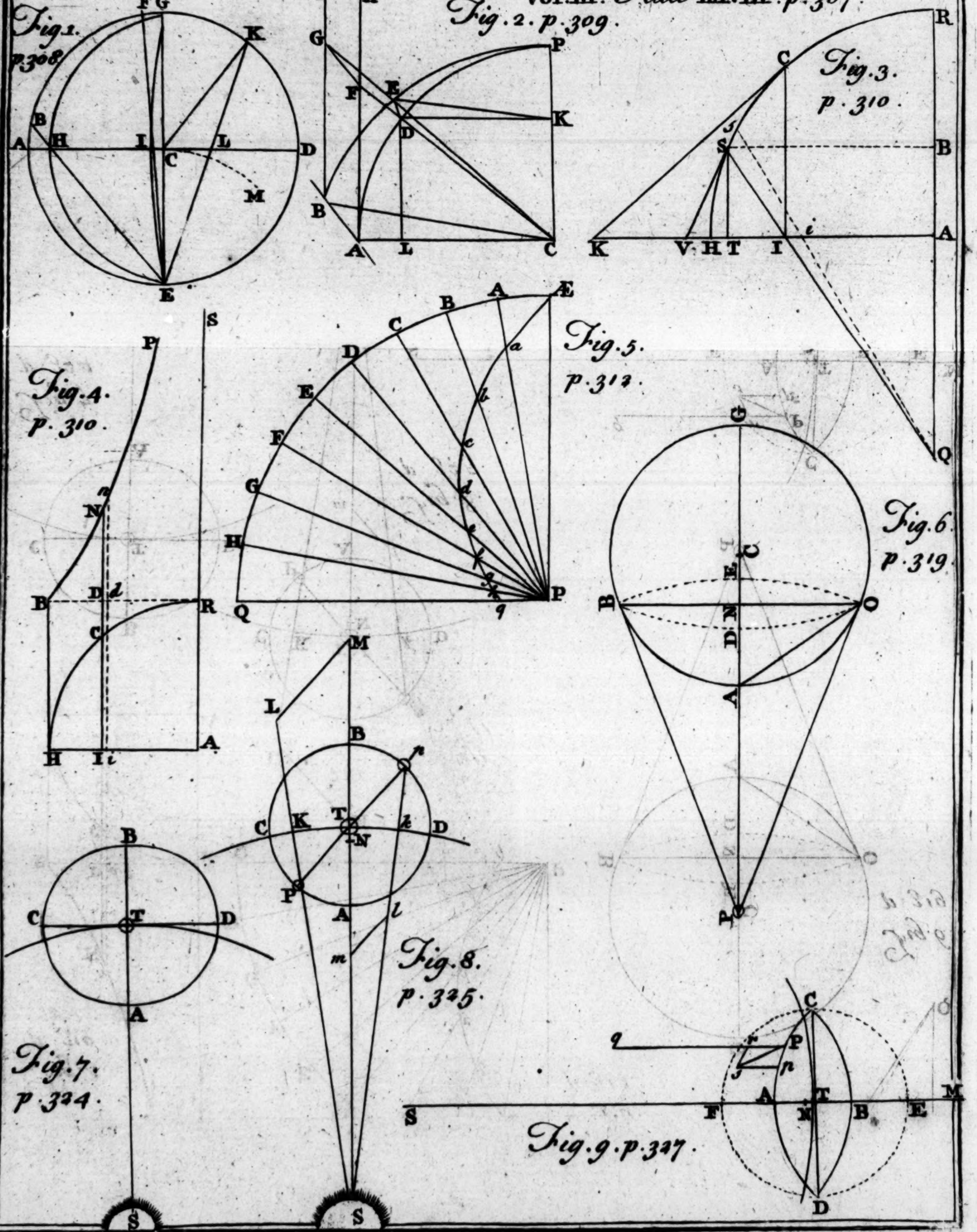
Z A N the Angle of *Poſition* or *Bearing* of Z from A.

2. After the ſame Manner may Problems of NAVIGATION be ſolved; and indeed the only true and natural Way of SAILING is upon the *Arch* of a *Great Circle*, which gives the neareſt Diſtance between any two Places on the Surface of the Globe; and therefore the nearer a Ship keeps to the Arch of a Great Circle, the ſhorter will her Way or Paſſage be from one Place to another. Thus in the ſame Triangle Z N A, if it be propoſed to ſail from Z to A, the Ship ought to be directed upon the Arch Z A. But in order to be acquainted with this Method of Sailing, the Doctrine of the Sphere muſt be well underſtood; therefore I ſhall refer the Reader who deſires it, to Vol. II. of my *Young Trigonometer's Guide*.

3. However, I ſhall here ſubjoin the *Philoſophical Principles* of all Kinds of Geographical and Nautical MAPS and CHARTS: And firſt I ſhall ſhew the Nature of what is call'd the ORTHOGRAPHIC PROJECTION of

Fig. 7. the SPHERE. Let A B D E be the Primitive Circle, or Plane of the Projection, which we may ſuppoſe to be a Meridian; and let A E D be a Great Circle elevated above

Vol. III. Plate LXVIII. p. 307.



above the Plane in any Angle BAE. Suppose this Circle to be projected on the Plane into the Curve AFD, by Perpendiculars passing through every Point thereof; it is required to find the Nature of the projected Curve AFD.

4. In order to this, let EF and IG be two Perpendiculars; draw GI parallel to CE, and HI parallel to CF, and G*g* to CI; and from *g* let fall the Perpendicular *gh*; then is the Right-angled Triangle GHI equal and similar to *gh*C, and *gh*C is similar to EFC. Wherefore putting $AC = EC = a$, $CI = x$, $GI = y$, $CE = b$, and $HI = \mathrm{y}$; then by the Property of the Circle we have $AI \times ID = GI^2$, that is, $yy = aa - xx$, and $y = \sqrt{a^2 - x^2} = GI$; but GI : HI :: (*g*C : *h*C ::) EC : FC, that is, $y : \mathrm{y} :: a : b$; therefore $\mathrm{y} = \frac{by}{a} = \frac{b}{a}\sqrt{a^2 - x^2}$, which shews the Curve AFD to be an *Ellipse*, whose Semi-axes are AC and CF.

5. Hence the Circles of a Sphere viewed at an infinite Distance are projected into *Ellipses*. Thus the *Circle of Illumination* on the Disk of the Moon is an *Ellipsis*, as observed *Annot.* CXXXV. 23. Thus also a Sphere set in the Sun-Beams will have its Circles all projected into Elliptic Shadows. And hence it is we construct the ORTHOGRAPHIC PROJECTION, call'd the ANALEMMA; which see in my fore-cited Book.

6. Now because CE : CF :: Radius : Co-sine of ECF, it appears that the Semidiameter CE of every Circle is projected into the Co-sine FC of its Elevation above the Plane of Projection. Hence also it appears, that in this Projection the same Number of Degrees in a Right Circle, as BCE, will be projected into very different Portions of the Diameter of the Plane BE. Thus 10 Degrees from the Pole of the Primitive will be projected into the Arch CK, but 10 Degrees from the Periphery will be projected into EM. But CK is to EM as the *Right Sine* of 10 Degrees to the *Versed Sine* of the same; that is, as 1736 to 152, or nearly as 12 to 1. Hence the Reason why the Spots in the Sun appear to move so much faster over the Middle Parts

of the Disk than on the Outside, and why their Motion is always unequal; with other Phænomena of the like Nature.

Plate LXVIII. Fig. 1. 7. The STEREOGRAPHIC PROJECTION of the SPHERE is that on which our Maps are commonly made, and depends on this Principle, That if the Plane of any Meridian be supposed the Plane of the Projection, then an Eye placed in one Pole of that Meridian will project all the Circles in the opposite Hemisphere into circular Arches on the said Plane. Thus let A G D E be any Meridian; then the Diameter A D, dividing it into the upper and nearer Hemisphere, is called the *Line of Measures*; and an Eye placed at the Pole E will project every Point B, F, G, in the opposite Semicircle into the Points H, I, C, into the Line of Measures A D, by the Visual Rays E B, E F, E G.

8. Hence if the Arch A B = F G = 10 Degrees, then will their Representatives in the Line of Measures be A H and I C; and the Points H and I are those through which the Circles of 10 Degrees and of 80 Degrees do pass in Projection, *viz.* the Circles G H E and G I E, as is evident from considering the Figure. Hence the Reason why the Meridians do all lie nearer to each other in the middle Parts of the Map than on the Outsides; and consequently, why the several Parts of the Earth cannot be duly represented on such Maps, either in respect of Magnitude or Position.

9. On E as a Centre describe the Arch C M, and draw the Line E K; the Arch G K will be projected into the Line C L, which is the Tangent of the Angle C E L. But the Angle C E L is equal to half the Angle G C K, or Arch G K; therefore any Arch G K is projected into a Line C L equal to the Tangent of half that Arch. Hence the Line C D is called the *Line of Half-Tangents* in respect to the Quadrant G K D.

10. On this Projection are usually made all the Maps of the World in two Hemispheres. There is also another call'd the GLOBULAR PROJECTION, wherein all the Meridians are equally distant, as they are on the Globe itself. They are circular Arches here, as in the last Projection, and are drawn after the same Manner, but are

not

not projected by the Eye on the Surface as they are. By this Sort of Maps the several Parts of the Earth have their proper Proportion of Magnitude, Distance, and Situation assigned nearly as on the Globe itself. As this Sort of Map is for that Reason very useful, and not common, I have given one here for the Reader's Use, corrected from the latest Observations. I have also just published a New PLANISPHERE on the *Globular Projection*, with the Solution of *Geographical* and *Astronomical Problems* thereby. Plate LXXII.

11. Besides the foregoing, there is another very useful Projection, generally made use of for Charts, and sometimes for Maps; it goes by the Name of MERCATOR'S PROJECTION, but was first invented by Mr. *Wright* long before. In this the Meridians and Parallels are straight Lines, and the former equidistant from each other. Hence in this Way the Degrees of Longitude in every Parallel are the same, and equal to those in the Equator; also the Degrees of Latitude are all unequal; both which are contrary to what they are on the Globe. Therefore Maps of this Sort do not exhibit the true Dimensions or Proportions of the several Parts of the Earth; however, they are very useful on divers Accounts; and that which I have given from Dr. *Halley* to illustrate the Account of the Winds is of this Kind.

12. But the greatest Use of this Projection is in SAILING; I shall here therefore shew how it is constructed in the following Manner. Let A B be an Arch of the Equator contained between any two Meridians A P, B P, meeting in the Pole P of the Sphere whose Centre is C. Upon the Points A and B let there be erected the Perpendiculars A H and B I, and let D E represent an Arch of any Parallel between the same Meridians; draw C A and C B, K D and K E perpendicular to P C; through D and E draw C F, C G, and join F G; lastly, let fall the Perpendicular D L. Plate LXVIII. Fig. 2.

13. Now the Arch A B in the Equator is to the similar Arch of the Parallel D E as A C to D K, or as Radius to the Co-sine of the Latitude A D. Suppose now the Meridians A P, B P, to be in part projected into the Perpendiculars A H and B I; then will the Arch

D E be projected into F G = A B; but in this Case D E, *the natural Length of the Arch*, *is to* F G *its protracted Length*, *as the Radius* C D *to the Secant* C F *of the Latitude*, *or as the Co-sine* L C *to the Radius* C D; for C F : (C D =) A C :: D C : L C.

14. But in whatever Proportion the Degrees of any Parallel are increased or diminished by a Projection *in Plano*, in the same Ratio ought the Degrees of Latitude also to be increased or diminished; otherwise the *true Bearing and Distances* of Places would be lost, as in the Case of the *Plain Chart*, where the Degrees of Latitude are all equal. *The Degrees*, therefore, *of Latitude in* Mercator's *Chart increase in Proportion of the Secant of the Latitude to the Radius*.

Plate LXVIII. Fig. 3.

15. But that the Reader may see how such a Meridian is projected, let R C H be a Quadrant of the Primitive Circle, and R Q a Diameter; draw Q S; then will the Arch S H be projected into H I, and R S into A I; but A I is the Tangent of $\frac{1}{2}$ R S (by *Art.* 9.). Let S T and C L be perpendicular to A H, and draw the Tangent S V, C K, to the Points S and C meeting A H produced in V and K. And let H I $= x$, H S $= z$, and A H $= 1$.

16. Then because A T : A H :: A H : A V, it is A T × A V = A H^2 : for the same Reason it is A I × A K = A H^2 = A T × A V. Wherefore A V : A K :: A I : A T (= S B) :: Q I : Q S. Let Q s be drawn infinitely near to Q S, then S $s = \dot{z}$, and I $i = \dot{x}$; and because the Angle A I Q = T I S = I S V = Q s S, therefore the Triangles Q I i and Q S s are (in their nascent State) similar, and therefore Q I : Q S :: I i : S s :: $\dot{x}$: $\dot{z}$ :: A V : A K; consequently, it is A V $= \dot{z} =$ A K × $\dot{x}$.

Fig. 4.

17. But A K × $\dot{x}$ is the Fluxionary Rectangle of what is called a *Figure of Secants*, which may be thus explained. Let R C H be a Quadrant as before, H C an Arch, of which let the Secant be equal to I N rightly applied as an Ordinate to the Abscis H I $= x$; and if this be conceived to be done for every Point in the Quadrant, we shall have a Curve B N P described by the Point N, which appears to be a *rectangular Hyperbola* by compleating the Square A B. Now drawing i n infinitely

finitely near IN, we shall have $IN \times in = IN \times \dot{x}$ ($= AK \times \dot{x}$) = Fluxion of the Area $IHBN$, which is composed of all the Secants belonging to the Arch HC, and is therefore called a *Figure of Secants*.

18. Now the Fluxion of the Area $IHBN$ is to the Fluxion of the Rectangle $IHBD$ as $IN \times \dot{x}$ to $ID \times \dot{x}$, that is, as IN to $ID = AR$, *viz.* as the Secant to the Radius. Therefore the Areas themselves are in the same Ratio; that is, the Area $IHB : R \times x :: S : R :: Z : z$, supposing Z represents the Arch z protracted. In the same Manner it is shewn, that the Fluent or Area belonging to the Fluxion $AV \times \dot{z}$ is to $R \times z$ as $Z : z$; but this latter Fluent of $AV \times \dot{z}$ is equal to the Area $IHBN$, because their Fluxions are equal (by *Art.* 16.). Therefore $IHBN : R \times z :: Z : z$; consequently, $IHBN \times z = R \times z \times Z$; whence $IHBN = Z$ when $R = 1$.

19. But the hyperbolical Area $INBH$ is the Logarithm or Measure of the Ratio of AH to AI, that is, of $\frac{1}{1-x} = \frac{1}{t}$, supposing t = Tangent of $\frac{1}{2}$ the Complement of z. But any Hyperbolical Logarithm is to the Tabular Logarithm of the same Ratio, as 2,302585, &c. to 1; therefore the Tabular Logarithm of $\frac{1}{t} \times 2{,}302585 = INBH = Z$ gives the Length of the protracted Meridional Arch, answering to the Natural Arch z or HS.

20. Therefore, if A and a denote a greater and a Lesser Arch, beginning from the Equator, then the Length of their Difference $A - a$ will be $\frac{2{,}302585}{T} - \frac{2{,}302585}{t}$, or $2{,}302585 \times \overline{\frac{1}{T} - \frac{1}{t}}$, or $2{,}302585 \times \overline{t - T}$. That is, *From the Tabular Logarithm of the Tangent of $\frac{1}{2}$ the Complement of the Lesser Arch a, subduct that of the Tangent of the Greater Arch* A; *the Difference multiplied by* 2,302585 *will give the Meridional Parts of the Arch* $A - a$.

21. As I am upon a Subject of this Nature, it will be proper to observe, that since the Ship's Course is or

ought to be upon a *Rhumb-Line*, which makes equal Angles with every Meridian, therefore the Differences of Longitude will be the Logarithms of the Tangents of the Half-Complements of the Latitudes, as may be thus shewn. Let Æ Q be a Quadrant of the Equator, P the Pole of the World; P Æ, P A, P B, *&c.* the several Meridians projected *in Plano*, and Æ *a b c*, &c. the Rhumb-Line making equal Angles, Æ *a* A, Æ *b* B, *&c.* with every Meridian.

Plate LXX. Fig. 5.

22. Then if we make Æ A = A B = B = B C, *&c.* and very small, then may the Triangles Æ P *a*, Æ P *b*, Æ P *c*, &c. be esteemed rectilineal, and will be similar; and therefore Æ P : P *a* :: P *a* : P *b* :: P *b* : P *c*, and so on. Now if Æ A expound the Ratio of *a* P to Æ P, then because the Ratio of *b* P to P Æ is double the Ratio of *a* P to P Æ, and Æ P = 2 Æ A, therefore Æ B will expound the Ratio of *b* P to Æ P. Again, because *c* P : Æ P = 3 × $\overline{a\,\mathrm{P} : \mathrm{Æ\,P}}$, and Æ C = 3 Æ A, therefore Æ C expounds the Ratio of *c* P to Æ P; and so of the rest.

23. Therefore the Arches Æ A, Æ B, Æ C, *&c.* are the Logarithms of *a* P, *b* P, *c* P, *&c.* in respect of P Æ. But Æ A, Æ B, *&c.* are the Differences of Longitude made in sailing from Æ to *a*, or *b*, &c.; and *a* P, *b* P, *&c.* are Tangents of half the Complements of the Latitude A *a* B *b*, *&c.* (See Art. 9.). *Therefore the Differences of Longitude in sailing on any Rhumb are the Logarithms of the Tangents of the Half-Co-Latitudes.*

24. Hence the Rhumb-Line has acquired the Name of the *Logarithmic Spiral*. Hence also it follows, that *any Table of Logarithmic Tangents is a Scale of the Differences of Longitude on some Rhumb or other*. Thus the Tabular Logarithms of Tangents in present Use are Differences of Longitude on that Rhumb which makes an Angle of 51° 38′ 9″; and the Rhumb which makes an Angle of 71° 1′ 42″, is the same for *Neper*'s Logarithmic Tangents. They who would see the Demonstration of this, as also how a Table of Meridional Parts is from hence constructed, and likewise how all the Problems of Navigation may be solved by the common Table of Logarithmic Tangents only, may consult my LOGARITHMOLOGIA. See also *Philosophical Transactions*, N° 219, where

where the Theory is given at large by its Inventor Dr. *Halley*.

SCHOLIUM.

25. I have here added a Table of *Meridional Parts*, calculated for the Oblate Spheroid by the Rev. Mr. *Murdoch*, in his new and learned Treatise of *Mercator*'s *Sailing applied to the true Figure of the Earth*. The Errors of the common Spherical Projections are not so very small in many Cases, as to be inconsiderable and not dangerous. For instance, if a Ship sails from South Latitude 25° to North Latitude 30°, and the Angle of the Course be 43°; then the Difference of Longitude by the common Table would be 3206′, exceeding the true Difference 3141 by 65′ or Miles. Also the Distance sailed would be 4512, exceeding the true Distance 4423, by 89′ or Miles; which Differences are too great to be neglected. For other Instances of such a Correction of the Charts, I refer to the Author abovementioned. See also my New Treatise of GEOGRAPHY and NAVIGATION lately published.

26. *A* TABLE *of Meridional Parts to the Spheroid and Sphere, with their Differences.*

D.	*Spheroid.*	*Sphere.*	*Diff.*	*D.*	*Spheroid.*	*Sphere.*	*Diff.*
1	58.7	60.0	1.3	9	530.4	542.2	11.8
2	117.3	120.0	2.7	10	589.9	603.0	1 1
3	176.1	180.1	4.0	11	649.7	664.1	14.4
4	234.9	240.2	5.3	12	709.6	725.3	15.7
5	293.8	300.4	6.6	13	769.8	786.8	17.0
6	352.7	360.6	7.9	14	830.2	848.5	18.3
7	411.8	421.0	9.2	15	890.9	910.5	19.6
8	471.0	481.5	10.5	16	951.8	972.7	20.9

D.	Spheroid.	Sphere.	Diff.	D.	Spheroid	Sphere.	Diff
17	1013.1	1035.3	22.2	39	2497.2	2544.9	47.7
18	1074.8	1098.3	23.5	40	2573.9	2622.6	48.7
19	1136.8	1161.6	24 8	41	2651 8	2701.5	49.7
20	1199.2	1225.2	26.0	42	2730.9	2781.6	50.7
21	1262.0	12 9.2	27.2	43	2811.3	2863.0	51.7
22	1325.3	1353.7	28.4	44	2893.1	2945.8	52.7
23	1389.0	1418.6	29.6	45	2976.2	3029.9	53.7
24	1453.3	1484.1	30.8	46	3060.9	3115.5	54.6
25	1518.0	1550.0	32.0	47	3147.2	3202.7	55.5
26	1583.3	1616.5	33.2	48	3235.1	3291.5	56.4
27	1649.1	1683.5	34.4	49	3324.8	3382.1	57.3
28	1715.6	1751.2	35.6	50	3416.3	3474.5	58.2
29	1782.7	1819.5	36.8	51	3509.7	3568.8	59.1
30	1850.5	1888.4	37.9	52	3605.3	3665.2	59.9
31	1919.0	1958.0	39.0	53	3703.1	3763.8	60.7
32	1988.2	2028.3	40.1	54	3803.1	3864.6	61.5
33	2058.3	2099.5	41.2	55	3905.7	3968.0	62.3
34	2129.0	2171.4	42.3	56	4010.9	4073.9	63.0
35	2200.8	2244.2	43.4	57	4118.9	4182.6	63.7
36	2273.4	2317.9	44.5	58	4229.8	4294.2	64.4
37	2347.0	2392.6	45.6	59	4344.0	4409.1	65.1
38	2421.6	2468.3	46.7	60	4461.5	4527.3	65.8

D.	*Spheroid.*	*Sphere.*	*Diff.*	D.	*Spheroid.*	*Sphere.*	*Diff.*
61	4582.7	4649.2	66.5	76	7136.2	7210.0	73.8
62	4707.8	4775.0	67.2	77	7393.0	7467.1	74.1
63	4837.1	4904.9	67.8	78	7670.1	7744.5	74.4
64	4971.0	5039.4	68.4	79	7970.9	8045.6	74.7
65	5109.8	5178.8	69.0	80	8300.2	8375.2	75.0
66	5254.0	5323.6	69.6	81	8663.8	8739.0	75.2
67	5403.9	5474.0	70.1	82	9070.0	9145.4	75.4
6[illegible]	5560.2	5630.8	70.6	83	9530.2	9605.8	75.6
69	5723.5	5794.6	71.1	84	10061.1	10136.9	75.8
70	5894.4	5965.9	71.5	85	10688.7	10764.0	75 9
71	6073.7	6145.5	71.9	86	11456.5	11532.5	76.0
72	6262.4	6334.7	72.3	87	12446.0	12522.1	76.1
73	6461.6	6534.3	72.7	88	13840.4	13916.4	76.0
74	6672.6	6745.7	73.1	89	16223.8	16299.5	75.7
75	6896.8	6970.3	73.5	90	∞	∞	37.75

APPEN-

APPENDIX I.

CONTAINING A

Physico-Mathematical Theory

OF

LUNAR Motions and Irregularities

OF THE

MOTION of the EARTH'S AXIS,

AND

PRECESSION of the EQUINOXES;

AND THE

Computation of the QUANTITY of Matter, DENSITY, WEIGHT of BODIES, &c.

On the SURFACE of the

SUN, EARTH, JUPITER, and *SATURN*.

AS the Subjects treated of in the ensuing Appendix *could not well be brought into the Body of the Book among the* Annotations, *and are the most important Part of the* Newtonian Philosophy, *they could not on any Account be omitted, and therefore I have here annexed them, to complete a System of that Science. And I have taken such a Method as I hope will be found not only more natural and concise, but much more adapted to render those difficult and intricate Ideas easy to be apprehended by the intelligent Reader.*

APPENDIX I.

1. THE Motion of the Moon about the Earth is similar to that of the Waters of the Ocean revolving about the Earth's Centre. To shew this, some Things must be premised; as, first, *That the Attraction of the Earth upon any Particle of Water is the same as it would be, were the same Quantity of Matter contracted into a Point in its Centre.* For let ABGO be the Earth, C its Centre, P a Particle at any Distance PA from its Surface; let PG be drawn through P and C, and BO the Diameter of any Circle BDOE, or Section of the Sphere perpendicular to the Axis PG. Plate LXVIII. Fig. 6.

2. Now put $PN = a$, $BP = x$; then $PB^2 - PN^2 = x^2 - a^2 = BN^2$, which is as the Area of the Circle BDOE; the attracting Force whereof is $2x\dot{x}$, and is proportional to the Quantity of Matter or Number of Particles which act on the Corpuscle P in the Periphery of the Area, and in

in Directions similar to P B. And since the Force of Attraction is as the Number of Particles ($2\dot{x}$) multiplied by the Force of each Particle, which is as some Power (n) of the Distance (x), therefore $2\dot{x} \times x^n = 2\dot{x}\,x^{n+1}$ will be as the whole or absolute Force of these Particles, that is, in the Directions P B.

3. BUT the Force represented by P B is resolvable into two Forces P N and B N, of which the former only causes the Corpuscle at P to approach the Sphere. Therefore as $PB : PN :: x : a :: 2\dot{x}\,x^{n+1} : 2a\dot{x}\,x^n$, the Force with which the Particle at P is attracted in the Direction P N; the Fluent of which $\frac{2ax^{n+1}}{n+1}$ (when corrected) is the whole Force of all the Particles in the Area of the Section B D O E, to attract the Corpuscle P in the Direction P C.

4. I say, the Fluent $\frac{2ax^{n+1}}{n+1}$ must be corrected, for it is at present too great; because when the Area of the Section becomes a Point, or $x = a$, then this Fluent has the Value $\frac{2a^{n+2}}{n+1}$, which therefore must be deducted from the General Fluent

2 a x

$\frac{2ax^{n+1}}{n+1}$, and their Difference $\frac{2ax^{n+1}-2a^{n+2}}{n+1}$

or $\frac{ax^{n+1}-a^{n+2}}{n+1} = \frac{PN \times PB^{n+1} - PN^{n+2}}{n+1}$,

will be as the Forces of any circular Areas B D O E attracting the Corpuscle P in the Direction of its Axis P C.

5. Now since in Natural Bodies this Power in any single Particle is *inversely as the Squares of the Distance*, therefore $n = -2$, and so the above Expression of the Force will become $1 - \frac{PN}{PB}$.

6. Now if we put $AC = r$, $PA = c$, $PC = c + r = \frac{b}{2}$, $PB = c + x$, and $PN = y$; then $AG \times AN = 2r \times \overline{y - c} = AO^2$, and $PA^2 + AO^2 + 2PA \times AN = (cc + 2ry - 2rc + 2cy - 2cc =)\ PO^2 = c^2 + 2cx + x^2$. Hence $2ry + 2cy = 2c^2 + 2rc + 2cx + x^2$, and $y = c + \frac{2cx + x^2}{2c + 2r} = \frac{cb + 2cx + x^2}{b}$ (because $b = 2r + 2c$). And because the Force of Attraction in the circular Plane whose Diameter is B O is $1 - \frac{PN}{PB} = 1 - \frac{cb + 2cx + x^2}{cb + bx} = \frac{2rx + x^2}{b \times \overline{c+x}}$, if we multi-

ply this by the Fluxion of the Diſtance, *viz.* $\dot{y} = \frac{2c\dot{x} + 2x\dot{x}}{b}$, we ſhall have $\frac{4rx\dot{x} + 2xx\dot{x}}{bb}$ whoſe Fluent $\frac{2rx^2 + \frac{2}{3}x^3}{bb}$, is proportional to the Attraction of any Segment BOA of the Globe upon the Particle P.

7. HENCE, when $x = 2r$, the Expreſſion will become $\frac{5r^3}{3bb}$ or ſimply $\frac{r^3}{bb}$, for the Attraction of the whole Globe. Whence it appears, that *the attractive Forces of ſpherical Bodies are to one another in a Ratio compounded of the Quantities of Matter directly, and as the Squares of the Diſtances from their Centres inverſely.* And therefore ſince the Number of Particles only, and their Diſtance from the Centre enter the Expreſſion of the Force, it is plain the Effect will be the ſame upon a Corpuſcle P placed any where without the Surface of the Globe, as if the whole Maſs of Matter were contracted into a Point at its Centre. *Q. E. D.*

8. To apply this: If all the Matter of the Earth were contracted into the Centre, and the Waters of the Ocean were to continue their diurnal Rotation the ſame as they now do, they would then be affected in the ſame Manner by the Earth and Moon

Moon as they now are, and have all the ſame Phænomena. And therefore if a Body, inſtead of revolving at the Diſtance of the Earth's Surface about its Centre, were to revolve at the Diſtance of the Moon, every Thing would happen in a ſimilar Manner, and the Effects of the Earth and Sun in diſturbing the Motion of the Satellite would be like thoſe which are produced in the Motion of the Water by the Earth and Moon, but only in a leſs Degree.

9. ANOTHER Thing to be premiſed is, that the Moon revolves not about the Centre of the Earth as the Centre of its Motion; and therefore in order to conſider its Motion in the beſt Manner, we muſt determine the Diſtance to which the Moon muſt be removed from the Centre of the Earth at reſt, (and conſidered as the Centre of its Motion) that it may revolve about it in the ſame periodical Time that it takes up now, together with the Earth, in revolving about the common *Centre of Gravity*. See *Annot*. XXXVI.

10. IN order to this, let D be the Diſtance of the Moon from the common Centre of Gravity, and d that of the Earth from it; then will $D + d$ be the Diſtance of the

Moon from the Earth, which, at a Mean, is $60\frac{1}{2}$ Semidiameters. Now let $x =$ Diſtance required; then becauſe the attracting Forces (F and f) in any two different Diſtances are as the Squares of thoſe Diſtances inverſely, we have $F : f :: x^2 : \overline{D+d}^2$. Again, becauſe the Periodical Time is given, or the ſame in both Caſes, we have the Forces proportional to the Diſtances from the Centres of Motion; (See *Annot.* XXXIV.) therefore $F : f :: D : x$. Conſequently $D : x :: x^2 : \overline{D+d}^2$, therefore $x^3 = \overline{D+d}^2 \times D$; and multiplying by $D + d$, we have $x^3 \times \overline{D+d} = \overline{D+d}^3 \times D$; whence $D + d : D :: \overline{D+d}^3 : x^3$; therefore $\sqrt[3]{\overline{D+d}} : \sqrt[3]{\overline{D}} :: D + d : x$. But $D + d : D ::$ the Quantity of Matter in the Earth and Moon together: the Quantity of Matter in the Earth alone; that is, as 40,31 to 39,31. Whence $\sqrt[3]{40{,}31} : \sqrt[3]{39{,}31} :: 60{,}5 : 60 = x$, *the Diſtance at which the Moon would revolve about the Earth at reſt in the ſame Time it now does.*

Plate LXVIII. Fig. 7. 11. THESE Things premiſed, let S be the Sun, T the Earth, and P a Satellite revolving about it, and let SK be the Mean Diſtance of the Satellite or Moon from the Sun; and expound the accelerative Force by

by which it is attracted towards the Sun S. And take $SL : SK :: SK^2 : SP^2$, and Sl : $Sk :: S^2 k : Sp^2$; then shall S L, or S *l*, expound the accelerative Attraction in any Distance of the Satellite S P or S*p*. That is, the Force at P is to the Force at *p* as S L is to S *l*; for $SK = Sk$, and $SK^3 = SL \times SP^2 = Sl \times Sp^2$; therefore $SL : Sl :: Sp^2 : SP^2$.

12. JOIN PT and *p*T, and draw parallel thereto the Lines L M and *l m*, meeting ST in M and *m*. And the Attraction S L, S *l*, is resolvable into two others S M and L M, and S*m* and *l m*. Hence the Body P is urged with a *Threefold* Force, *viz.* (1.) That by which it is attracted or tends towards T, arising from the mutual Attraction of the Bodies T and P. (2.) The Force L M, or *l m*, by which it is likewise urged towards T. (3.) The Force S M, S *m*, by which it is urged towards S, or attracted in Directions always parallel to S T. Plate LXVIII. Fig. 8.

13. By the first of these Forces the Satellite ought to describe an Ellipsis about T in one of its *Foci*, and therefore Areas proportional to the Time, as is evident from what was demonstrated in *Annotat.* CXL. This is upon Supposition the Body T was

 fixed;

fixed; but the Caſe is the ſame, ſuppoſing it moveable with the Body P about a common Centre (which is really the Caſe of the *Earth* and *Moon*) as Sir *Iſaac Newton* has ſhewn in *Theor.* XX. and XXI. *Lib.* I. of the *Principia.*

14. THE ſecond Force L M, as it conſpires to impel the Body in the Direction P T, is to be added to the former, and cauſes that the Body ſhall ſtill deſcribe *Areas proportional to the Time.* But becauſe this Force is not in the inverſe Ratio of the Square of the Diſtance, it will, compounded with the former, cauſe the Curve which the Satellite deſcribes to deviate from an *Elliptic Form*, and the more ſo, *cæteris paribus*, the greater the Proportion is which this Force bears to the former. Theſe Forces L M, $l\,m$, have been ſhewn (*Annot.* LXXXIV. 9.) to be as $\frac{PT}{SP^3}$ and $\frac{pT}{Sp^3}$; and therefore increaſe and decreaſe with the Diſtance P T or p T.

15. LASTLY, the third Force S M impelling the Body P in Directions parallel to T S, will compound a Force with the former two, that is not directed from P to T, and ſo will cauſe that the Body P ſhall no longer *deſcribe Areas proportional to the Times* (as

(as we have ſhewn). It will alſo augment the Aberration of the Orbit from an Elliptic Form, on a double Account, *viz.* both becauſe it is not directed from P to T, and alſo becauſe it is not inverſely as the Square of the Diſtance P T. For the Forces S M : S m :: $\frac{1}{SP^3}$: $\frac{1}{Sp^3}$ :: Sp^3 : S P^3. Theſe Errors therefore are leaſt of all when the ſecond and third Forces (eſpecially the third) are ſo, the firſt Force remaining the ſame.

16. LET S N expound the Force by which the Body T is accelerated towards S; then if the Forces S M and S N are equal, they, by attracting the Bodies T and P equally, and in parallel Lines, will cauſe no Alteration in their Site or Poſitions in reſpect of each other. But when the Force S M is greater, or S m leſſer than the Force S N, the Difference N M, or N m, will be that alone by which the Proportionality of the Times and Areas, and alſo the Elliptic Form of the Orbit, will be diſturbed. Hence when N M or N m is nothing, or leaſt of all, the aforeſaid Perturbations will vaniſh, that is, when the Body P is nearly in the Points C and D of its Orbit.

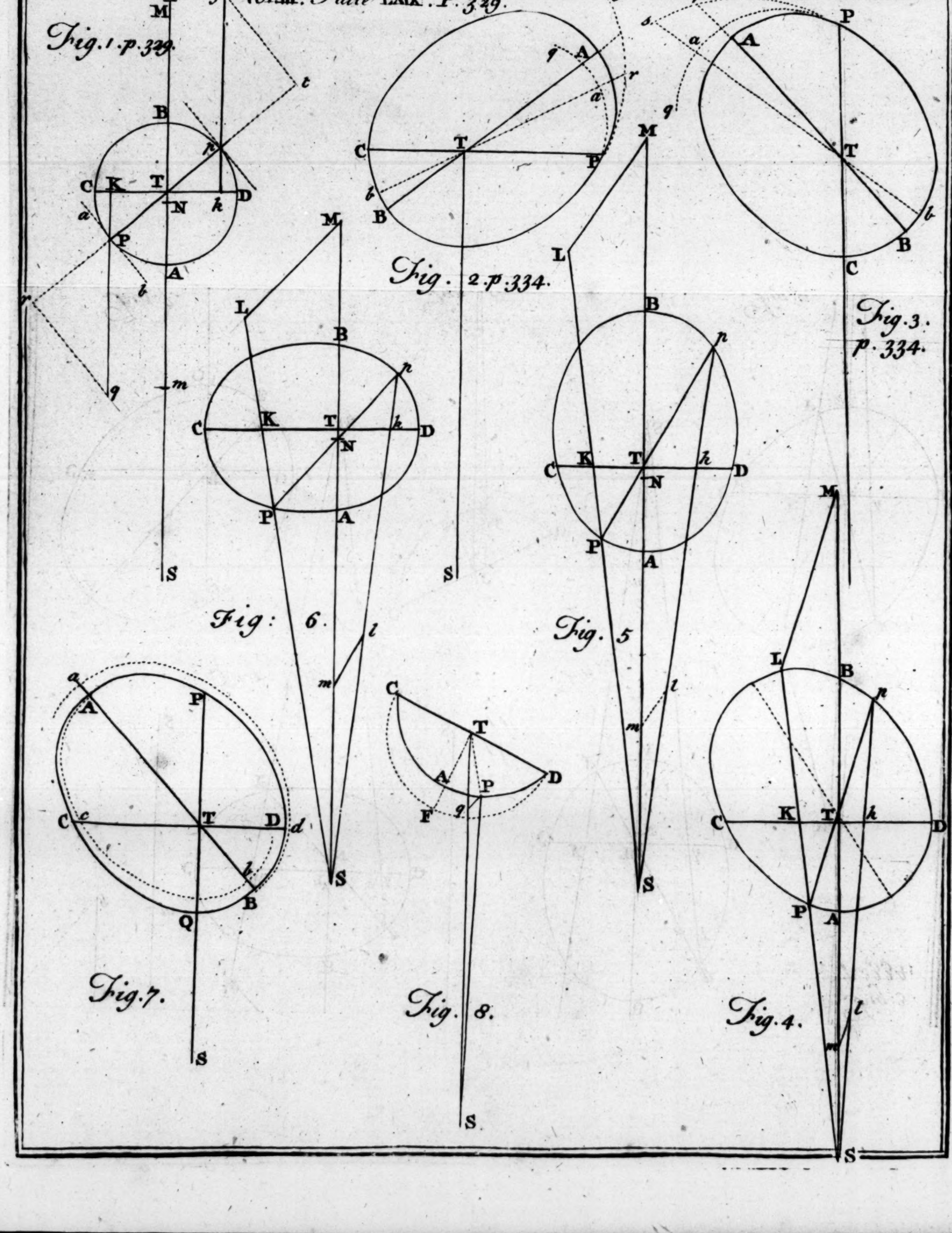
17. WE have hitherto ſuppoſed the Body P revolving about T in the ſame Plane with Plate LXVIII. Fig. 9.

with S; let us now ſuppoſe it to revolve in a different Plane, and let the Semi-Orbit C A D be above, and C D B below the Plane, in which are the Bodies S and T. In this Caſe the Force L M will have the ſame Effect as before, *viz.* will only tend or impel the Body P from P to T. But the other Force N M, by acting in a Direction parallel to S T, and therefore (when the Body P is not in the Nodes C, D,) inclined to the Plane of the Orbit P A B, will, beſides the above-mentioned Error in Longitude, induce an Error in Latitude, or diſturb the Inclination of the Orbit.

18. FOR let P *q* be drawn parallel to N M, and let P *p* be the Space through which the Satellite P would move in its Orbit in a ſmall Particle of Time, excluſive of the Force N M; and by the Force N M alone ſuppoſe it in the ſame Time moved through the Space P *r*; then compleating the Parallelogram P *r s p*, and drawing the Diagonal P *s*, that will repreſent the real Motion, and *s* the true Place of the Satellite at the End of the ſaid Time: But 'tis evident the *Lineola* P *s* is not in the Plane of the Orbit C A D.

19. HENCE it follows, that by the Force N M the Body P will be accelerated in its Motion

Vol. III. Plate LXIX. P. 329.



Motion from C to A, and from D to B, and retarded as it paſſes from A to D, and from B to C. For let P *q* be drawn parallel to N M, and expound that Force, then continuing T P to *r*, and drawing *q r* perpendicular thereto, the Force P *q* becomes reſolved into the two Forces *r* P and *r q*; of which the former, acting in the Direction P T, does not diſturb the Planet's Motion in Longitude, nor the equable Deſcription of Areas; but the other Part *r q*, acting in the Direction *r q*, conſpires with the Motion of the Satellite P in its Orbit, (as being parallel to the Tangent *a b*) and therefore accelerates its Motion in Longitude. By the ſame Way of Reaſoning, by making the like Conſtruction between A and D it will appear that the Motion of the Satellite will be there retarded, the Force *r q* being on that Side in a contrary Direction. Plate LXIX. Fig. 1.

20. AGAIN, as the Planet paſſes from D to B, it will be again accelerated; for let *p s* here expreſs the Force N *m*, which is now Negative, or acts in a contrary Direction to the former N M, that is, from *p* to *s*, ſuppoſing *p s* parallel to N *m*; for then *t s* conſpires with the Motion of the Planet in the Direction of the Tangent *c d*. In the ſame Manner it is ſhewn the Planet is

retarded

retarded in going from B to C, by the contrary Direction of *t s*.

21. HENCE also it appears, that since the Body P is constantly accelerated from C to A, and from D to B, the Velocity of the Satellite will be greater in the Points A and B *(cæteris paribus)* than in the Points C and D.

22. The Orbit also will *(cæteris paribus)* be more convex in C and D than in the Points A and B; for the swifter Bodies move, the less they deflect from a Right-Line Course in a given Time. Moreover, in the Points A and B, the Force L M and N M are directly contrary to each other, and their Difference N M − L M = K L, will be as the Force which draws the Body from T towards S; and since this Force K L is greater when the Body is at A, than when at C or D, the Body will there be less urged towards T, and so will less deflect from a Right-Line. The same may be shewn when the Body is in the Point B by the Force *k l*. See *Annot.* LXXXIV. 20.

23. WHENCE the Body P will *(cæteris paribus)* recede farther from T in the Points C and D than at A and B; as is easy to observe from the Figure of the Orbit, which is less curved, and therefore nearer to T at A and

A and B, than at C and D. What is here ſaid is upon ſuppoſition that the Orbit (excluſive of the perturbating Forces) is a *Circle*, and not an *Ellipſis*, which Caſe will be conſidered by and by.

24. BECAUSE the centripetal Force of the central Body T, by which the Body P is retain'd in its Orbit, is augmented in the Points C and D by the addititious Force LM, and diminiſhed in the Points A and B by the Force KL; and becauſe KL is always greater than LM from C to A (and double thereto at A: See *Annot.* LXXXIV. 20, 21, 22.) and from A to D, (where it becomes equal to it) therefore the centripetal Force (F) is upon the whole diminiſhed by the Action of the Body S. And becauſe $F : \frac{a}{P^2}$, (by *Annotat.* XXXIV. 6.) therefore the Radius TP (a) remaining the ſame, the Periodical Time (P) of the Planet will be augmented by the Action of the Power KL; and becauſe in that Caſe $P : \frac{1}{\sqrt{F}}$, it appears the Periodical Time will be increaſed in the *Subduplicate Ratio* by which the Force F is decreaſed.

25. AGAIN, ſuppoſing the centripetal Force F to remain the ſame, (as we may when

when K L is very ſmall with reſpect to it, *Annotat.* LXXXIV. 22.) then however the Diſtance P T (a) may vary, we have $r P^2 : a^3$, or $P : a\sqrt{a}$. Therefore when neither the Diſtance (a) nor the centripetal Force F are conſtant, we have $P : \frac{a\sqrt{a}}{\sqrt{F}} : \sqrt{\frac{a^3}{F}}$; that is, the Periodical Time (P) will be in a Ratio compounded of the Seſquiplicate Ratio of the Diſtance $\sqrt{a^3}$, and the Ratio $\frac{1}{\sqrt{F}}$, which is ſubduplicate of that by which the central Force F is increaſed or diminiſhed by the Decreaſe or Increaſe of the Action of the Diſtant Body S.

26. FROM what has been ſaid, it follows alſo, that the Axis of the Ellipſis deſcribed by the Body P, or Line of the *Apſides*, has an angular Motion backwards and forwards by Turns, but its *Progreſs* exceeds the *Regreſs*; and by that Exceſs it is upon the whole carried forwards, or *in Conſequentia*. For the Force by which the Body P is urged towards T in the Points C and D, where the Force M V vaniſhes, is compounded of the Force L M, and the centripetal

tripetal Force or Attraction of the Body T. The former Force L M, if the Distance P T be increased, increases nearly in the same Ratio; and the latter Force (F) is inversely as the Square of that Distance, *viz.* as $\frac{1}{PT^2}$; wherefore the whole Force is as $PT + \frac{1}{PT^2}$

27. Now $F : TP + \frac{1}{PT^2}$ is a less Ratio than $F : \frac{1}{PT^2}$. For Example: Let the Ellipsis (when the Satellite is in the Quadratures) be A P B, and the Axis be A B; the Distances from the central Body T let be $PT : CT :: 6 : 5$; then the centripetal Force at P will be to that at C as 25 to 36; but the additional Force (L M) at P is as $PT = 6$, and at C as $TC = 5$; therefore the compound Forces at P and C are as $25 + 6 : 36 + 5$; or as 31 to 41, which is a less Ratio than 25 : 36. For as 25 : 36 $:: 31 : \frac{36 \times 31}{25} = 44{,}64$. But the Ratio 31 : 41 is less than the Ratio 31 : 44,64.

28. HENCE, since when the Force at P is as $\frac{P}{PT^2}$, the Planet describes an *Ellipsis*

PAB;

Plate LXIX. Fig. 2. P A B; and when the said Force is as $\frac{1}{PT^3}$, the Curve is the *Equiangular Spiral* P *r s*, (by *Annot.* CXL.) 'tis evident the Satellite will with a Force as $PT + \frac{1}{PT^2}$, describe an Oval P *a q* still more curved than the Ellipsis, and therefore will lie within it. Now were the Planet P to set out from any Point P (in which the Radius T P cuts all the three Curves in one common oblique Angle) and to proceed first in the Spiral Path from P towards *s*, the Radius T P would constantly intersect the said Curve in the same Angle as at P. But secondly, if it proceeded in the Elliptic Arch from P towards A, the Angle T P A would continually be altering and approaching nearer to a Right
Fig. 3. Angle, which it would make when it arrived in the Point A. Lastly, if it set out in the Oval P *a q*, the said Angle T P *q* would alter much faster, and approach more quickly to a Right Angle, which happens in the Point *a*, because of its greater Curvity, or Deviation from the Spiral P *s*.

29. THEREFORE by this compound Force the highest Apsis A will be removed backwards to *a*, or the Axis of the Ellipsis A B will

will recede into the Poſition *a b*; and this will be the Caſe every Time the Line of the Apſides comes into Square with the Sun.

30. ON the other hand, when the Satellite is in the Syzygical Line C P it is urged with a Force in the lower Apſis C, which is equal to the Difference between the centripetal Force and that expreſſed by K L; and in the upper Apſis it is equal to the Difference between the central Force and *k l*; which *k l* is as P T or A T, as being double thereof; therefore the compound Force about the upper Apſis is as $\frac{1}{PT^2} - PT$, which is a greater Ratio than that of $F : \frac{1}{PT^2}$; or, in Numbers, $25 - 6 : 36 - 5 :: 19 : 31$. But $19 : 31$ is a greater Ratio than 25 to 36; whence the Path of the Satellite P *a q* is not ſo much curved as the Ellipſis P A B, and therefore lies between it and the Spiral P *r s*; and therefore as the Radius moves from P towards A, it ſooner makes a Right Angle with the Ellipſe at A, than with the Oval P *a q*, which happens at *a*. The Line of the Apſides A B therefore goes forwards in this Caſe, and becomes *a b*.

31. AND

31. AND because the ablatitious Part $k\,l$ is twice as great as the addititious Part $l\,m$ for the upper Apsis, and $KL = 2\,LM$ for the lower; therefore the Ratio of the compound Forces, which is greater than the Ratio of the Squares of the Distances inversely, will upon the whole prevail, and cause a progressive angular Motion of the Line of the Apsides.

32. HENCE 'tis evident there is a certain Point between the Quadratures and Syzygies, where the Apsides are quiescent; to find which, let P be the Place of the Satellite in the Apsis required; through P draw $P\,q$ equal and parallel to N M or T M, and produce it to K, then is $P\,q = 3\,PK$. (*Annot.* LXXXIV. 21.) From q let fall the Perpendicular $q\,r$ upon P T produced, and the Force $P\,q$ is resolved into two others $P\,r$ and $q\,r$; of which $q\,r$, by acting perpendicularly to the Radius, does neither accelerate nor retard the Motion of P towards T; but the other Part $P\,r$, acting directly contrary thereto from P towards r, diminishes the central Force of P towards T. But the Force L M or P T augments it; the Point therefore where $P\,r = PT$ is that required. Now because of similar Triangles T P K and $q\,P\,r$, we have PT : PK

$PK :: Pq\ (= 3\,PK) : Pr = PT$, in the Caſe propoſed. Therefore $3\,PK^2 = PT^2$; whence $PT : PK :: \sqrt{3} : 1$.

33. HENCE we have this Analogy; As $\sqrt{3} : 1 ::$ Radius PT : Sine PK of the Arch $CP = 35^\circ\ 16'$. The Point, then, where the central Force is neither increaſed nor diminiſhed by the Force of the Sun, and conſequently where the Apſides are at reſt, is at $35^\circ\ 16'$ on each Side the Quadratures, or at $54^\circ\ 44'$ from the Syzygies on each Side; ſo that the Apſides do in each Revolution of the Planet *(cæteris paribus)* go backwards through $141^\circ\ 4'$, and forwards through $218^\circ\ 56'$.

34. SINCE the Progreſs or Regreſs of the Apſides depends on the Decrement of the central Force in a greater or leſſer Ratio than that which is duplicate of the Diſtance in going from the lower Apſis A to the upper one B, and alſo on a ſimilar Increment in returning from B to A, and is therefore greateſt when this Proportion of the Force in the upper Apſis to that in the lower Apſis does moſt of all recede from the inverſe duplicate Ratio of the Diſtances; it is evident that the Apſides in their Syzygies by the ablative Force K L Plate LXIX. Fig. 4.

 will

will go forwards more ſwiftly, and more ſlowly in their Quadratures by the additi-tious Force L M.

35. For let the abſolute Force of Attraction in T be $= a$, then becauſe this is every where in the Ratio of $\frac{1}{PT^2}$ at the Body P, the Force by which the Body P is attracted towards T will be as $\frac{a}{PT^2}$. Again, if the Satellite P be within 54 Degrees of the Syzygies A or B, its Force is diſturbed by an extraneous Force (b), which is every where as K L or $k\,l$; therefore this perturbating Force is $b \times KL$, or $b \times kl$; ſo that the Force upon P in the Points P and p (within that Limit) is every where in the Ratio of $\frac{a}{PT^2} - b \times KL$ to $\frac{a}{pT^2} - b \times kl$; which Ratio, when P is in the Points A and B, becomes $\frac{a}{AT^2} - b \times AT$ to $\frac{a}{BT^2} - b \times BT$ (becauſe then $KL : kl :: AT : pT$, and $TP = AT$, and $pT = BT$, as has been ſhewn). Now this reduced to a common Denomination is $\overline{TB^2} \times a - \overline{b \times AT^3}$ to $\overline{AT^2} \times a - \overline{b \times TB^3}$.

36. Now

36. Now this Ratio recedes so much the more from the Ratio of TB^2 to AT^2, or $\frac{1}{AT^2}$ to $\frac{1}{TB^2}$, by how much $\overline{a-b} \times AT^3$ recedes from an Equality with $\overline{a-b} \times TB^3$, or by how much A T is less than T B; that is, when the Line of the Apsides is in the Syzygies as in *Fig.* 5. In this Position Plate therefore the Apsides will go forwards swifter LXIX. than in any other.

BUT when (in this Case) the Body P is in the Quadratures C and D, the additional Force L M becoming equal to C T = T D, and C T + T D being here less than in any other Situation of the Apsides, (as *Fig.* 4.) from the Nature of an Ellipsis, therefore the Ratio or Quantity of the perturbating Force thence arising will be least of all; and consequently the Apsides will recede slower in this than in any other Situation. Hence, upon the Whole, the Excess in the Progress of the Apsides will in this Situation be greater than in any other.

38. IF the Line of the Apsides be situ- Fig. 6. ated in the Quadratures, then for just contrary Causes the contrary Phænomena will happen; that is, they will recede most swiftly when the Satellite is in the Quadra-

 tures,

tures, and proceed most slowly when it is in the Syzygies. So that in this Case the Regress might exceed the Progress, and the Apsides upon the Whole be moved *in Antecedentia*, were it not that the Force K L, by which they go forwards at A, is near twice as great as L M, by which they go backwards when the Body is at C. See *Art.* 24.

39. THE Excess of the progressive above the regressive Motion of the Apsides will be augmented, if the Bodies P and S move both towards the same Parts; for then the Apsides will continue a longer Time in and near the Syzygies, than if the Body S were fixed: And on the contrary, as their Motion would be contrary to that of S when P is in the Quadratures, so the Time of the Regress will be shorter; therefore the Time by which they go forwards will, upon the Whole, be from hence very much increased.

Plate LXIX. Fig. 7.

40. FROM what we have demonstrated (*Art.* 28, 29, 30.) it is evident, that if a Body in descending from the upper to the lower Apsis be urged by a centripetal Force, which increases more than in a duplicate Ratio of the diminished Distance from the Centre, it will describe a Curve A *c b* interior

rior to the Ellipſe ACB, and conſequently more eccentric, inaſmuch as the Ratio of TB to TA is increaſed by being changed to the Ratio of T*b* to TA.

41. ON the contrary, if a Body ſets out from the lower Apſis B towards the upper A, and is attracted every where with a Force that decreaſes more than in the duplicate Ratio of the increaſing Diſtance; then, being leſs attracted than it would be in the Ellipſe, it will deſcribe an Orbit exterior to the Ellipſe, as B*da*; which alſo is more eccentric than the Ellipſe, becauſe T*a* to TB is a greater Ratio than TA to TB.

42. BY the ſame Way of Reaſoning we ſhew, that if in the Deſcent the Force be increaſed in a Ratio leſs than that of the Square of the diminiſhed Diſtance, or in the Aſcent it be diminiſhed in a Ratio leſs than the Square of the increaſed Diſtance, the Orbit deſcribed will be leſs eccentric than the Ellipſe.

43. THEREFORE when the Satellite P is in the Quadratures C and D, if the abſolute central Force be to the abſolute additional Force as *a* to *n*, we ſhall have the whole Forces at C and D, in the Ratio of

$\frac{a}{CT^2} + n \times CT$ to $\frac{a}{TD} + n \times TD$; which is as $\overline{TD^2 \times a + n \times CT^3}$ to $\overline{TC^2 \times a + n \times TD^3}$. But this is a leſs Ratio than that of TD^2 to TC^2, becauſe CT is greater than TD. Therefore in that Part of the Orbit where the addititious Force LM takes place, the Eccentricity will be diminiſhed, by *Art.* 42.

44. AGAIN; ſuppoſing the Satellite in the Syzygies PQ, then the Force in Q will be to that at P as $\overline{TP^2 \times a - b \times TQ^3}$ to $\overline{TQ^2 \times a - b \times TP^3}$, which Ratio is greater than that of TP^2 to TQ^2, becauſe TQ is leſs than TP; wherefore in and near the Syzygies the *Eccentricity* of the Orbit will be increaſed (by *Art.* 40, 41). The Eccentricity therefore of the Orbit will be twice changed in every Revolution of the Satellite.

Plate LXIX. Fig. 6. 45. If the Apſides be ſituated in the Quadratures, then, becauſe the Ratio of TD to TC is greateſt of all, the Eccentricity of the Orbit will be leaſt of all (*Art.* 43). Again; when the Apſides are in the Syzygies, the Eccentricity is the greateſt of all for the ſame Reaſon, *viz.* the greateſt Diſparity of AT and TB. Hence the

Fig. 5.

Eccentricity

Eccentricity of the Orbit is continually increasing as the Apſides paſs from the Quadratures to the Syzygies, and *vice verſa*.

46. IT has been already ſhewn, (*Art.* 17, 18.) that if the Plane of the Satellite's Orbit be inclined to the Plane in which are the Bodies S and T, the Motion of the Satellite in Latitude will in no wiſe be diſturbed by the Part L M of the extraneous Force, but only by the other Part N M, and not by that neither when the Nodes are in the Syzygies; but when they are in the Quadratures this Perturbation is greateſt of all.

47. FOR let P be the Satellite in its Orbit C A D, inclined to the immoveable Plane C F D in any Angle A D F; and let P S expound the Force of the Body S, attracting the Satellite in the Direction P S. From P let fall the Perpendicular P q to the Plane C F D, and draw the Right Line T q S; then the Force P S is reſolvable into the Forces S q and P q; of which the former, being in the ſaid Plane C F D, does not diſturb the Satellite's Motion in Latitude; but the other Force P q, being perpendicular to the Plane C F D, is wholly ſpent in drawing the Satellite from its Orbit C A D towards it, and therefore is proportional Plate LXIX. Fig. 8.

to the Force by which the Motion in Latitude is disturbed. But the Force P q is evidently greatest when S T D is a Right Angle, and is Nothing when that Angle vanishes.

48. WHEN the Nodes are in the Quadratures C, D, as the Satellite P passes from the Quadratures to the Syzygies the Inclination of the Orbit is diminished, and it is increased in going from the Syzygies to the Quadratures. For let P q (as before) represent the extraneous Force N M, and the Direction of its Action; we have shewn that the Body P will describe the *Lineola* P s in a small Particle of Time by the compound Force, which *Lineola* P s is not in the Plane of the Orbit C P D, but deflects from it towards P q; so that the Satellite really moves in the Plane T P s, which produced will not meet the Plane E C F in C, but in another Point c, toward the Opposition B.

Plate LXX. Fig. 1.

49. FOR with the Radius T P describe the Circle E C F D in the fixed Plane passing through T and S, and in the Plane T P s the Arch of a Circle P c intersecting the other in c. Now because the Force N M is very small, compared with the central Force, therefore the Angle C P c, the Inclination

Plate LXX. P. 512.

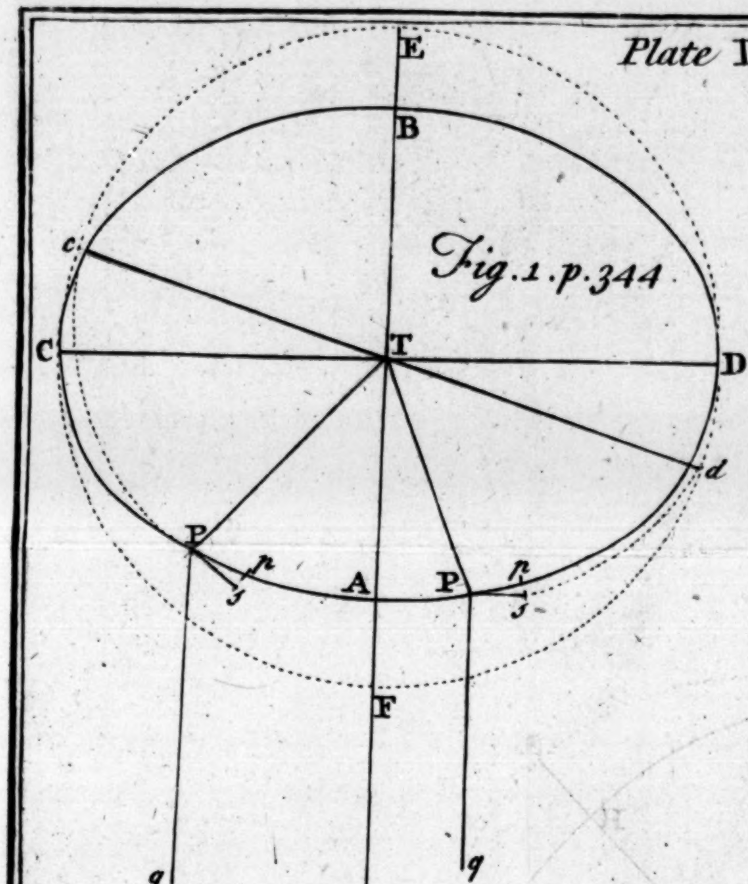


Fig. 1. p. 344

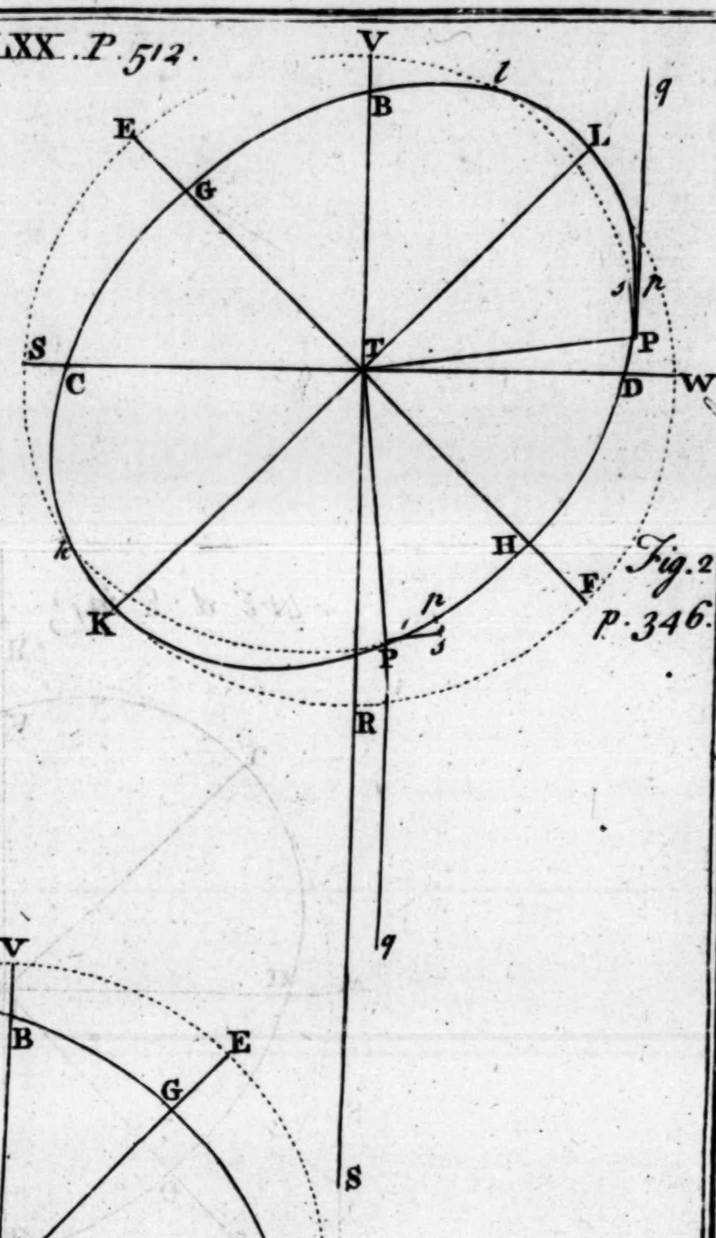


Fig. 2 p. 346.

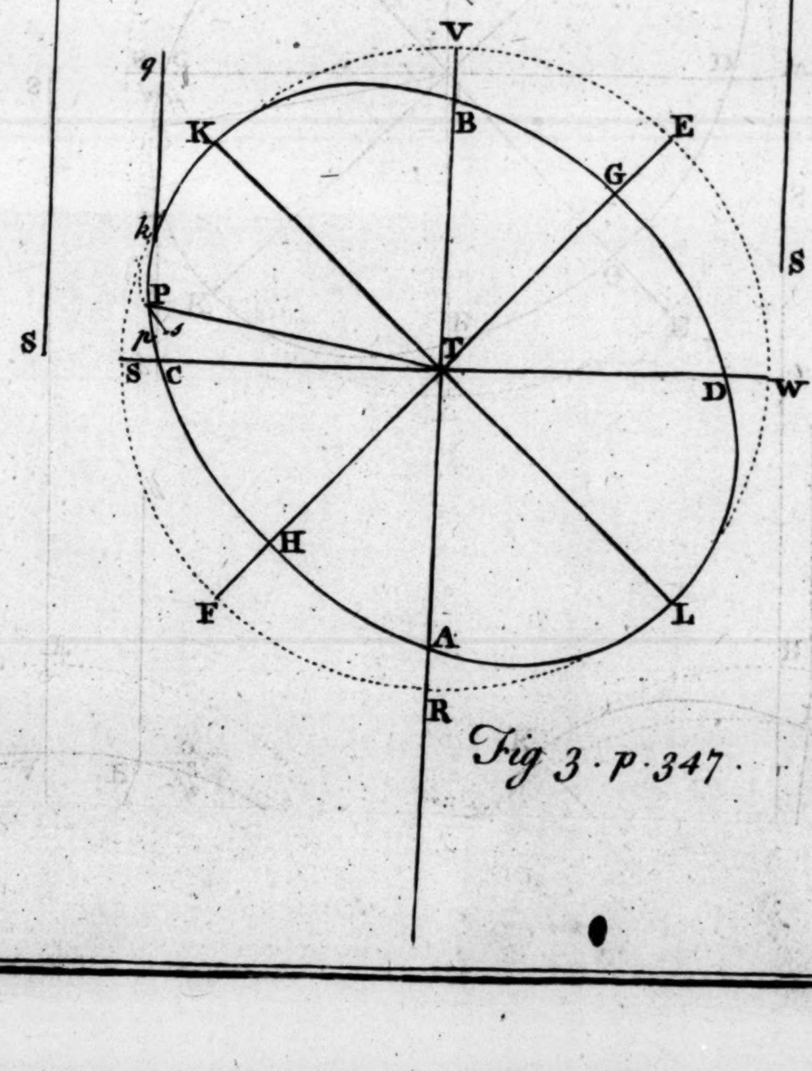


Fig 3. p. 347.

Inclination of the Planes C P T and *c s* T, is exceeding ſmall, and the Arch C *c* an infiniteſimal Quantity; therefore ſince P A is a finite Quantity, the Sum of the two Arches P C + P *c* is leſs than C A + A D, or a Semicircle; and hence in the ſpherical Triangle C P *c* the external Angle P C F is greater than the internal oppoſite Angle P *c* C. (See my *Young Trigonometer's Guide*, Vol. II.) That is, the Inclination of the Plane C A D to the Plane C F D is greater than the Inclination of the Plane *c* P T thereto; which was the firſt Thing to be ſhewn.

50. IN like Manner we prove, that as the Body P goes from the Conjunction A to the Quadrature D, the Inclination of the Orbit will be increaſed; for if, in this Caſe, through the Points P and *s* we deſcribe an Arch of a Circle in the Plane T P *s*, the ſaid Arch P *s d* will meet the Plane C F D in the Point *d* between F and D; and the exterior Angle P *d* F, the new Inclination of its Orbit, will be greater than the interior oppoſite Angle P D F, which was the Inclination when the Satellite was at A; which was the ſecond Thing to be ſhewn.

51. HENCE 'tis evident, that in this Situation of the Nodes the Inclination of the Orbit

Orbit is leaſt of all when the Satellite is in the Syzygies at A, and that it returns to its former Magnitude at the next Node; for the ſame Things are in the ſame Manner ſhewn when the Satellite paſſes through the remoter Part of its Orbit DBC.

52. HENCE alſo the Nodes in this Situation have a *retrograde Motion*, or are carried backwards from the Site DC to *dc*, in half a Revolution of the Satellite; and they recede as much more during the other Half-Revolution.

Plate LXX. Fig. 2.

53. IF the Nodes K, L, are in the Octants after the Quadratures C and D, then (1.) The Inclination of the Plane will be conſtantly diminiſhed in paſſing from the Node K to the 90th Degree at H or G. (2.) It will be increaſed during the Motion from that Point to the next Quadrature D or C. (3.) During both theſe Tranſits, or the Motion from K to D, or from L to C, the Nodes go backwards. (4.) In paſſing from the Quadratures to the next Node the Inclination of the Orbit is diminiſhed, and the Nodes go forwards. The firſt, ſecond, and third, are ſhewn as before, (*Art.* 49---52.) and the fourth is thus demonſtrated.

54. WHEN the Satellite P has paſs'd the Quadrature D, the Power NM becomes negative,

negative, or acts in a contrary Direction with respect to T, and hence the *Lineola* P *s* described by the compound Motion deflects from the Arch of the Orbit P *p* towards the Side B A; therefore 'tis plain, the Arch of a Circle P *s l*, described with the Radius T P in the Plane T P *s*, will meet the Circle F L B in a Point *l* between L and B; then, as before, we shew the Angle P *l* F is less than the Angle P L F; and the Node L has, during the Motion through D *l*, gone forwards to *l*. The same Things happen in the Transit from C to K.

55. FROM what we have demonstrated it appears, that during the whole Transit from the Node K to the Node L, the Inclination of the Orbit is more diminished than increased, and the same Thing happens on the other Side in going from L to K; therefore the Inclination is always less in the subsequent than in the preceding Node. And this will be the Case, more or less, wherever the Node K is placed between R and S.

56. WHEN the Nodes are in the other Octants, *viz.* between S and V, and R and W; then, (1.) While the Body P is passing from the Node to the next Quadrature, Plate LXX. Fig. 3.

ture, the Inclination of the Orbit is increased, and the Nodes go forwards. (2.) In passing from the Quadrature to the 90th Degree from the Node H or G, the Inclination is diminished, and the Nodes go backwards. (3.) In passing from thence to the next Node, the Inclination is increased, and the Nodes still go backwards. The second and third are demonstrated altogether as before, (*Art.* 49.) and the first is thus shewn.

57. The Satellite being at P, between K and C, the Direction of the Force NM is that of P*q*; whence the *Lineola* P*s*, described by the compound Force, will deflect from the Arch P*p* of the Orbit towards the Side VR; and consequently a circular Arch described on the Centre T through the Points *s* and P, in the Plane T*s*P, will meet the primitive Circle VSR in a Point *k* between K and S. Therefore the Angle *sk*F is greater than the Angle PKF; and the Node K is carried *in Consequentia* from K to *k*. The same Thing is shewn for the other part of the Orbit LGK.

58. HENCE it appears, that since the Nodes go forwards only while the Satellite is between the Node and the next Quadrature,

ture, and backwards while it paſſes from thence to the next Node, *the Nodes in each Revolution go backwards more than forwards*; and therefore, upon the Whole, *the Motion of the Nodes is abſolutely backwards*, unleſs they happen to be in the Syzygies, where they are quieſcent; becauſe in that Caſe the Motion in Latitude is not at all diſturbed by the Force N M, and conſequently where the Inclination of the Orbit is the greateſt of all. (See *Art.* 46.)

59. ALL the Errors in the Satellite's Motion hitherto deſcribed are a little greater in the Conjunction of the Bodies P and S, than in their Oppoſition; becauſe the generating Forces N M and L M in the former Caſe are greater than N *m* and *l m* in the latter; as we have ſhewn in *Annotation* LXXXIV. *Art.* 9, 10, 11, 12. Alſo it is there ſhewn, that each of the diſturbing Forces N M and L M is inverſely as the Cube of the Diſtance, and therefore become greater than the Diſtance S T is leſs, viz. *in Perihelio*, and leſs as the Diſtance increaſes, viz. *in Aphelio*.

60. OF theſe diſturbing Forces, ſince N M is near twice as great as L M, therefore the Diminution of the central Force will exceed its Augmentation doubly; and ſo,

ſo, upon the Whole, the Satellite P will be leſs attracted towards T by the joint Forces of S and T, than by the Body T alone; conſequently the Satellite deſcribes a larger Orbit, and its Period of Revolution is greater.

61. In all that has been ſaid, if S be the Sun, T the Earth, and P the Moon, the Theory of the Lunar Motions and Irregularities is contained in the foregoing Articles. And as this Theory reſults from the Laws of Attraction, and was firſt excogitated by Sir *Iſaac Newton* by reaſoning *à priori*; ſo it is found no leſs conſonant to the Experience and Obſervation of Aſtronomers: For from thence it appears, (1.) That the Moon deſcribes not a *Circle* but an *Ellipſe* about the Earth. (2.) That the Eccentricity of the Lunar Orbit is variable, being when leaſt but 43619; when mean, 55237; and when greateſt, 66854 of ſuch Parts as the Radius contains 1000000. (3.) That the Moon's Apogee goes forwards in the Syzygies, and backwards in the Quadratures; but upon the Whole it goes forwards, ſo as to complete a Revolution in about nine Years. (4.) That the Moon's Orbit is inclined to the Plane of the Ecliptic in a certain Angle. (5.) That this

Plate LXXI. P. 351

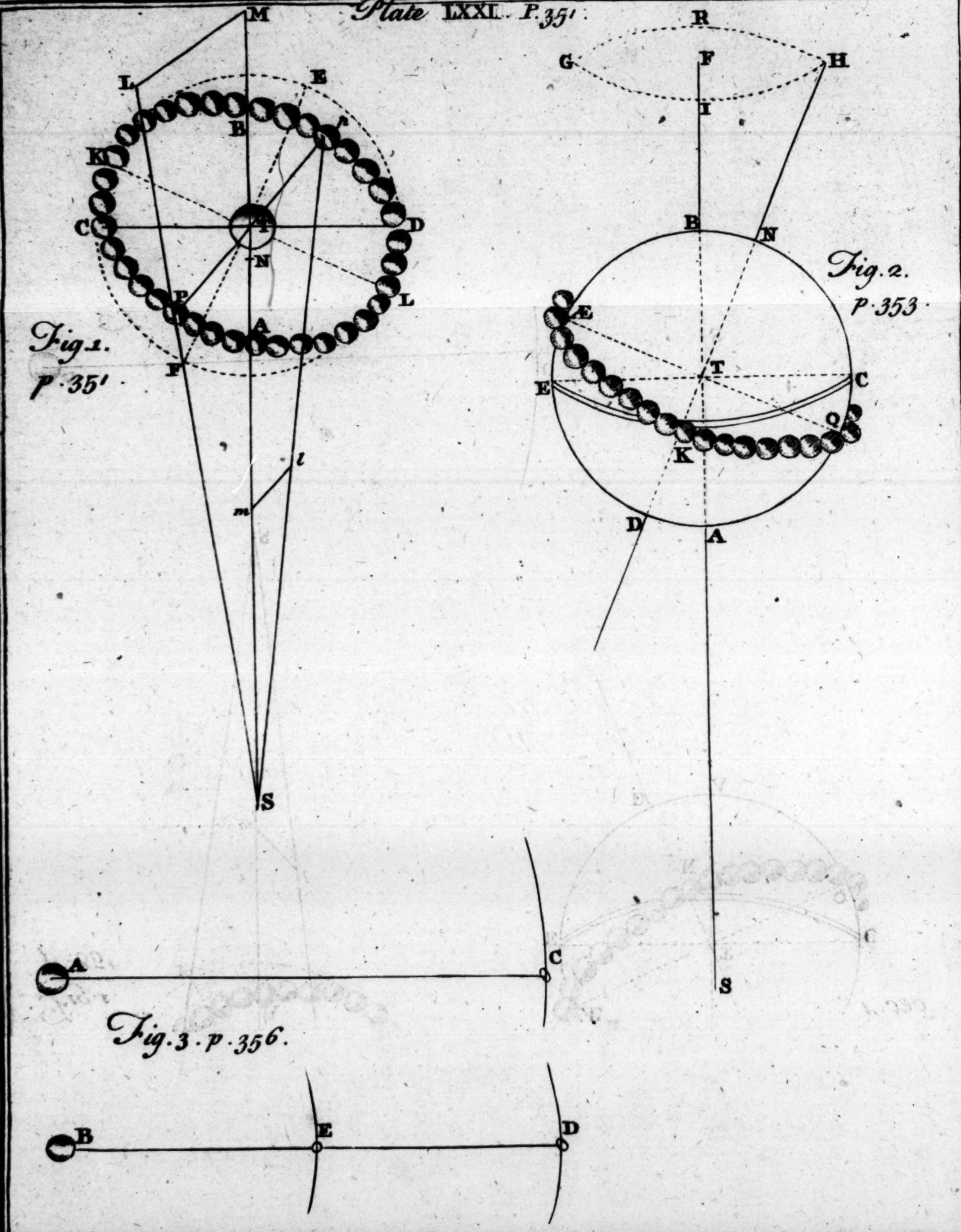


Fig. 1. p. 351

Fig. 2. p. 353

Fig. 3. p. 356.

this Inclination of the Lunar Orbit is variable, being when leaſt 5°, and when greateſt 5° 18′. (6.) That the Nodes of the Moon go ſometimes backwards, ſometimes forwards, and are in the Syzygies quieſcent. (7.) That the Motion of the Lunar Nodes is upon the whole *backwards*, at the Rate of 20° *per Annum*, and ſo as to complete a Revolution in about 18 Years and a half. Such is the *Surprizing Harmony of the* Newtonian *Theory with Aſtronomical Obſervation, even in this moſt difficult Part*, that *Halley* might well ſay,

Intima panduntur victi penetralia Cœli,
Nec latet extremos quæ vis circumrotat Orbes.

And,

Diſcimus hinc tandem quâ cauſâ argentea Phœbe
Paſſibus haud æquis graditur; cur ſubdita nulli
Hactenus Aſtronomo numerorum fræna recuſet;
Cur remeant Nodi, curque Anges progrediuntur.

62. THE ſame Method of Reaſoning, by which we have explained the *Tides*, and the *Lunar Theory*, does alſo furniſh us with a *Phyſical Explication* of the *Motion of the Earth's Axis*. For let us conceive numerous Bodies, ſuch as P, to revolve about the Earth T, at an equal Diſtance, in equal Times, Plate LXXI. Fig. 1.

Times, and in a Plane inclined to the Plane of the Ecliptic, 'tis evident each one will be affected with the ſame Motions as the Body P. Again, Let us ſuppoſe their Number ſo increaſed as that they become contiguous to each other, and thereby form a Fluid *Annulus* or Ring of cohering Bodies.

63. THEN ſince each Part of the Ring obſerves the ſame Laws of Motion with P, and becauſe while one Part is ſo attracted as to augment the Inclination of the Plane, the contrary Part is affected by a contrary Force to diminiſh it, therefore the Inclination of the Plane will always be variable, and governed by the Difference of the Forces which act upon it in contrary Parts.

64. THERFFORE ſince the greater Force always prevails, the Parts of the Ring which are in the Conjunction and Oppoſition will move more ſwiftly, and accede nearer to the Body T than thoſe in the Quadratures (by *Article* 21, 22.). And the Nodes of this Ring will be quieſcent in the Syzygies, but in any other Situation will go backwards, and ſwifteſt of all in the Quadratures (by *Article* 47---58.). Laſtly, the Inclination of the Ring will be every where analogous to that of the Lunar Orbit;

bit; and consequently its Axis will in each Revolution oscillate to and from the Axis of the Ecliptic, and be carried backward by the Retrocession of the Line of Nodes.

65. IF the Quantity of Matter in the Ring were to be diminished in any Ratio, the Motions will all remain the same, as depending on the attractive Force of the central Body T, which is still the same. If the Diameter of the Ring be diminished, the Motions will be in the same Ratio diminished also; for Effects will be as their Causes. But L M : $\frac{PT}{TS^3}$; and because T S is constant, LM is as PT. Also M S = $\frac{ST \times LM}{PT}$ = S T; therefore M S is as S T, a given Quantity. (See *Annotation* LXXXIV. 9, 11.) Consequently the Motions of the Ring will be every where as the diminished Distance P T.

66. SUPPOSE therefore the Diameter of the Ring to be diminished so far as to be equal only to the Diameter of the Earth, and the Body T to be spherical, and every Way enlarged till it equalled the Bulk of the Earth; then would the Ring of Bodies coincide with, and be contiguous to the Plate LXXI. Fig. 2.

 Surface

Surface of the Earth, and would alſo cohere to it. And ſuppoſe the Plane of the Ring made an Angle with the Plane of the Ecliptic of 23 Degrees and a half, then would all the Motions of the Ring continue, only in a leſſer Degree; and would be communicated to the Earth, becauſe it adheres firmly thereto; for the Earth equilibrated in Æther will yield to any Motion impreſſed upon it from without. But the Motions of the Ring being now communicated to the Body of the Earth, will be farther diminiſhed in Proportion as the Maſs of Matter to be moved is augmented.

67. Now this Circle or Ring of Bodies encompaſſing the Earth by Suppoſition is actually the true State of the Earth; for we have ſhewn its Diameter through the Equator ÆQ exceeds the Length of the Axis N D, (*Annot.* CXLVIII.) and therefore it is ſurrounded by a Zone of Matter upon the Equator analogous to this feigned Ring of Bodies, and which muſt of courſe produce the ſame Effects.

68. HENCE in the Equinoxes, that is, when the Earth's Nodes are in the Syzygies, or when the Line of the Nodes (*viz.* the Equinoxes) paſs through the Earth and Sun, the Inclination of the Equator and Ecliptic, that is, the Angle Æ T E or F T H is

is greateſt of all; and from this Time it grows leſs till the Sun arrives at the 90th Degree (or *Solſtice*), when the Line of Nodes are in the Quadratures, and then it is leaſt of all.

69. THEREFORE twice in a Year the Inclination of the Ecliptic and Equator is diminiſhed, and twice again reſtored; and the Nodes (or Equinoxes) conſtantly go backwards, and carry the Axis of the Earth T H with a retrograde Motion about the Axis of the Ecliptic T F, tracing out the Circle or rather vermicular Curve H I G R in the Heavens among the Fixed Stars.

70. AGAIN; the Plane of the Equator is inclined to the Plane of the Moon's Orbit, for the latter makes an Angle of but about 5 Degrees with the Plane of the Ecliptic; and therefore the Moon (though a leſs Body than the Sun, yet being nearer) produces a greater Effect than the Sun on the Equatorial Ring or Zone of Matter, and ſo augments all the aforeſaid Motions of the Earth's Plane and Axis. Sir *Iſaac Newton* has ſhewn (*Prop.* XXXIX. *Lib.* III.) that the Part of the annual *Receſſion of the Equinoxes*, which is owing to the Sun, is $9'' \; 7''' \; 20''''$, and that which is owing to the Moon is $40'' \; 52''' \; 52''''$; therefore by the

 joint

joint Influence of the Sun and Moon the Equinoxes recede yearly about 50″ 00‴ 12⁗; which is likewiſe verified by the Obſervations of Aſtronomers for 2000 Years paſt. See *Annotation* CXLI.

71. I SHALL now explain the Method uſed by Philoſophers for computing the Quantities of Matter, Denſities, Weight of Bodies, &c. in the *Sun*, the *Earth*, *Jupiter*, and *Saturn*, by Means of Satellites revolving about them. In order to this let

Plate LXXI. Fig. 3. Q, *q*, expreſs the Quantities of Matter in the two Bodies A, B; alſo let G, *g*, be the reſpective Forces of Gravity at the equal Diſtances A C and B D. Let T, *T*, be the Periodical Times of Bodies revolving about A and B at thoſe equal Diſtances; and let T, *t*, be the Periodical Times of Bodies revolving at the unequal Diſtances A C and B E, which call D and *d*.

72. THEN in the given Diſtances A C, B D, we have $Q : q :: G : g$ (*Art.* 7.) But

$G : g :: \frac{1}{T^2} : \frac{1}{\mathcal{T}^2}$ (by *Annot.* XXXIV. 6.)

Whence $Q : q :: \frac{1}{T^2} : \frac{1}{\mathcal{T}^2}$; and multiplying the latter Ratio by D^3, we have $Q : q :: \frac{D^3}{T^2} : \frac{D^3}{\mathcal{T}^2}$. But becauſe $\mathcal{T}^2 : t^2 :: D^3 : d^3$, (ibid.

Art.

Art. 11.) therefore $\frac{D^3}{T^2}=\frac{d^3}{t^2}$; conſequently, $Q:q::\frac{D^3}{T^2}:\frac{d^3}{t^2}$. That is, *The Quantities of Matter in any two Bodies are in the compound Ratio of the Cubes of the Diſtances directly, and Squares of the Periodical Times inverſely, of Bodies revolving about them.*

73. IN this Calculation the Bodies A and B are ſuppoſed at reſt. We conſider the Sun at reſt with reſpect to *Venus*, and *Jupiter* and *Saturn* in reſpect of their Secondaries; and we have reduced the Diſtance of the Moon to 60 Semidiameters, at which ſhe would revolve about the Earth at reſt. Now let the Diſtance of the Earth from the Sun be put — — 1000

Sun be put — —	1000
then *Venus* revolves about the Sun at the Diſtance —	723
the 4th Satellite of *Jupiter* at the Diſtance — —	12,4775
the 4th Satellite of *Saturn* at the Diſtance — —	8,5107
the Moon at the Diſtance	3,054
The Periodical Time of *Venus* is	19414160″
of the *Jovian* Sat.	1441929″
of the *Saturnian* Sat.	1377674″
of the Moon,	2360580″

 74. Now

74. Now suppose the Quantity of Matter in the Sun be 10000, then for that in *Jupiter* say, As $\frac{\overline{723}^{3}}{\overline{194141\,60''}^{2}} : \frac{\overline{12,4775}^{3}}{\overline{1441929'}^{2}}$ :: 10000 : 9,305, (by *Art.* 71.) the Density of *Jupiter* compared with that of the Sun. By the same Analogy the rest are found, and in each they are as follow.

In the *Sun*, *Jupiter*, *Saturn*, *Earth*, *Moon*,
10000. 9,305. 3,250. 0,0512. 0,0013.

75. Now if these Quantities of Matter are divided by the Squares of the Diameters of these Bodies, the Quotients will be as the *Weight of Bodies on their Superficies*, (by *Annot.* XIX. 3.) The Diameters of the Sun and Planets see in *Annot.* CXXXV. Then these Gravities will be as follow.

In the *Sun*, *Jupiter*, *Saturn*, *Earth*, *Moon*,
10000. 936. 519. 431. 146.

76. In homogeneous, unequal, spherical Bodies, the *Gravities on their Surfaces are as the Diameters*, if the Densities are equal (*Annotation* XIX. 3.) But if the Bodies be equal, *the Gravities will be as the Densities*, because they will be as the Quantities of Matter, which in this Case are as the Densities (*Annot.* XVII.) Therefore in Bodies of unequal Bulks and Densities, *the Gravities will be in a compound Ratio of the Diameters*

ameters and Denſities. Conſequently, *the Denſities will be as the Gravities divided by the Diameters*; and therefore in the ſeveral Bodies as follows.

In the *Sun*, *Jupiter*, *Saturn*, *Earth*, *Moon*,
10000. 9385. 6567. 39539. 48911.

77. As it is not likely that theſe Bodies are homogeneal, the Denſities here determined are not to be ſuppoſed the *truc*, but rather *mean Denſities*, or ſuch as the Bodies would have if they were homogeneal, and of the ſame Maſs of Matter and Magnitude.

78. LET F, *f*, be the Forces of the Sun and Moon to move the Sea; D, *d*, their Diſtances from the Earth; then $F : f : : \frac{Q}{D^3} : \frac{q}{d^3}$. (See *Annotation* XIX. and LXXXIV. 9, 11.) Let B, *b*, be the Bulks; R, *r*, the Diameters; and N, *n*, the Denſities of the Sun and Moon: then will $Q : q : : BN : bn : : R^3 N : r^3 n$; (*Annot.* XVII. and XIX.) wherefore $F : f : : \frac{R^3 N}{D^3} : \frac{r\,n}{d^3}$. Laſtly, let A, *a*, be the apparent Diameters of the Sun and Moon; then will $A : a : : \frac{R}{D} : \frac{r}{d}$; becauſe any Body ap-

pears larger the bigger it is, and leſs in Proportion to the increaſing Diſtance; therefore $A^3 : a^3 :: \frac{R^3}{D^3} : \frac{r^3}{d^3}$. Hence $F : f :: A^3 N : a^3 n$. Conſequently, $N : n :: F a^3 : f A^3 :: \frac{F}{A^3} : \frac{f}{a^3}$.

79. BUT (according to Sir *Iſaac Newton*) $F : f :: 1 : 4{,}4815$. See *Annot*. LXXXIV. 28.) And $A : a :: 32' \; 12'' : 31' \; 16\frac{1}{2}''$ (at a Mean, by Obſervation.) That is, $A : a :: 3864 : 3753$. Therefore $N : n :: \frac{1}{3864^3} : \frac{4{,}4815}{3753^3} :: 10000 : 48911$, the Ratio of the Sun and Moon, as above ſhewn, *Art*. 76.

80. The Quantities of Matter being $Q : q :: R^3 N : r^3 n$ (*Art*. 78.) and with reſpect to the Earth and Moon, $N : n :: 39539 : 48911$; and $R : r :: 109 : 30$, (*Annot*. CXXXVI. 4.) therefore $Q : q :: 109^3 \times 39539 : 30^2 \times 48911 :: 32{,}31 : 1 :: 0{,}0512 : 0{,}0013$, as determined in *Art*. 74.

81. THE Weight of Bodies on the Surface of the Earth and Moon are *in the compound Ratio of the Diameters and Denſities*, (*Art*. 76.) that is, in the Ratio of 109×39539 to 30×48911, or as 431 to 146, (as *per Art*. 75.) or as 3 to 1 nearly.

82. HAVING the Quantities of Matter in the Earth and Moon, the Diſtance of the common Centre of Gravity is determined: For the Diſtance of the Moon from the Earth's Centre is to this Diſtance as 40,31 to 1; which Ratio is more accurate than that of 41 to 1, made uſe of in *Annot.* XXXVI. *Art.* 2.

83. THE Theory we have here been explaining is applicable to any Syſtem of three or more Bodies, as well as to the *Sun*, the *Earth*, and *Moon.* Thus the perturbating Forces and Irregularities of Motion in the Syſtem of the *Sun*, *Jupiter*, and any of his Moons, may be eſtimated in nearly the ſame Manner, *(mutatis mutandis)* as alſo thoſe of the *Sun*, *Saturn*, and his *Satellites*; and laſtly, between the *Sun*, and *primary Planets*, by putting the Caſe more generally, (as Sir *Iſaac* does) in ſuppoſing both S and P to revolve about the fixed central Body T, which we may ſuppoſe to be the Sun, and S and P any two of the Planets at pleaſure. Therefore, to uſe the Author's own Words (in another Caſe) for Concluſion: *Uſus igitur hujus Theoriæ latiſſimè patet; & latè patendo, Veritatem ejus evinçit.*

THE GLOBULAR PR

OR

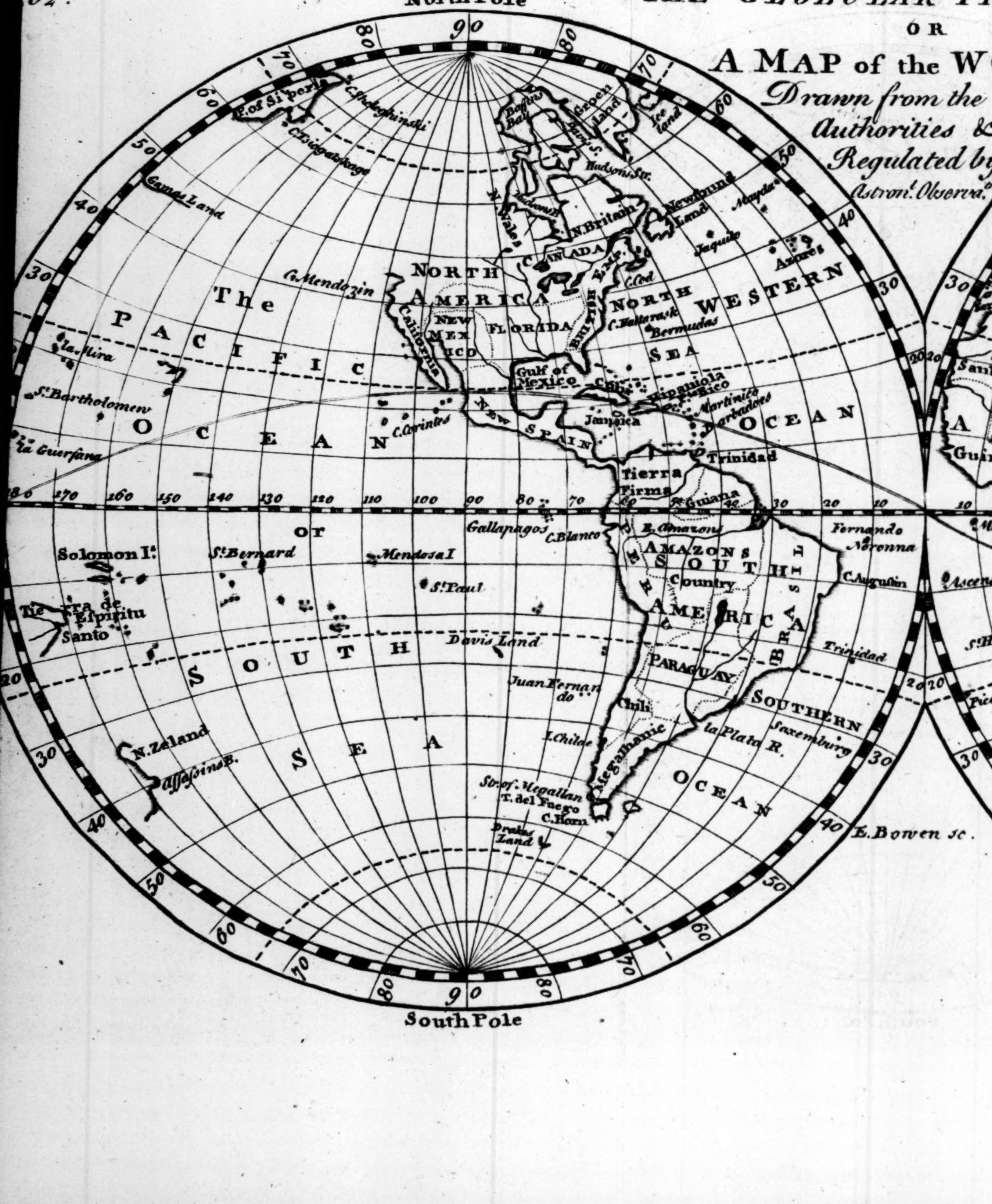
A MAP of the WO

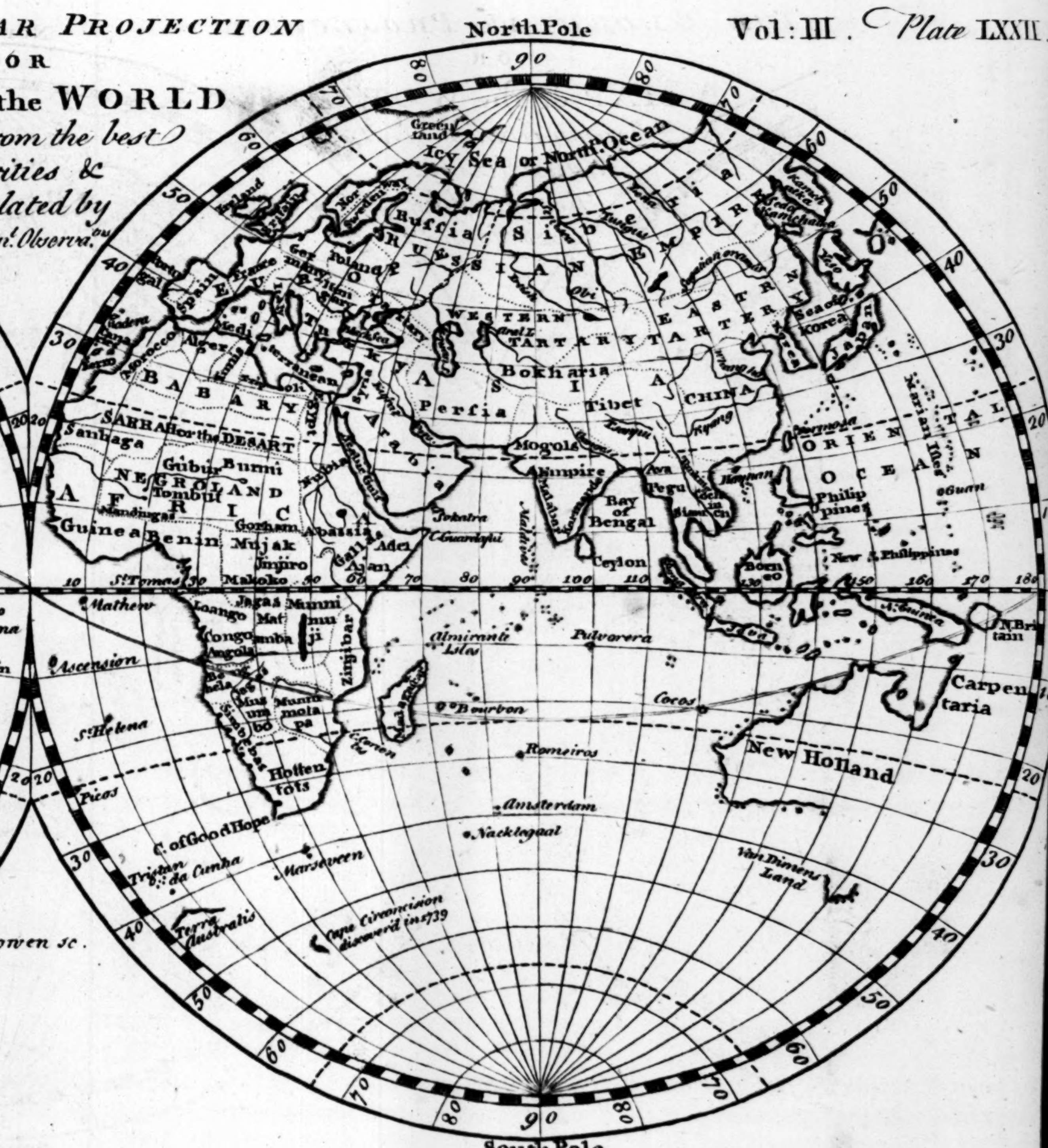
Drawn from the

Authorities &

Regulated by

Astron. Observa.




AR PROJECTION
OR
the WORLD
om the best
rities &
lated by
n. Observa.
Vol: III. Plate LXXII.
North Pole
South Pole
Icy Sea or North Ocean
Russia
RUSSIAN EMPIRE
Siberia
EUROPE
Spain
France
Germany
Poland
Turkey
Barbary
Morocco
Algeria
Mediterranean
ASIA
WESTERN TARTARY
EASTERN TARTARY
Bokharia
Persia
Arabia
Tibet
CHINA
Japan
Korea
Sea of Korea
Mogols Empire
Bay of Bengal
Ceylon
Peguu
Siam
Borneo
Philippines
ORIENTAL OCEAN
New Philippines
N. Guinea
N. Britain
Carpentaria
New Holland
Van Dimens Land
AFRICA
SARRA or the DESART
Sanhaga
Gubur
Burni
NEGROLAND
Tombut
Guinea
Benin
Nujak
Abassia
Congo
Angola
Zanguebar
Hottentots
C. of Good Hope
Madagascar
Ascension
St. Helena
Tristan da Cunha
Bourbon
Amsterdam
Cape Circoncision discover'd in 1739
Terra Australis
Iceland
Britain
Egypt
Nubia
Greenland
Kamchatka
Marian Isles
Guam
Cocos
Romeiros
Maldivia
Socotra
Pulvorera
owen sc.

APPENDIX II.

Concerning the IMPROVEMENTS *in* OPTICAL INSTRUMENTS.

I. *Of the* Univerſal Compound MICROSCOPE. II. *Of a new conſtructed* SOLAR MICROSCOPE. III. *A new* REFLECTING TELESCOPE *and Megalaſcope in one Inſtrument.* IV. *The Theory and Application of a* NEW MICROMETER, *by Means of a* divided Glaſs. V. *The* New Improvements *in Refracting Teleſcopes conſidered.* VI. *Of the latter Improvements of* REFRACTING *and* REFLECTING TELESCOPES. VII. *The Nature and new Conſtruction of* VISUAL GLASSES *explained.* VIII. *A Deſcription of the* EQUATORIAL TELESCOPE.

I. *Of the* UNIVERSAL MICROSCOPE.

IN the firſt Edition of this Work a Plate of the univerſal Compound Microſcope was deſigned, but through Inadvertency was left out; and ſince that Time

Time having made considerable Improvements, both in its Form and Use, I have supplied that Deficiency in the Print of one of the most elegant Construction I can think of, and of the most extensive Use; there being no small Object of any Kind but what may be readily and easily viewed by it. It has all the Apparatus of Glasses, reflecting Specula, adjusting Screws, *&c.* in common with others. Besides which, the Contrivance of a moveable Stage for the Objects, and the Joint on the Top by which it is placed, either in a perpendicular or horizontal Position, are Advantages peculiar to this Construction of a Microscope. The Description of which will be easy to understand as follows:

Plate I. A B C is the Body of the Microscope.

A is the Magnifier, of which there are four of different Powers.

B is the large Middle-Glass for amplifying the Field of View.

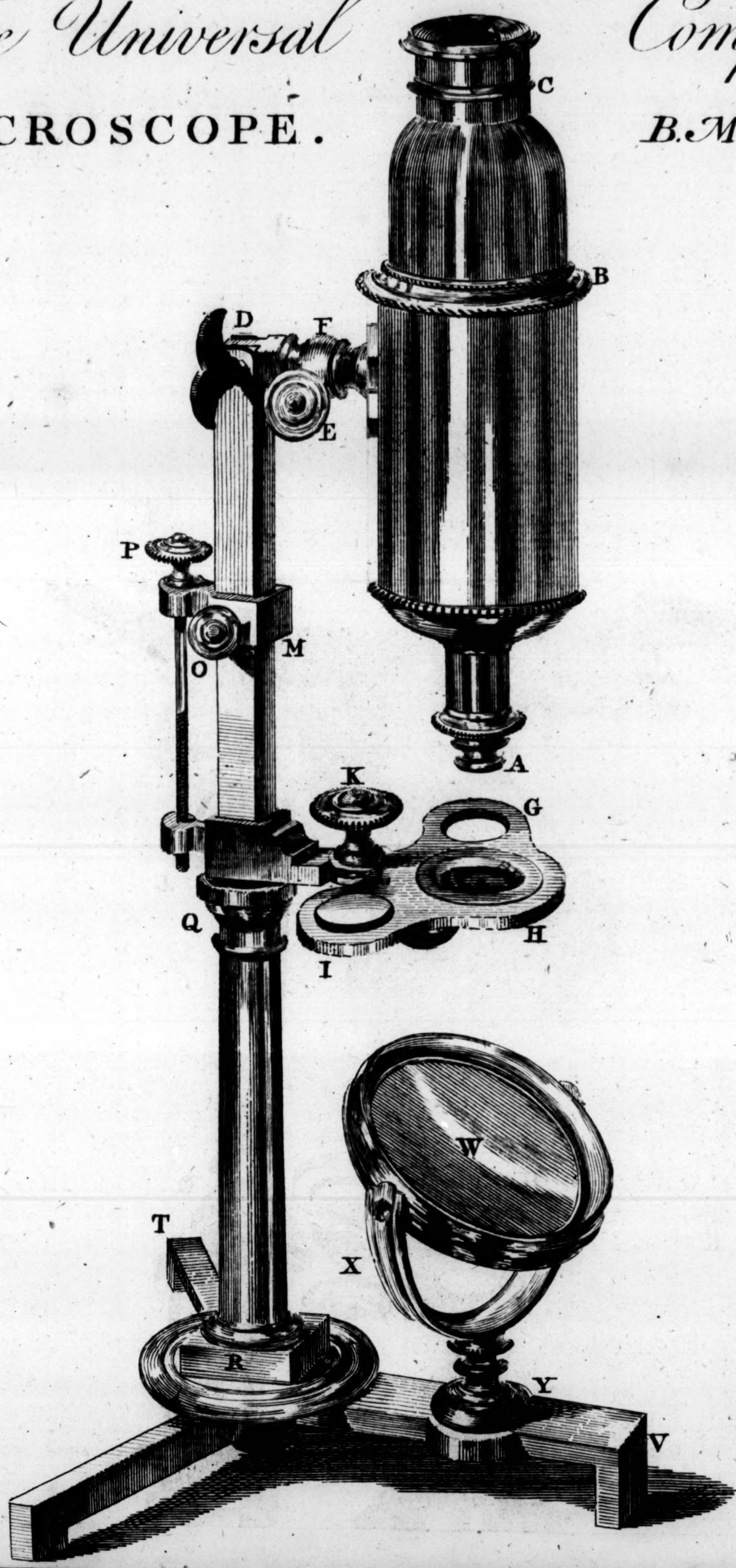
C is the Eye-Glass at the Top.

D the Joint on which the Microscope is moveable on a perpendicular, oblique, or horizontal Position at pleasure.

E is a Screw, and F the Socket by which it is taken off and put on as Occasion requires.

The Universal Compound MICROSCOPE.

B. MARTIN, Inv.



GHI is the moveable Stage on which Objects are placed to be viewed, and has a Motion about the Centre.

K is the Screw in the Centre L, by which that Motion is adjusted and regulated.

MN are two Pieces of Brass moveable upon a square Stem, to adjust the Stage with its Objects to a proper Distance from the Magnifier A.

O is a Screw for fixing the Piece M to the Stem.

P the adjusting Screw for moving the Part N up and down to its proper Situation.

QR the Brass Pillar on which the Whole is supported.

STV three strong Feet, serving as a firm Basis to the Microscope.

W a reflecting Mirror, moveable vertically in a Semicircle X, and horizontally in a Socket at Y.

THE Body of the Microscope, the Stage, and the Reflecting Mirror are taken off from the Stem, and the Legs folded together, so as to be conveniently placed, and take up but little Room in a Shagreen Case, which is therefore of a portable Size.

II. *Of the New* SOLAR MICROSCOPE.

THE Plate of the Solar Microſcope, given in the firſt Edition, being imperfect on two Accounts, *viz.* That it exhibited a View only of Part of the Inſtrument, and that of a very bad Conſtruction, I have thought it proper here to add the Figure of this extraordinary Inſtrument, with all the Improvements I have hitherto been able to make in it, which are as follow:

FIRST, The Common Solar Microſcope was made of *Wilſon*'s, fixed to a proper Apparatus: But as this Form admitted only one Magnifier in the Inſtrument at a Time, it is changed for another, in which all the Magnifiers are contained in a Slider; and ſo all the different Degrees of magnifying an Object in a dark Room are immediately at Hand.

SECONDLY, With *Wilſon*'s Part the Object was moved towards the Glaſs with a Screw; in this the Glaſs is moved towards the Object by Teeth and Pinion, which is found to be much the better Way.

THIRDLY, In the old Form the whole Body of *Wilſon*'s Microſcope was obliged to

be

be turned round ſeveral Times in getting a true focal Diſtance, by which Means a Motion of the large confuſed Image became ſo great as to be very diſagreeable to the Spectators; which is entirely prevented in this new Form, where there is only a Motion of the Object backward and forward in a right Line.

FOURTHLY, Thoſe which were made in Wood had the Looking-Glaſs move by Wheel and Pulley, with Cat-gut Strings ſtrained tight upon them. But this Mechaniſm being often at Fault, and giving Gentlemen a great deal of Trouble, I have ſubſtituted in this new Form a Wheel and Pinion with Teeth, as well in thoſe made of Wood as Braſs, by which the Motion of the Glaſs is rendered always conſtant, certain, and eaſy.

FIFTHLY, The Looking-Glaſs in the old Form is raiſed and depreſſed by a ſliding Wire; but in this new one, whether in Braſs or Wood, the ſaid Glaſs is moved by a Screw, and therefore will always ſtand in the Poſition it is ſet, and give no Trouble in the Experiment.

SIXTHLY, The illuminating Glaſs was ſcrewed on the Outſide of the Frame of this Inſtrument; but in this new Conſtruction

ſtruction it is placed within the End of the large Tube, which is much more convenient.

UPON the whole, the *old Conſtruction* of the Solar Microſcope rendered it a heavy, clumſy, and ill-contrived Inſtrument; whereas every one who has ſeen the Form which I have here given in the Plate allow it to be the moſt neat and eaſy, light and uſeful that can be contrived. The ſeveral Parts of which are as follow:

Plate II. ABCD is the ſquare Plate ſcrewed on to the Window-ſhutter.

EF is the Looking-Glaſs on the Outſide.

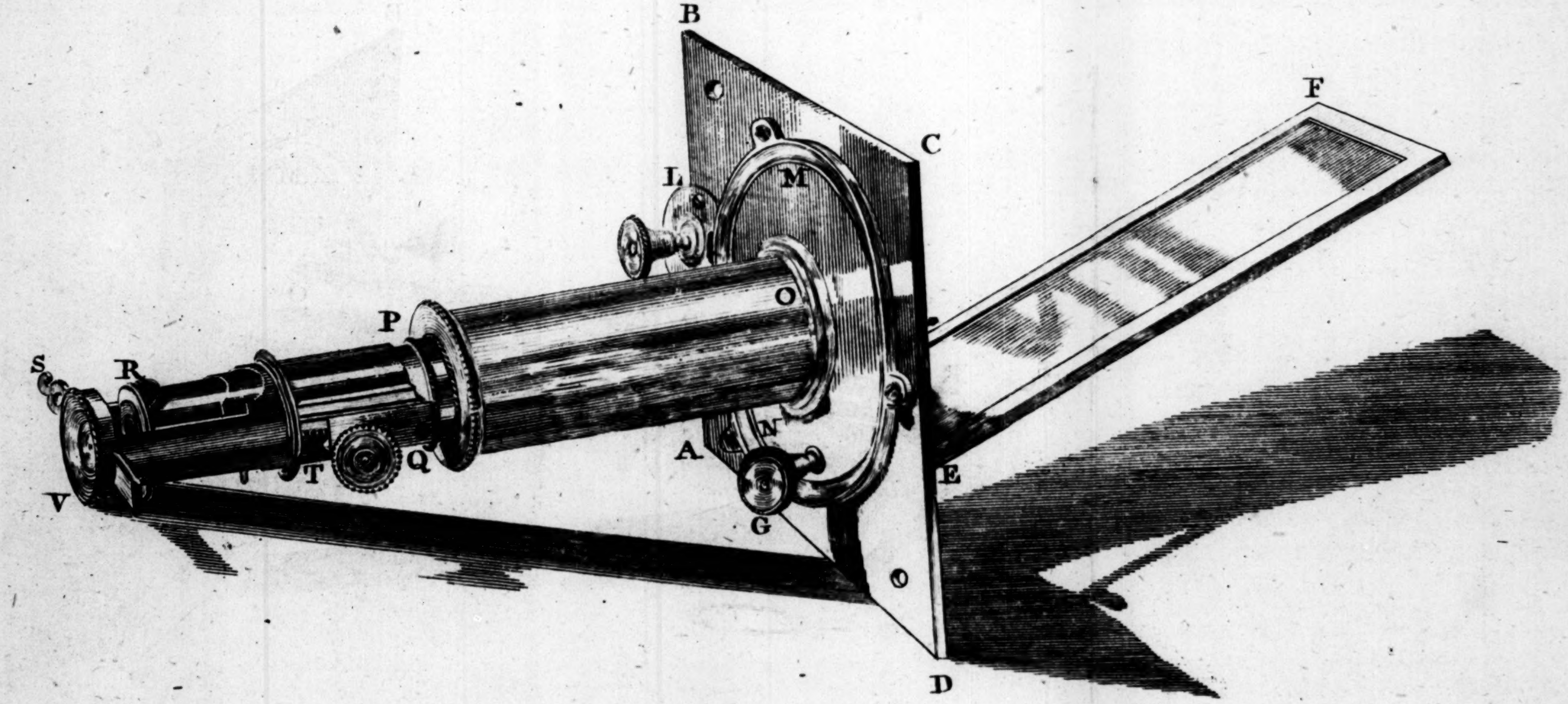
G is the Pinion, which, by an End-leſs Screw on the other Side the Plate, moves the reflecting Glaſs up and down.

L is the Pinion by which the Wheel contained under the Piece

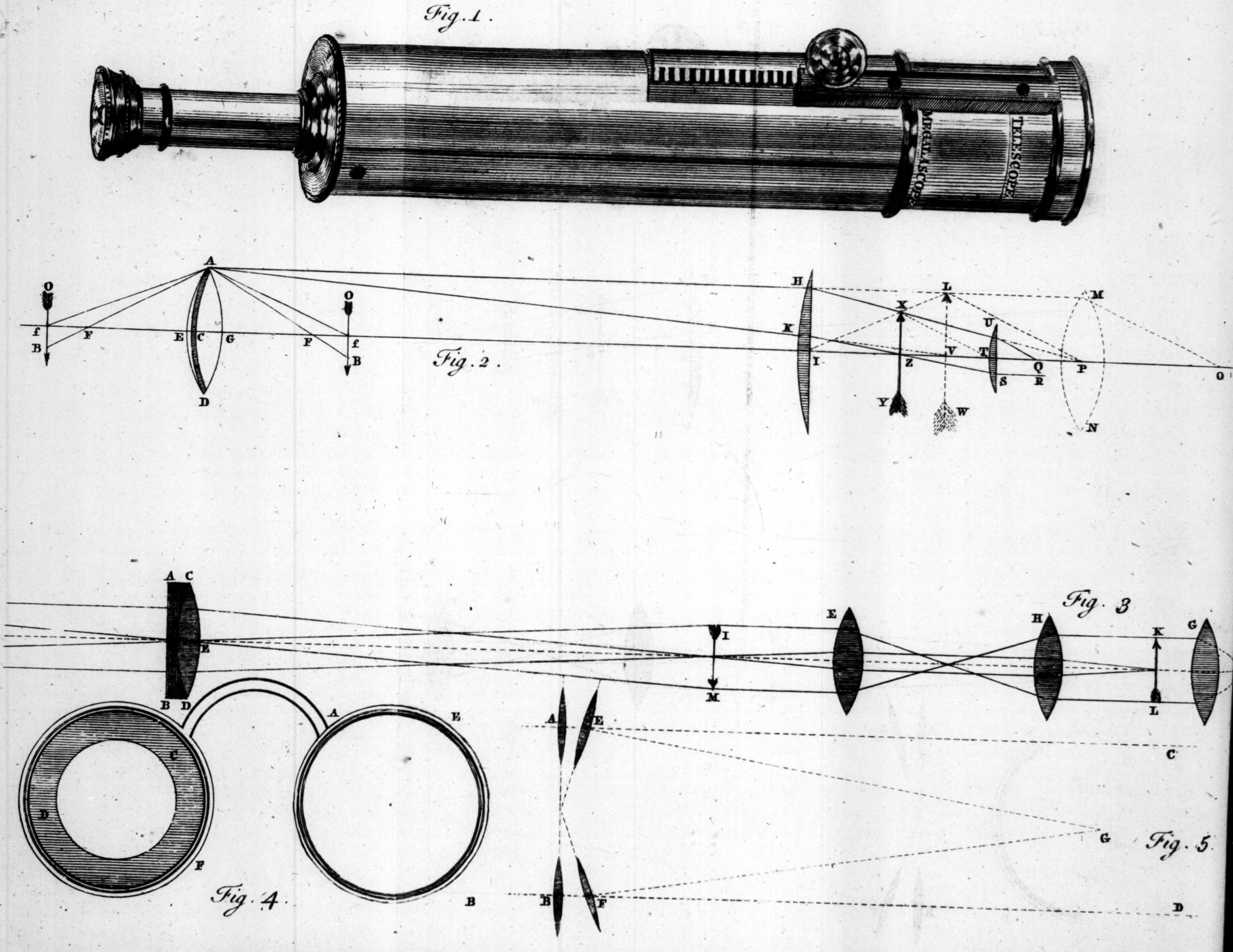
MN is moved, and by which the Looking-Glaſs EF has a circular Motion given it.

OP is the Tube containing the Illuminating Glaſs at O, and a Drawer at P, by which a due Quantity of Light is thrown upon the Object.

The New. SOLAR MICROSCOPE.



Appendix II . Plate IV.



QR is a Part ſubſtituted inſtead of *Wilſon*'s Microſcope, the inner Part of which is fixed, and the outer Part moveable for the adjuſting a due focal Diſtance of the Object from the Glaſs.

R is the Part conſiſting of three Braſs Plates, in which the Sliders and Tubes containing the Objects are placed, as in *Wilſon*'s Microſcope, and kept together by a ſpiral Spring Wire within.

S is the Braſs Slider containing the ſeveral magnifying Glaſſes, which may be ſucceſſively applied to the Object, by which it may be inſtantly magnified to any Degree at Pleaſure,

T is the Pinion by which the external Part QV is moveable backward and forward, for procuring a true focal Diſtance, and conſequently the Image of the Object with the utmoſt Facility and Exactneſs.

III. *Of a New* REFLECTING TELESCOPE.

HAVING long obſerved the great Perfection and Uſefulneſs of Reflecting Teleſcopes above Refractors, ariſing from their Nature and Conſtruction, I have been Plate IV. Fig. 1.

more ſolicitous to cultivate and improve that Inſtrument than any other; eſpecially when I ſaw much Room left for that Purpoſe. The reflecting Teleſcope, as uſually made, has not half the Uſes it is capable of; for when it is well conſidered, it will be found to anſwer the Purpoſes, not of a TELESCOPE only for viewing diſtant Bodies, but likewiſe it is the moſt compleat MEGALASCOPE, or Inſtrument for ſhewing all Objects at a very near Diſtance, and all the larger Sort of ſmall Objects, in the greateſt Perfection. In the third Place, it gives the moſt perfect View of any ſmall Object, opake or tranſparent; and therefore is a MICROSCOPE of the very beſt Kind, as far exceeding common Refracting Microſcopes, as Viſion by reflected Light is more exquiſite and perfect than that by Refraction. Indeed there are two different Ways by which the Reflecting Teleſcope may be uſed as a Microſcope, each of which has its peculiar Advantage. In the fourth and laſt Place, this Inſtrument is a SOLAR MICROSCOPE, by being ſcrewed into the Plate A B C D of that Inſtrument above deſcribed. In all theſe ſeveral Ways I have uſed this Inſtrument with great Succeſs, and Pleaſure. I have, for theſe Purpoſes, contrived

trived it in a different Form from that in which they are uſually made, and by which Means they are eaſily applicable to the foregoing Purpoſes; and even thoſe that will ſhew *Jupiter*'s Moons are hereby rendered portable, or adapted to the Pocket.

THESE Pocket Reflecting Teleſcopes have a Drawer, in the End of which the ſmall Speculum is fixed, and adjuſted by Teeth and Pinion. They are equally applicable to Uſe at Sea as at Land; and, notwithſtanding many People have been perſuaded, that Refracting Teleſcopes are moſt uſeful at Sea, the Abſurdity of this Notion will evidently appear to any one who conſiders what we have in the foregoing Lecture ſufficiently demonſtrated, *viz.* That the natural Cauſe of indiſtinct Viſion is vaſtly leſs in a Reflecting Teleſcope than in a Refractor; and this upon the Suppoſition that the Quantity of Light and magnifying Power are the ſame in both. To conclude, I am thoroughly ſatisfied that if the Nature of this Inſtrument was well conſidered and underſtood, and People could be made ready at the Uſe of it, they who could afford to purchaſe it, would never have any other. But though the practical Application be very eaſy, yet as it is

 new

new and unuſual, and requires ſome ſmall Degree of Dexterity, I have no great Expectation of its being ſoon made public or brought into common Uſe, when I find by Experience, how eaſy, cheap, and common every Thing muſt be for that Purpoſe, and how few there are to encourage any Thing of a different Kind in the preſent Age. I had Thoughts of illuſtrating the particular Uſes of this *general Catoptric Inſtrument*; as likewiſe to ſhew how a *Reflecting Teleſcope*, a *Reflecting Microſcope*, and a *Reflecting Solar Microſcope*, by metalline Speculums of a different Form and Poſition, and of a more perfect Nature, might be conſtructed and applied to Uſe, but have declined that alſo for the Reaſons above mentioned.

IV. *The Theory of a* NEW MICROMETER.

AS the Perfection of Aſtronomy depends upon the moſt accurate Methods for meaſuring the Diameters of the celeſtial Bodies by Means of thoſe Inſtruments we call MICROMETERS, applied in Teleſcopes for that Purpoſe, it has given the ſagacious Aſtronomers Occaſion to contrive

trive them in many different Forms; but all of them, till very lately, were applied in the Focus of the Object-Glass, to measure the Images of the heavenly Bodies formed there, and they may be justly reckoned among the most exquisite Inventions of modern Mechanics.

BUT as the Diameter of the Sun and Moon are so much larger than the Planets, the Micrometer which serves for one, could not be so well adapted to measure the other; which put some Philosophers upon thinking of a Method by which they might measure the apparent Diameter of the Sun, without being obliged to take the Whole of the Sun's Disk into the Telescope; and this they at length ingeniously contrived by forming a Telescope in such a manner as to make two Images of the Sun in the Focus of the Telescope at the same time, and bringing them into Contact with each other. And this Instrument they properly call a HELIOMETER.

THE first Instrument of this Kind is of a late Date, and was the Contrivance of the late celebrated *Servington Savory*, Esq; of *Exeter*, who, in the Year 1743, in a Letter to Mr. *George Graham*, dated *November* 30th, gives an Account of a new Way

of meaſuring the Difference between the apparent Diameter of the Sun at the time when the Earth is neareſt to and fartheſt from the Sun, with a Micrometer placed in the Teleſcope invented for that Purpoſe; though the Charge, or magnifying Power of the Teleſcope is ſo great, that the Whole Sun's Diameter does not appear therein at one View.—" This, ſays Mr. " *Savory*, I doubt not, will at firſt Sight " ſeem impoſſible, ſince only a Part of the " Diameter appears, and no viſible Mark " or Point therein from which ſuch Mea- " ſure can be taken. And, indeed, it is " ſo by Obſervation with our common Te- " leſcopes, whether refracting or reflecting " ones. I have therefore contrived ſome " Dioptric Teleſcopes, and a reflecting one; " either of which, by repreſenting the Ob- " ject double, will, if well made, anſwer " the Deſign."

AFTER this he proceeds to ſhew at large, by cutting of a large Object-Lens into four Parts or Segments, and combining the two outer ones together, or the two middle ones inverted, how he could form this double Image of the Sun; but as thoſe Images were not well defined and free from Colours, he proceeded to a third Method of forming thoſe Images by Means of two Object-

Object-Lenſes entire, but of ſmall Diameter; but in this Caſe the Difficulty lay in getting Object-Glaſſes exactly centered, and of the ſame focal Length in ſo ſmall a Size.

He then had recourſe to reflecting Mirrors, cut them aſunder, and combined their Fruſtums to anſwer the ſame Purpoſe. However, all his Attempts of this Kind left the Inſtrument but imperfect; and after this we find the ſame Thing attempted that M. *Bouguer* read before the Royal Academy of Sciences at *Paris*, in the Year 1748, a *Memoir*, in which he deſcribes an Heliometer with two Object-Glaſſes for meaſuring the Diameter of the Sun and Planets. It is probable this was taken from Mr. *Savory*'s Experiments ſome Years before, but of this we have no particular Account.

However, it was ſoon conſidered, that an Object-Glaſs divided into two Segments through the Centre or Pole, would more conveniently anſwer the Purpoſe in thoſe two Parts, than could be done by two whole Glaſſes: Foraſmuch as each Segment would form an equal Image of the Sun, and at an equal Diſtance from the End of the Teleſcope; or in other Words, the focal

Diſtance of each Segment would be exactly the ſame, and the Images of the Sun and Planets formed exactly in the ſame Plane in the Focus of the Teleſcope.

AND ſince the Centres of the Sun, each Segment of the Glaſs, and its Image are all found in one Right Line, therefore if one Segment of the Glaſs be fixed in the End of the Teleſcope, the Image formed in it will alſo be fixed, or remain conſtantly in the ſame Part of the Focus; and if the other Segment be moved in the ſame Plane over it, as its Centre or Middle Point is removed from that of the fixed Segment, ſo likewiſe is the Centre of the moveable Image equally removed from the Centre of the fixed one; or the Centres of the two Images will always be equidiſtant with the Centres of the two Object-Glaſſes, and conſequently the Diameter of the Sun or Planet (being equal to its two Semidiameters) muſt be equal to the Diſtance between the two Centres of the Segments, when thoſe two Images are in Contact or touch each other in the Focus of the Teleſcope. And this is the whole Ground or Reaſon of the new Micrometer, which will be beſt illuſtrated by Figures, as it is applied to the Refracting or Reflecting Teleſcope, as follows:

LET

Appendix II. Plate III.

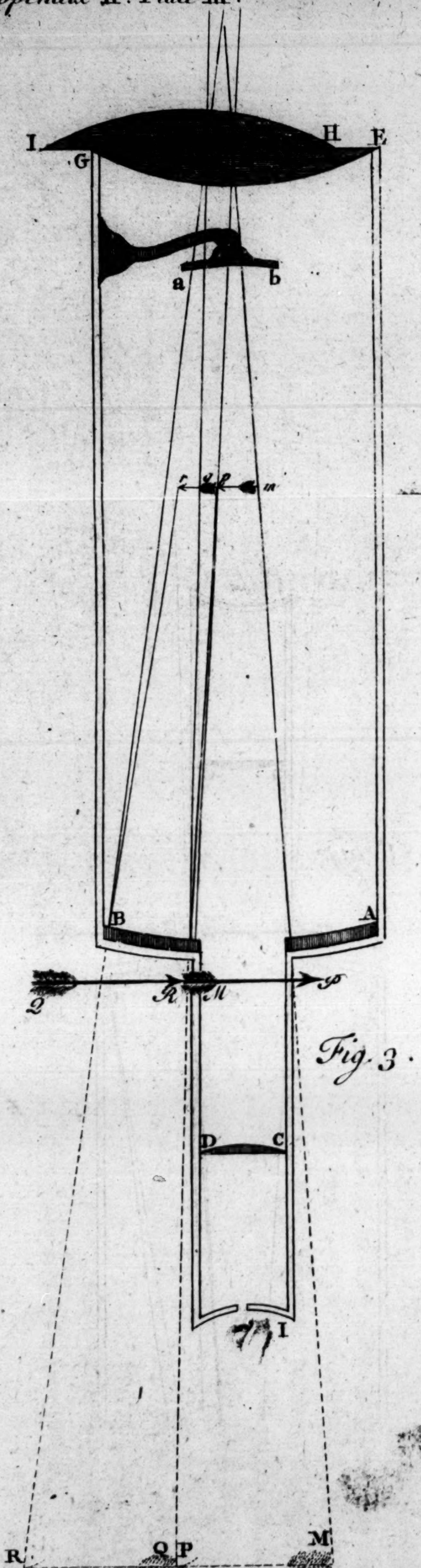


Fig. 3.

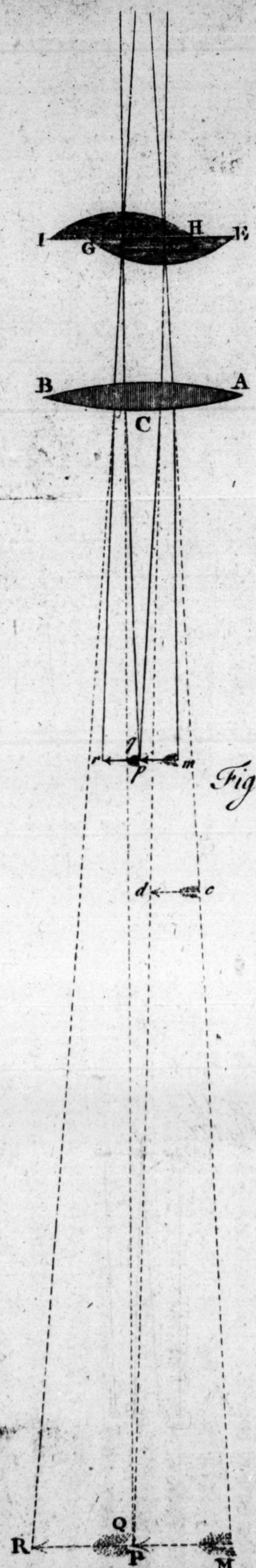


Fig. 2.

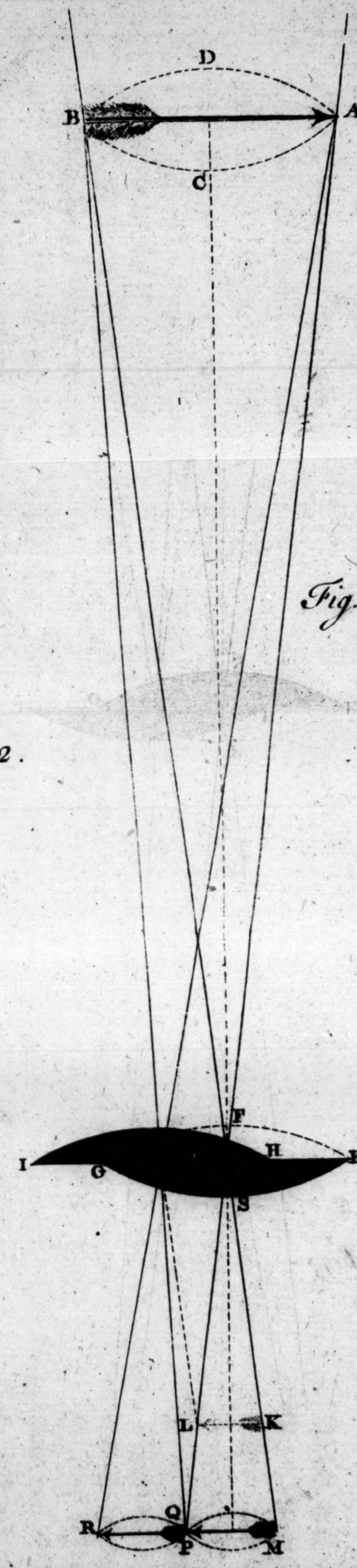


Fig. 1.

Let A B C D represent any very distant Object, as the Sun, &c. and A B its Diameter; also let E F G S represent the Object-Glass consisting of two Segments E F G and E S G divided through the Centre N in the Right Line E G. The Angle under which it appears at the End of the Telescope will be A N B equal to the Angle K N L, under which the Image K L is contained. Now, suppose the moveable Segment E F G were by a mechanical Contrivance drawn off to the Position H I, the Distance of their Centres would be N O; and the two Lines A N and B O passing through the Centres N, O, of the Segments, if produced, meet at the Focus in L; and since B L and B K do also pass through the Centres N and O, and the Object being at an indefinitely great Distance, the Line O L will be parallel to N K, and consequently the Angle N L O is equal to the Angle K N L or A N B; that is to say, *the Angle under which the Object appears from the End of the Telescope (or to the naked Eye) is equal to the Angle under which the Distance between the two Centres of the Segments appear from the Solar Focus of the Telescope.* Plate III. Fig. 1.

AND

Lenſes, they will likewiſe be in Contact in the ſhortened compound Focus. And as the Centres N and O of the two Semi-Lenſes G E and I H are ſeparated farther from, or brought nearer to each other, the Images in either Focus will be moved in ſimilar Manner; and when the Centres N and O coincide, the Images in each Focus reſpectively will alſo coincide, or become one entire Image, the Difference in every Caſe being only as to large and ſmall, greater or leſſer Diſtance. Conſequently in the Micrometer by which thoſe two Semi-Lenſes are moved by each other, the ſame Turns of the Screw which meaſures the Angle O P N, and which brings the Images into an exact Contact in the ſingle Focus at Q, will be neceſſary for the ſame Purpoſe in the compound Focus alſo; ſo that by this Means we have an Opportunity of meaſuring the ſaid Angle O P Q, without being obliged to have ſo great and ſo unmanageable a Length of the Teleſcope.

HOWEVER, the larger the focal Diſtance of the Lens A B is, the more diſtinct the Contact of the Images will appear; and becauſe this is the Point on which the whole Perfection of this Micrometer depends,

pends, it will be likewiſe neceſſary to have it ſo contrived, when applied to a Teleſcope, that the Centres N O may be equally diſtant from the Axis of the Teleſcope or Centre of the Aperture on either Side; becauſe in this Caſe the Point of Contact in the two Images will be juſt in the Centre of the Focus, and therefore the moſt diſtinct that it poſſibly can be.

BUT the Application of this Micrometer to Refracting Teleſcopes will be leſs convenient than when it is applied to a Reflecting Teleſcope; for if it be placed on the open End of the Reflecting Teleſcope, as in *Fig.* 3, then will the Rays that tend to form the larger Images R Q and PM be incident upon the larger Speculum AB, and from thence reflected to a compound Focus, where the ſimilar Images *r q* and *pm* will be formed as before; the Rays proceeding from theſe two Images to the ſmaller Speculum a b, will be reflected back through the Hole of the larger, to form the Images *QR* and *P M*, which likewiſe will ſtill be in Contact in the Focus of the Eye-Glaſs D C, where it will be diſtinctly perceived by the Eye at I. This Contact will likewiſe be ſhewn in the middle of the Focus of the Eye-Glaſs, if the Centres Plate III. Fig. 3.

O and

O and N are properly diſpoſed, as before mentioned.

From what has been ſaid, the general Rationale of this Micrometer will, I preſume, evidently appear; but one Thing muſt not paſs unregarded in an Affair of ſuch Moment and Conſequence as the meaſuring theſe ſmall Angles in the Science of Aſtronomy, which has been cuſtomary to ſuppoſe, that ſo far the Focus of a Lens, or the focal Diſtance of Rays parallel to its Axis, is equal to the Radius in a double and equally convex Lens. But this is too great an Error not to be noticed here; for in different Sorts of Glaſs there is found a different refractive Power, and the Focus of parallel Rays is at a different Diſtance in each; but this Diſtance in no Sort of Glaſs is equal to the Radius, but falls ſhort of it more or leſs. Now the foregoing Demonſtration regards the Radius, and not the focal Diſtance of parallel Rays. But this Affair will be beſt illuſtrated by Example, as follows:

Let the Sine of Incidence be to the Sine of Refraction out of Air into Glaſs as m to n; and let $\frac{m-n}{n}=a$, then will a expreſs the refractive Power of the Glaſs; and

and the Theorem for a double and equally convex Lens will be $\frac{dr}{2ad-r}=f$, which for parallel Rays, becomes $\frac{r}{2a}=f$; whence $r=2af$; or $f:r::1:2a$.

THEREFORE to repreſent this Matter in its proper Light, we will ſtate the Proportions of Page 378 from this correct Theorem, and they will ſtand thus, $d:f::2ad-r:r$; and $d+f:f::2ad:r$; that is, AQ : QN :: $2a\times$ AN : NL :: AB : NO : Hence NO : AB :: $r:2ad::f:d$. *Or, the Diſtance of the Centres* NO *has the ſame Ratio to* AB, *the Length of the Object, as the focal Diſtance of parallel Rays has to the Diſtance* AN, *or as the Radius has to the ſaid Diſtance multiplied by* $2a$. Whence 'tis evident we cannot uſe the Radius without conſidering the Refraction of the Glaſs; but the *focal Diſtance of parallel Rays* ſerves equally for all Sorts of Lenſes, is moſt eaſily found, and therefore fitteſt for the Analogy, as before determined, in meaſuring Angles and Diſtances.

WITH regard to the Planets, as *Jupiter* is the largeſt of all, and ſubtends an Angle to the Eye of 3′ 12″, the Diameter of his Image in the Focus of a 50 Foot Glaſs will be

be about half an Inch, and that will be the utmoſt Diſtance to which the Centres of the Segments will be required to be ſeparated for meaſuring the apparent Diameters of the Planets.

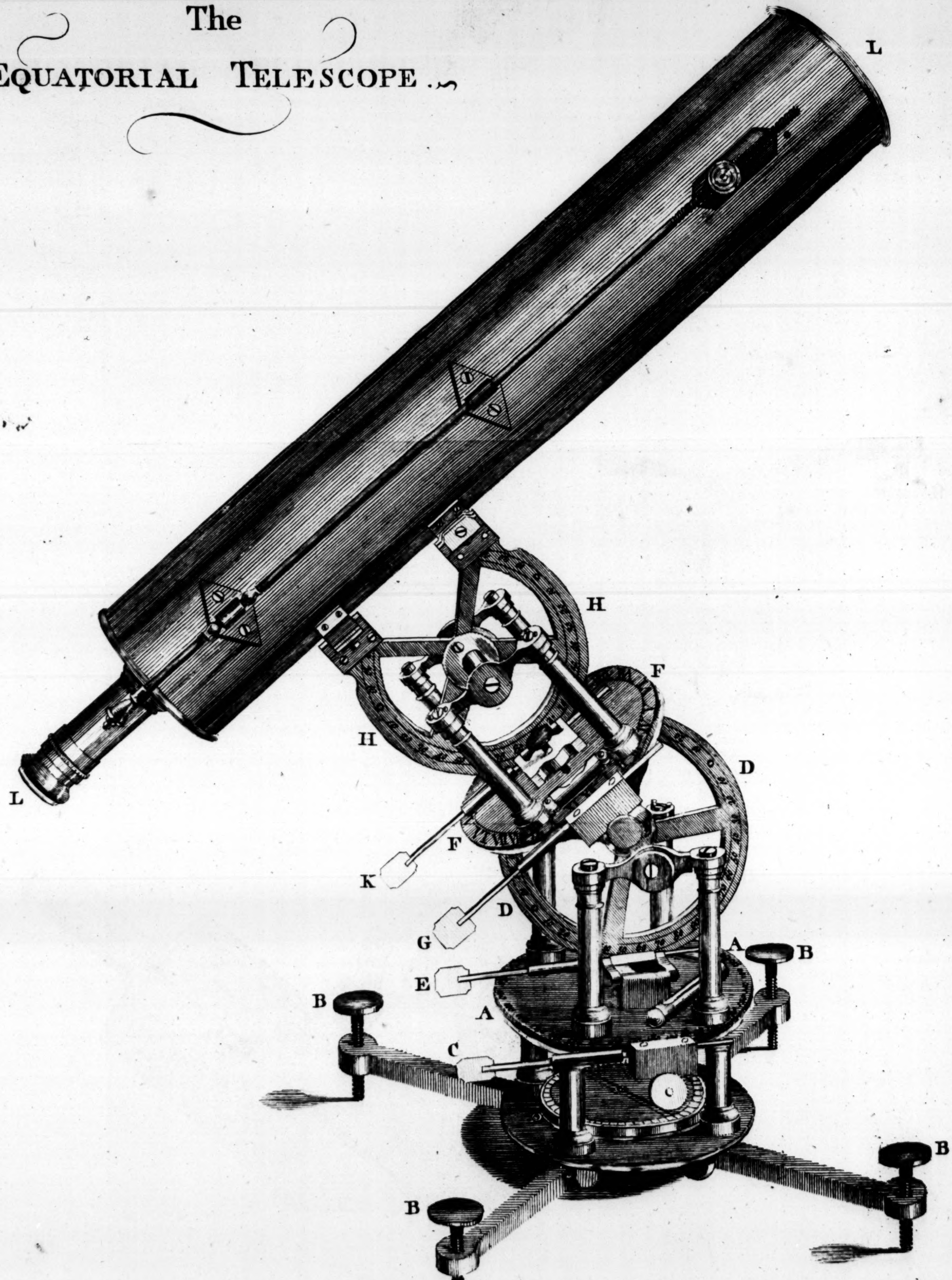
BUT for a HELIOMETER, the Diameter of the Sun being near ten Times as great as that of *Jupiter*, will require the Centres of the Segments in a Glaſs of 40 or 50 Feet Focus to be removed from each other at leaſt to the Diſtance of 4 or 5 Inches; and to take in the whole Syſtem of *Jupiter's* Moons, the Diſtance of the Centres will be required much larger; and therefore for ſuch Purpoſes the Segments of Glaſſes of a leſs focal Length muſt be uſed.

As this Micrometer has many Advantages above that which is in common Uſe, ſo on the other hand it is liable to ſome Objections; for ſince the Images of Objects can never be well defined but in the very large focal Diſtances, and ſince thoſe Glaſſes are not made true without great Care and Difficulty, they will come dear, and then we muſt run the Riſque of ſpoiling them in cutting them aſunder; for if they are not cut nicely through the Centre, it will derogate from the Value and Good-

neſs

The

EQUATORIAL TELESCOPE.



neſs of the Inſtrument. Alſo ſince at the ſame time a very great magnifying Power is required in the Inſtrument to which it is applied, it can be applicable for Aſtronomical Purpoſes in only Reflecting Teleſcopes, becauſe of the Conveniency of adapting it to the Tube of the Teleſcope, and moving it at the ſame time you are obſerving the Object through it.

V. *Concerning the Improvement of Refracting* MICROSCOPES *and* TELESCOPES *by a Compoſition of Glaſſes.*

IT has been already ſhewn in *Annotation* CXXVIII. *Art.* 18. *that the Errors ariſing from the different Refrangibility of the Rays of Light will be as the Cubes of the lineal Aperture of the Glaſs directly, and the Squares of its Diſtance inverſely*; and conſequently in Glaſſes of the ſame Aperture the Indiſtinctneſs of Viſion will be inverſely as their focal Diſtances; wherefore if, while the magnifying Power and Aperture of the Lens continue the ſame, we can increaſe this Ratio of the focal Diſtance to the ſaid lineal Aperture, we ſhall in that Proportion increaſe the Diſtinctneſs of Viſion, which is the very Point in which the Perfection of Microſcopes and Teleſcopes conſiſt.

 To

Plate IV. Fig. 2.

To illuſtrate this, let A E D G be a Lens, F the Focus of parallel Rays, f the Place of an Object O B; through the Point F draw a Line to interſect the Object in the Point B, and falling upon the Lens in the Point A. This Ray will be refracted into A M, parallel to the Axis of the Lens F C O; from f draw the Line F A, which by the Lens will be refracted to its reſpective Focus V; then will L V repreſent the Image of Part of the Object B f; let M N be an Eye-Glaſs by which that Image is to be viewed, which therefore muſt be placed at its Focal Diſtance P V; from it draw P L, and parallel thereto draw M O, then will the Angle M O P be the viſual Angle under which the Image L V will appear in the Eye at O.

We are now to ſhew how this Image may be viewed with more Diſtinctneſs, and under the ſame Angle and Aperture as before; and this is effected by ſubſtituting two Glaſſes H I and U S inſtead of the ſingle Glaſs M N; for by this Means we ſhall make the ſame Angle by two Refractions which before was made by one, and that will produce the deſired Effect; for let the Glaſs H I be placed at its focal Diſtance I P from the Centre of the firſt Glaſs M N, then will the Ray A H (which before

before proceeded to M) be now refracted into H P, and the Ray A V will at K be refracted into K Z; draw the Line I L, and it will interſect the Ray H P in X, and X Z will be the Image of the Object B f. All this we have demonſtrated in the Optical Lecture.

LET this Image be viewed by the Eye-Glaſs U S, whoſe focal Diſtance T Z or T P may be ſuch that the Ray H P may be now a ſecond Time refracted at U into U Q; parallel to M O or L P draw T X, which will alſo be parallel to U Q, then will the Image Z X, or rather the Part of the Object B f, be ſeen under the Angle T Q U equal to M O P, as at firſt.

Now it is evident, that were the Glaſs U S alone by itſelf, the Indiſtinctneſs of Viſion would be inverſely as T Q, for that would then be the focal Diſtance of the Glaſs; but now this Glaſs T U, in Compoſition with the other Lens H I, ſhews the ſame Image under the ſame Angle and Aperture with the focal Diſtance T P; the Indiſtinctneſs of Viſion therefore now will be inverſely as T P; or, in other Words, the Diſtinctneſs of Viſion in the firſt Caſe by a ſingle Glaſs will be to the Diſtinctneſs of the Glaſſes in Compoſition as T Q to T P, or the Image will appear ſo much

diſtincter by the two Glaſſes H I and U S together, than it would do by the ſingle Eye-Glaſs M N alone, in Proportion as T P is greater than T Q; all which is evident from the foregoing Principles of Optics.

With reſpect to the Lens A E D, if it be a ſmall one, and the focal Diſtance C F be ſhort, then will it repreſent the Object-Glaſs of a Microſcope, and the ſmall Object O B will be viewed to much greater Advantage by Means of the two Eye-Glaſſes H I and U S, than by a ſingle Eye-Glaſs alone M N. Whence the Reaſon is evident of the uſual Conſtruction of this Inſtrument, as we have before ſhewn.

But if A E D G be the Object-Glaſs of a Teleſcope, and F C its Focus, then will O B repreſent a diſtant Object, whoſe Image W L is that which is viewed by a ſingle Eye-Glaſs M N in what is called the *Aſtronomical Teleſcope*; but this Image is more diſtinctly viewed by two Glaſſes I and T, than by one alone; and therefore ſuch a Teleſcope ſhould conſiſt at leaſt of three Glaſſes.

In the common Teleſcope the Image L W is conſidered as an Object, and another Image is formed of it, as was ſhewn in Figure 8 of Plate XLVIII. And as we have now ſhewn that each Image requires the

the Addition of a Lens to ſhew it more diſtinctly, inſtead of the three Eye-Glaſſes which you there ſee repreſented in a common Teleſcope, there ought to be the Addition of two more, or five in all, next the Eye, to ſhew the Object to the beſt Advantage. Theſe, together with the Object-Glaſs, make ſix Glaſſes in all, in Teleſcopes of the beſt Sort.

IF A C D repreſent the ſmall Speculum of a Reflecting Teleſcope, then will O B be the Image formed by the large Speculum before it, and I, T, will be the two Glaſſes contained in the Eye-Piece of a Reflecting Teleſcope. And, indeed, with a little Reflection it will appear, that if two Glaſſes, by breaking the Refraction into two Parts, is an Advantage, three Glaſſes will be more ſo, and therefore in ſome Caſes might be ſucceſsfully applied.

Alſo ſince the Diſtinctneſs of Viſion is by a Compoſition of Glaſſes promoted, the Aperture of the Glaſs next the Eye may be ſomewhat encreaſed, and conſequently the Field of View thereby enlarged, which is the great Advantage of the Six-Glaſs Teleſcope.

We ſhall now give the analytical Inveſtigation of the focal Diſtance of the three Glaſſes; in order to which,

Let $F = IP$, the Focal Diſtance of the Glaſs HI.

$y = TP$, the focal Diſtance of the Glaſs US.

$x = PO$, the focal Diſtance of the Lens MN.

And becauſe the Image WL is to be conſidered as an Object with regard to the Glaſs HI, therefore put $IV = d$, and $IZ = f$, and we ſhall have $\frac{Fd}{F+d} = f$, (as we have ſhewn in *Annot*. CXXV.) and from hence we get $d = \frac{Ff}{F-f}$.

BUT becauſe the ſimilar Triangles IZX, IVL, and ITX, IPL, we have $f : d :: IZ : IV :: ZX : VL :: IX : IL :: IT : IP :: f + y : d + x$; conſequently $f : d :: y : x$. Therefore $d = \frac{fx}{y} = \frac{Ff}{F-f}$: Hence we get $f = \frac{Fx - Fy}{x} = D - y$ (by putting $D = f + y = IT$.) Whence laſtly, $x = \frac{Fy}{F+y-D}$; $y = \frac{Fx - xD}{F-x}$; $F = \frac{xD - xy}{x-y}$; $D = F + y - \frac{Fy}{x}$.

BY

By these Theorems we solve any Problem relating to the Composition of Glasses for any given Power of Magnifying. For Instance, suppose the focal Distance x of the Lens M N, were $1\frac{1}{2}$ Inch, and that of the large Glass H I, *viz.* F = 3, to find the focal Distance y of the Lens U S, that is to be combined with it at the Distance D = 2 Inches, that shall magnify just as much as the single Lens M N. We have the Answer by the second Theorem, where $x = 1,5$; $F = 3$, and $D = 2$; hence $y = \frac{3 \times 1,5 - 1,5 \times 2}{3 - 1,5} = \frac{4,5 - 3}{1,5} = 1$. So the focal Distance of the Lens U S must be one Inch. If $D = 1$, then $y = 2$; and if $D = 0$, then $Y = 3$.

VI. *Of the latter Improvement of* REFRACTING *and* REFLECTING TELESCOPES.

BESIDES the last mentioned Construction of Telescopes, which is to be looked upon only as a Correction of Dioptric Instruments, there is of late another Improvement still greater, which tends not only to lessen the Aberration of Rays in refracted Light, but even to prevent any sensible Effects arising therefrom; and this

is done by Means of a *double* or *treble Object-Glaſs* in all thoſe Inſtruments, where an Object is viewed by Means of its Image formed by refracted Light. The Conſtruction of a Teleſcope for this Purpoſe with two Object-Glaſſes in Fig. 3. where A B C D is a plane Concave Lens of what is called *white Flint* Glaſs, and C D E a double Convex Lens of *green* or *Crown* Glaſs; by Means of the different refractive Power of theſe two Sorts of Glaſs and their unequal Figures, it comes to paſs that all the Rays of Light incident upon thoſe Glaſſes from diſtant radiant Objects, will paſs through them in ſuch a Manner, that whatever Aberration is occaſioned in the heterogeneal Rays in Refraction through the firſt Glaſs is ſo far corrected by the ſecond, that thoſe Rays emerge from it nearly parallel among themſelves, and are converged to one Focus (F) forming an image I M, not ſenſibly compounded or coloured, and therefore more perfect and diſtinct when viewed by the Eye-Glaſſes than that can be which is made by a ſingle Object-Glaſs.

Plate IV. Fig. 3.

Now, if thoſe two Glaſſes could abſolutely prevent the Aberration of Rays, or produce an Image entirely free from Confuſion and Colours, then only three Eye-Glaſſes

Glaſſes would be neceſſary to view it, *viz.* E, G, H; for ſince, according to common Optics, the Lens H corrects the Errors ariſing from Refraction in the Lens E; therefore if the Image IM be perfect, the ſecond Image KL will be ſo likewiſe, and therefore may be viewed diſtinct and without Colours by the Eye-Glaſs, or Lens G.

But we always find in the Conſtruction of thoſe Teleſcopes *five Eye-Glaſſes*, as they are uſually thought to be; though if you examine them, the Poſition of the firſt will be found within the focal Diſtance of the Compound Object-Glaſs, and therefore co-operates with it in forming the Image of external Objects; which Image, therefore, cannot be ſo purely *achromatic* as it would be if it were formed by the Double Object-Glaſs alone.

Not only ſo, but the focal Diſtance of that achromatic Object-Glaſs is by means of this Lens ſomewhat ſhortened, and the magnifying Power of the Teleſcope thereby diminiſhed. I can ſee no Reaſon for the Poſition of this Lens within the ſaid Focus of the Object-Glaſs, ſince, if all the five Eye-Glaſſes were placed without it, the ſame extenſive and more perfect *Field of View* would be

be obtained, and at the ſame time the Teleſcope would magnify more, and be *truly achromatic*, which now, ſtrictly ſpeaking, it really cannot be. But for more Satisfaction in this Matter, I ſhall refer the Reader to my *New* ELEMENTS of *Achromatic* OPTICS, in Six PARTS.

VII. *The Nature of* VISUAL GLASSES *explained, and ſhewn to be an Improvement of* Common Spectacles.

IN the firſt Edition of this Treatiſe I had no Occaſion to make Uſe of Spectacles, and therefore did not ſo thoroughly conſider their Nature, Form, and Uſe, as ſince that I have done. It appears to me very wonderful, that an Inſtrument of the moſt common and neceſſary Uſe, ſhould have continued ſo long of a Form or Make quite contrary to that which the Theory of Optics and the Nature of Viſion require, and yet paſs unobſerved by thoſe who uſe them, and uncenſured by thoſe who have wrote on this Subject; eſpecially if it be conſidered that the erroneous Conſtruction is in itſelf moſt obvious, and in its Effects very prejudicial to the Sight.

THE

THE Fault of the common Spectacles consists in two Particulars, *viz.* 1st, The Largeness of the Aperture or Diameter of the Glasses; and, 2dly, their oblique Position to the Axis of the Eye; both which must in time have a very sensible Effect upon the Texture of that curious and delicate Organ of Sight, the EYE. But the Harm we receive being not immediately sensible, is not considered by common People, and therefore not regarded; and when at length they find their Eyes weakened and impaired, they reckon it as the natural Consequence of Age, without being apprized how far their Glasses have contributed to produce that Effect.

THAT the Area of the Glasses in common Spectacles is visibly larger than is necessary, no Body will dispute, when they consider how small the Pupil of the Eye is when compared therewith; and that no more Light can be useful than that which enters the Pupil. Suppose, for Instance, the Pupil of the Eye were $\frac{1}{5}$ Part of an Inch in Diameter (which is larger than any I have yet observed) and that the Glass be one Inch and a half in Diameter, then will the Area of the Glass be to that of the Pupil as 225 to 4, which is more than 50 to 1; so

ſo that there is at leaſt 50 Times more Light upon the Eye than is neceſſary for the Purpoſes of Viſion.

IF the Aperture of the Glaſs were no longer than that of the Pupil, though it would make the Object appear more diſtinct, there might not in all Caſes be Light enough to ſhew it ſo plainly as might be deſired; neither, as the Glaſſes are placed at a conſiderable Diſtance from the Eye, would there be a ſufficient Field of View. It is neceſſary, therefore, that the Aperture of the Glaſſes ſhould be larger than that of the Pupil; and it is well known by Experience, that if the Aperture of the Glaſſes be $\frac{3}{4}$ of an Inch, it will anſwer all Purpoſes of reading, working, &c. by them, and in this Caſe but $\frac{1}{4}$ Part of the Light can come upon the Eye as goes to it through a common Spectacle-Glaſs; and therefore by excluding three Parts out of four of the ſuperfluous Light, muſt tend greatly to the Safety of the Eye, and procuring diſtinct Viſion of the Object.

WHOEVER conſiders the great Force which the Action of Light has, and the exquiſitely tender and minute Veſſels in the Compages of the Eye (exceeding that of any other Part of the Animal Syſtem) will not

not wonder, that if by ſo great a Quantity of Light which is thrown upon old Eyes by very convex Glaſſes, they ſhould become debilitated, weak, and watery in the Courſe of a few Years. So potent a Cauſe as that of Light by the Conſtancy of its Operation produces very conſiderable Effects, though by very ſlow Degrees. The conſtant Dripping of Water will in a Courſe of Time excavate the hardeſt Stone, though its immediate Effects be not in the leaſt diſcernible, nor any Thing more conſidered than the Action of Light upon the Eye.

THE Difference then between the Viſual Glaſſes is as repreſented in Figure 4. Fig. 4. where AB is the open Glaſs, as in common Spectacles, and CD the Glaſs of a reduced Size in the *Viſual Form*; the dark Part between that and the Steel Frame EF is a Black Zone or Circle of Horn in which the Glaſs is placed, and ſerves at the ſame Time as a Safe-guard and Defence to the Eye againſt extraneous Light.

THE ſecond Thing in which the common Spectacles are moſt egregiouſly faulty, is their being placed direct before the Eye, or both in the ſame Plane, by which Means the Axes of the Glaſs and thoſe of the Eye make a conſiderable Angle with each other;

other; whereas they ought to coincide, or the Glaſſes ſhould be ſo placed before the Eye in two Planes equally inclined, that their Axes may both unite in the Object to which the Eye is directed, and they become one with the Axis of the Eye. To illuſ-
Fig. 5. trate this, let AB be the two *common Spectacle*-Glaſſes, whoſe Axes AC and BD are parallel, and thereby directed to Objects only at an infinite Diſtance, quite contrary to the Deſign of theſe Glaſſes, which is to ſhew a near Object, as ſuppoſe G; then the Glaſſes E, F, are ſo placed in the *new viſual Form* as to have their Axes tending to, and uniting in that Point G; by which Means they become coincident with the Axis of the Eye, and the Rays of Light are regularly and equally refracted to the Pupil; and conſequently the Viſion or Appearance of an Object will by this Means be rendered the moſt natural and eaſy that it poſſibly can be. And indeed the Difference is ſo conſiderable, not only in itſelf, but likewiſe by Experience, that I may venture to prognoſticate, that in a few Years time, after the Averſions which ariſe from Cuſtom, Intereſt, and Novelty are worn off, the *common Spectacles* will be looked upon as the *Opprobrium* of the Optical

Optical Science, and become equally in Contempt and Disuse; while the VISUAL GLASSES will approve themselves the genuine Result of Optical Philosophy, and be used at least by all the prudent and thinking Part of Mankind. Those who would see a larger Account of this Matter may consult my *Essay on Visual Glasses*, which has passed through four Editions.

VIII. *Description and Uses of the* EQUATORIAL TELESCOPE, *or* PORTABLE OBSERVATORY, *by Mr.* James Short, *F. R. S. who first adapted a Telescope to this Machinery.*

THIS Instrument consists of two circular Planes or Plates, marked A A, which are supported upon four Pillars; and these are again supported upon a Cross-foot, or Pedestal, moveable at each End by the four Screws B B B B: The two circular Plates A A are moveable, the one above the other, and are called the horizontal Plates, as representing the Horizon of the Plate; and upon the upper one are placed two Spirit-Levels, to render them at all times horizontal: These Levels are fixed at Right-Angles to one another: This upper Plate is moved by a Handle C, which is called

called the Horizontal Handle, and is divided into 360°, and has a *Nonius* Index divided into every three Minutes.

ABOVE this horizontal Plate there is a Semicircle D D, divided into twice 90°; which is called the Meridian Semicircle, as representing the Meridian of the Place, and is moved by a Handle E, which is called the Meridian Handle, and has a *Nonius* Index divided into every three Minutes.

ABOVE this Meridian Semicircle is fastened a circular Plate, upon which are affixed two other circular Plates F F, moveable the one upon the other, and are called the Equatorial Plates. One of them representing the Plane of the Equator, is divided into twice 12 Hours, and these are subdivided into every 10 Minutes of Time. This Plate is moved by a Handle G, called the Equatorial Handle, and has a *Nonius* Index for shewing every Minute.

ABOVE this Equatorial Plate there is a Semicircle H H, which is called the Declination-Semicircle, as representing the Half of a Circle of Declination, or horary Circle, and is divided into twice 90°, being moved by the Handle K, which is called the Declination-Handle. It has also a *Nonius*

a *Nonius* Index for ſubdividing into every three Minutes.

ABOVE this Declination-Semicircle is faſtened a Reflecting Teleſcope L L, of the *Gregorian* Conſtruction, the focal Length of its great Speculum being 18 Inches.

IN order to adjuſt the Inſtrument for Obſervation, the firſt Thing to be done, is to make the Horizontal Plates level or horizontal, by means of the two Spirit-Levels, and the four Screws in the Croſs-Pedeſtal. This being done, you move the Meridian Semicircle, by means of the Meridian Handle, ſo as to raiſe the Equatorial Plates to the Elevation of the Equator of the Place; which is equal to the Complement of the Latitude (and which, if not known, may likewiſe be found by this Inſtrument, as ſhall be afterwards ſhewn). And thus the Inſtrument is ready for Obſervation.

To find the Hour of the Day, and Meridian of the Place.

FIRST find, from aſtronomical Tables, the Sun's Declination for the Day, and for that particular Time of the Day; then ſet the Declination-Semicircle to the Declination of the Sun, taking particular Notice whether it is North or South, and ſet the Declination-Semicircle accordingly.

YOU then turn about the Horizontal Handle, and the Equatorial Handle, both at the ſame time, till you find the Sun preciſely concentrical with the Field of the Teleſcope. If you have a Clock or Watch at hand, mark that Inſtant of Time; and by looking upon the Equatorial Plate, and *Nonius* Index, you will find the Hour and Minute of the Day, which comparing with the Time ſhewn by the Clock or Watch, ſhews how much either of them differ from the Sun. In this Manner you find the Hour of the Day.

Now, in order to find the Meridian of the Place, and conſequently to have a Mark, by which you may always know your Meridian again, you firſt move the Equatorial Plate, by means of the Equatorial Handle, till the Meridian of the Plate or Hourline of 12 is in the Middle of the *Nonius* Index; and then, by turning about the Declination-Handle till the Teleſcope comes down to the Horizon, you obſerve the Place or Point which is then in the Middle of the Field of the Teleſcope; and a ſuppoſed Line drawn from the Centre of this Field to that Point in the Horizon, is your Meridian Line. The beſt Time of the Day for making this Obſervation for finding your Meridian, is about

about three Hours before Noon, or as much after Noon. The Meridian of the Place may be found by this Method ſo exact, that it will not differ at any time from the true Meridian above 10″ Time; and if a proper Allowance be made for the Refraction at the Time of Obſervation, it may be found much more exact. This Line thus found will be of uſe to ſave Trouble afterwards; and is, indeed, the Foundation of all aſtronomical Obſervations.

To find a Star or Planet in the Day-time, even at Noon-Day.

THE Inſtrument remaining as rectified in the laſt Experiment, you ſet the Declination-Semicircle to the Declination of the Star or Planet you want to ſee; and then you ſet the Equatorial Plate to the Right Aſcenſion of the Star or Planet at that time, and, looking through the Teleſcope, you will ſee the Star or Planet; and after you have once got into the Field, you cannot loſe it: For, as the diurnal Motion of a Star is parallel to the Equator, by your moving the Equatorial Handle ſo as to follow it, you will at any time, while

it is above the Horizon, recover it, if it be gone out of the Field.

THE easiest Method for seeing a Star or Planet in the Day-time is this: Your Instrument being adjusted as before directed, you bring the Telescope down so as to look directly at your Meridian Mark; and then you set it to the Declination and Right Ascension, as before-mentioned.

BY this Instrument most of the Stars of the first and second Magnitude have been seen even at Mid-day, and the Sun shining bright; as also *Mercury*, *Venus*, and *Jupiter: Saturn* and *Mars* are not so easy to be seen, upon account of the Faintness of their Light, except when the Sun is but a few Hours above the Horizon.

AND in the same Manner in the Night-time, when you can see a Star, Planet, or any new Phænomenon, such as a Comet, you may find its Declination and Right Ascension immediately, by turning about the Equatorial Handle, and Declination-Handle, till you see the Star, Planet, or Phænomenon; and then, looking upon the Equatorial Plate, you find its Right Ascension in Time; and you find upon the Declination-Semicircle, its Declination in Degrees and Minutes.

IN

IN order to have the other Uſes of this Inſtrument, you muſt make the Equatorial Plates become parallel to the Horizontal Plates; and then this Inſtrument becomes an *Equal Altitude Inſtrument*, a *Tranſit Inſtrument*, a *Theodolite*, a *Quadrant*, an *Azimuth Inſtrument*, and a *Level.* The Manner of applying it to theſe different Purpoſes is too obvious to need any Explanation.

As there is alſo a Box with a magnetic Needle faſtened in the lower Plate of this Inſtrument, by it you may adjuſt the Inſtrument nearly in the Meridian; and by it likewiſe you may find the Variation of the Needle: If you ſet the Horizontal Meridian, and the Equatorial Meridian, in the Middle of their *Nonius* Indexes, and direct your Teleſcope to your Meridian Mark, you obſerve how many Degrees from the Meridian of the Box the Needle points at; and this Diſtance or Difference is the Variation of the Needle.

AN

INDEX

TO THE

THREE VOLUMES.

N. B. *The* Numerical Letters *denote the* Volume, *and the* Figures *the* Page *of that Volume.*

INDEX.

B.

Pendant,

I N D E X.

the

I N D E X.

iii.

Fixity,

G.

Uni-

H.

I.

and

ibid.

INDEX.

N.

P.

INDEX.

INDEX.

Irregular,

60.

INDEX.

CONTENTS.

LECTURE XI.

ASTRONOMY.

CONTENTS.

CONTENTS.

CONTENTS.

AN

APPENDIX.

OF

CHRONOLOGY.

FINIS.